NOTA

I diagrammi di funzioni composte (prodotto di funzioni elementari), presentati nei fogli seguenti, sono tracciati con procedimento “non veloce”, cioè come somma di diagrammi di funzioni elementari.

Per il tracciamento dei diagrammi asintotici, funzioni contenenti termini di secondo grado, con poli / zeri complessi coniugati, sono approssimate sostituendo tali poli / zeri con i corrispondenti poli /zeri REALI COINCIDENTI alla medesima pulsazione, tralasciando eventuali effetti di risonanza o antirisonanza dovuti al fattore di smorzamento presente nel termine di primo grado.

Le funzioni sono preventivamente convertite nella forma di Bode, se non sono già assegnate in tale forma.

$F_2(s) = 0{,}8$

RISPOSTA IN FREQUENZA

$\overline{F}_2(j\omega) = 0{,}8$

MODULO

$F_2(\omega) = 0{,}8$

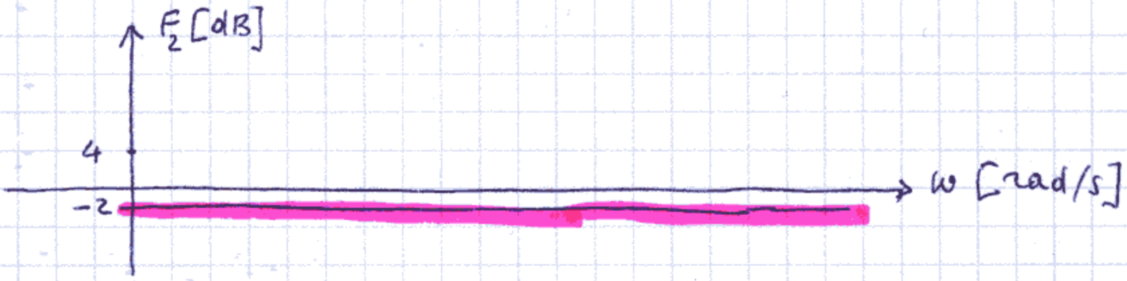
MODULO IN dB

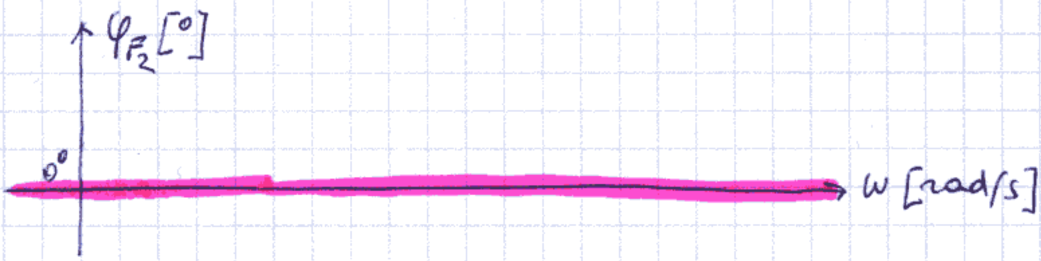
$$F_2[dB] = 20\lg_{10} F_2 = 20\lg_{10} 0{,}8 = 20\lg_{10}\frac{8}{10} = 20\lg_{10} 2^3 - 20\lg_{10} 10 = 3\cdot 20\lg_{10} 2 - 20 =$$
$$= 3\cdot 6 - 20 = -2\,dB$$

FASE

$\varphi_{F_2} = 0^\circ$

GRAFICI





$F_3(s) = -4$

RISPOSTA IN FREQUENZA

$\overline{F_3}(j\omega) = -4$

MODULO

$F_3(\omega) = 4$

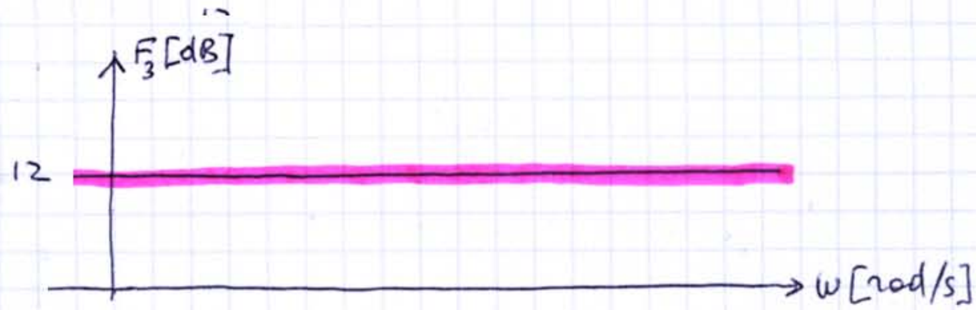
MODULO IN dB

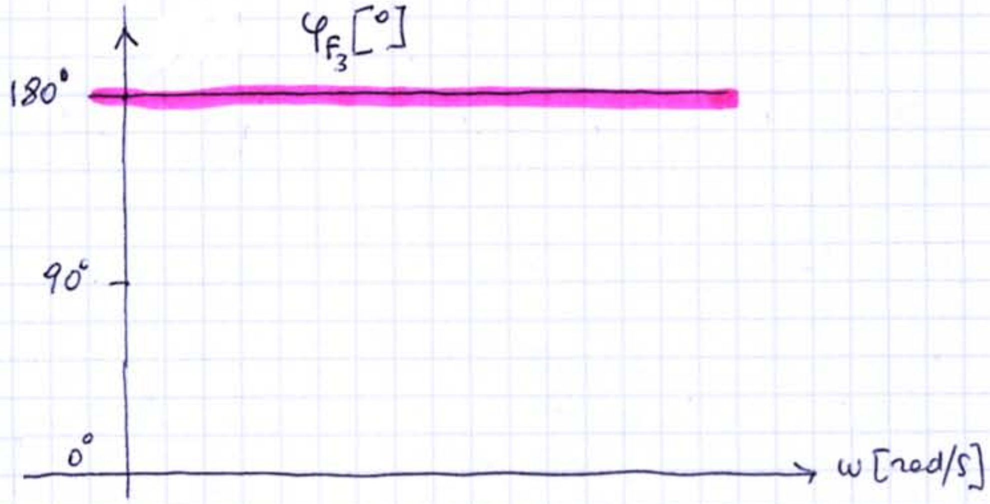
$F_3[dB] = 20 \lg_{10} F_3 = 20 \lg_{10} 4 = 20 \lg_{10} 2^2 = 2 \cdot 20 \lg_{10} 2 = 2 \cdot 6\,dB = 12\,dB$

FASE IN GRADI

$\varphi_{F_3} = \pm 180^\circ$

GRAFICI





$F_4(s) = -0{,}2$

RISPOSTA IN FREQUENZA

$\overline{F_4}(j\omega) = -0{,}2$

MODULO

$F_4(\omega) = 0{,}2$

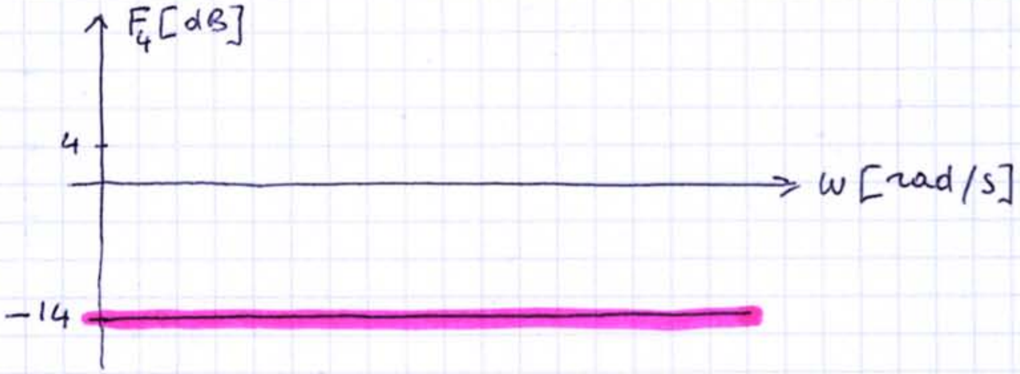
MODULO IN dB

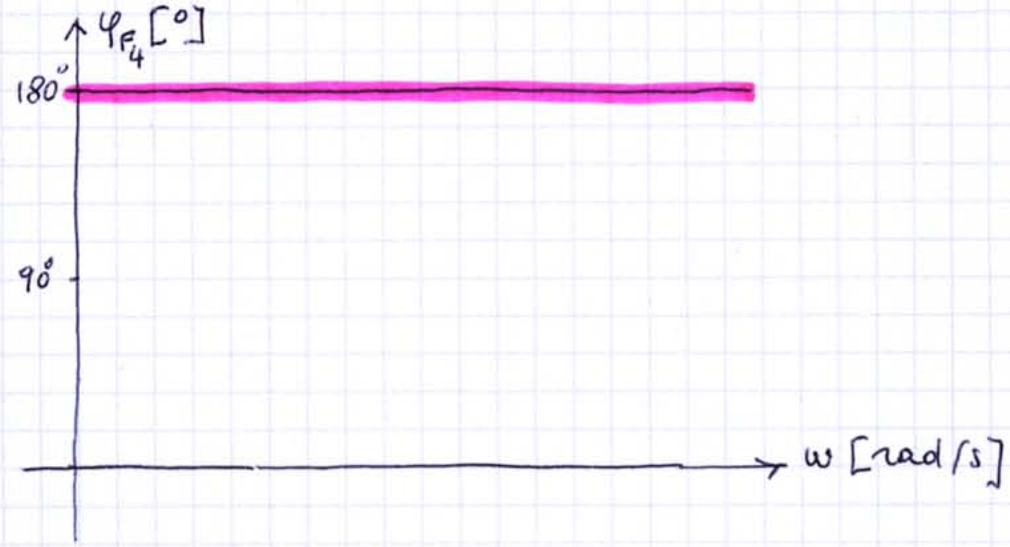
$F_4[dB] = 20\lg_{10} F_4 = 20\lg_{10} 0{,}2 = 20\lg_{10} \frac{2}{10} = 20\lg_{10} 2 - 20\lg_{10} 10 = 6 - 20 = -14\,dB$

FASE

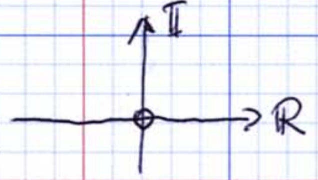
$\varphi_{F_4} = \pm 180°$

GRAFICI





$F(s) = \sqrt{10}\, s$ ZERO SINGOLO NELL'ORIGINE



RISPOSTA IN FREQUENZA

$\bar{F}(j\omega) = j\sqrt{10}\,\omega$

MODULO

$F(\omega) = \sqrt{10} \cdot \omega = F_1 \cdot F_2 \quad \begin{cases} F_1 = \sqrt{10} \\ F_2 = \omega \end{cases}$

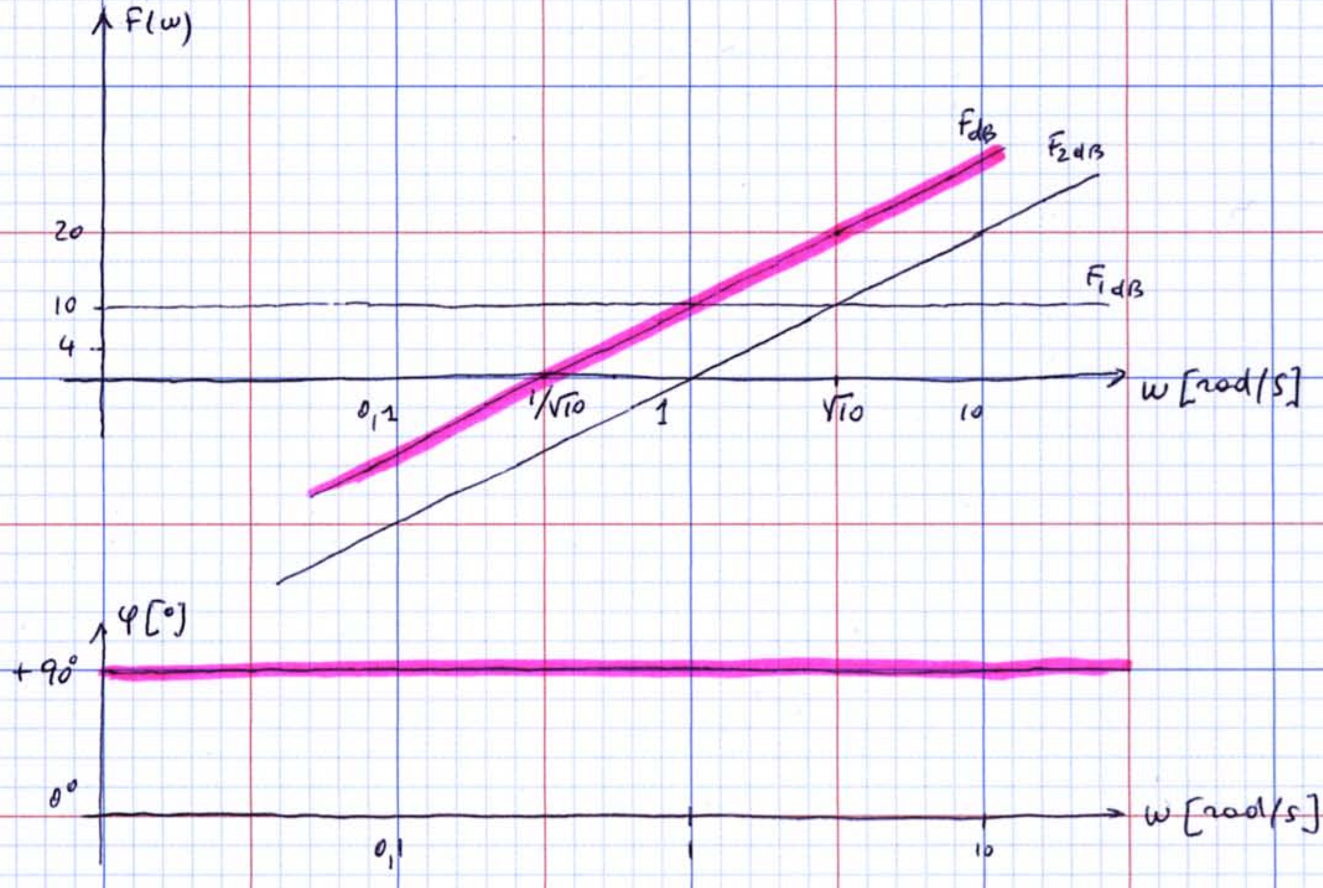
$F_1 = \sqrt{10} \qquad F_{1\,dB} = 20 \lg_{10} \sqrt{10} = 10\,dB$

$F_2 = \omega \qquad F_{2\,dB} = 20 \lg_{10} \omega$ PENDENZA +20 dB/dec

FASE

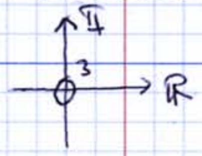
$\varphi(\omega) = +90°$

DIAGRAMMI DI BODE



$F(s) = s^3$

TRIPLO ZERO NELL'ORIGINE



RISPOSTA IN FREQUENZA

$\bar{F}(j\omega) = (j\omega)^3 = -j\omega^3$

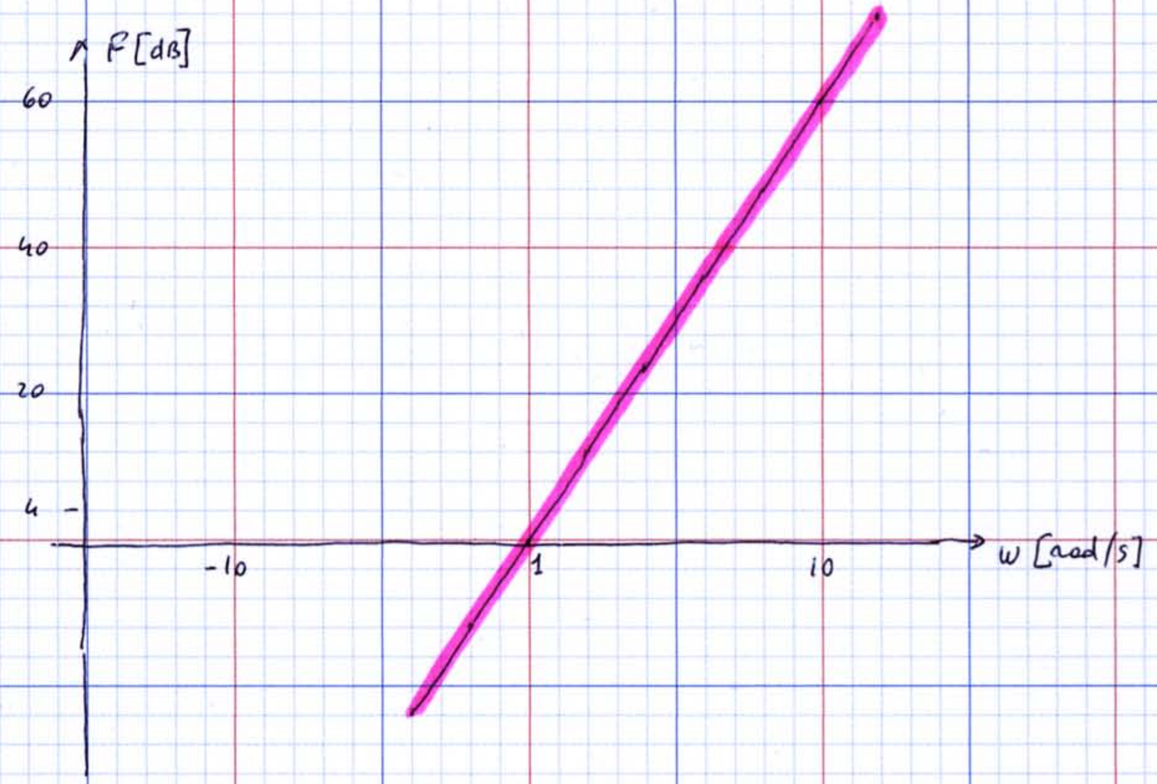
MODULO

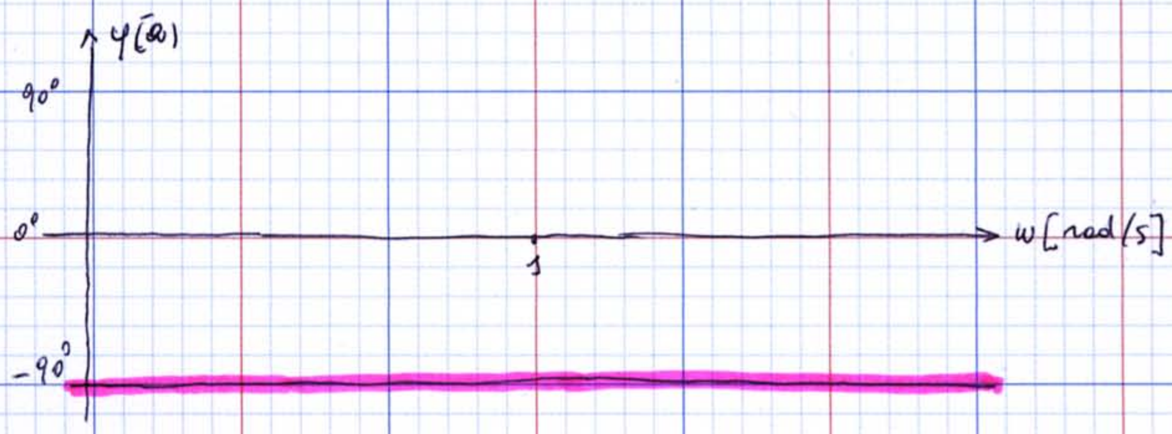
$F(\omega) = \omega^3$ $\quad F_{dB} = 20 \log_{10} \omega^3 = 60 \log_{10} \omega$ $\quad$ PENDENZA $+60$ dB/dec $(+3)$

FASE

$\varphi(\omega) = -90^\circ$

DIAGRAMMI DI BODE





$F(s) = \frac{10}{s}$ POLO SINGOLO NELL'ORIGINE

RISPOSTA IN FREQUENZA

$\bar{F}(j\omega) = \frac{10}{j\omega} = -j\frac{10}{\omega}$

MODULO

$F(\omega) = \frac{10}{\omega} = F_1 \cdot F_2 \quad \begin{cases} F_1 = 10 \\ F_2 = \frac{1}{\omega} \end{cases}$

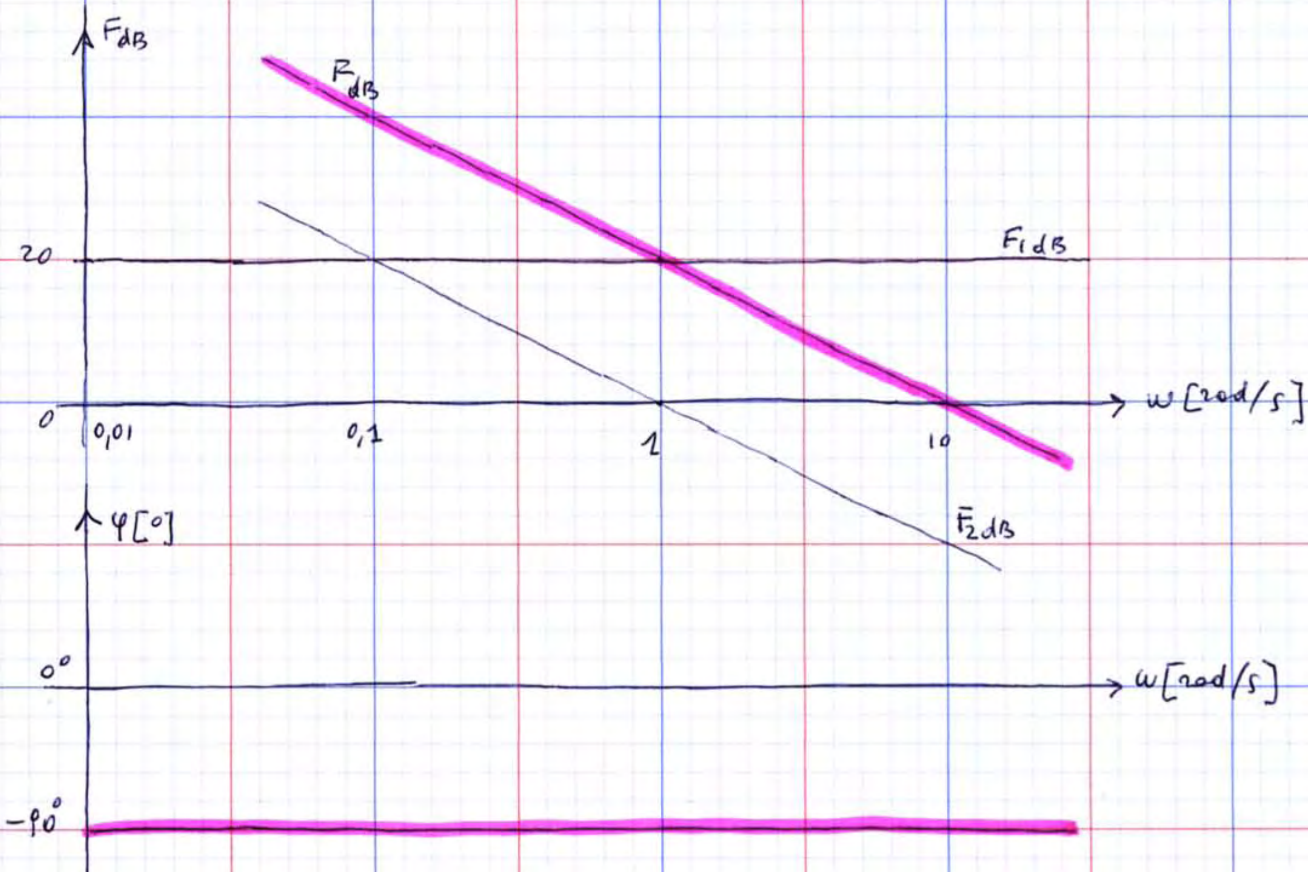
$F_1 = 10 \quad F_{1dB} = 20 \lg_{10} 10 = 20\,dB$

$F_2 = \frac{1}{\omega} \quad F_{2dB} = -20 \lg_{10} \omega$ PENDENZA -20 dB/dec

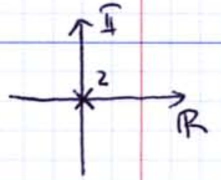
FASE

$\varphi(\omega) = -90^\circ$

DIAGRAMMI DI BODE



$F(s) = \frac{1}{s^2}$ POLO DOPPIO NELL'ORIGINE



RISPOSTA IN FREQUENZA

$\bar{F}(j\omega) = \frac{1}{(j\omega)^2} = -\frac{1}{\omega^2}$

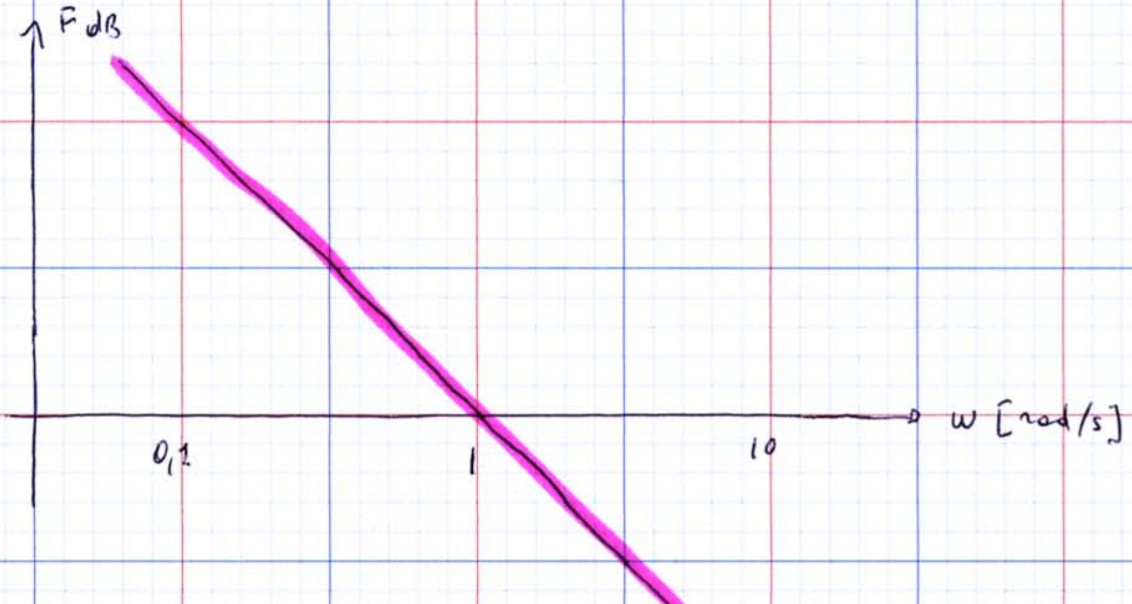
MODULO

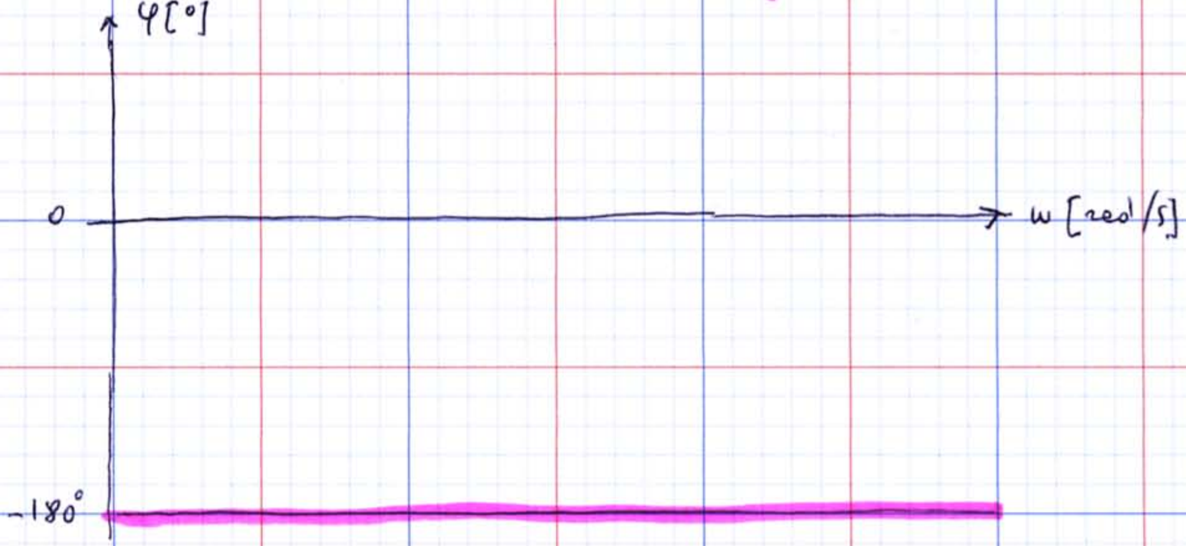
$F(\omega) = \frac{1}{\omega^2}$ $F_{dB} = 20 \log_{10} \frac{1}{\omega^2} = -40 \log_{10} \omega$ PENDENZA -40dB/dec (-2)

FASE

$\varphi(\omega) = \pm 180°$

DIAGRAMMI DI BODE





$F_1(s) = 1 + 2s$ ZERO $1+2s=0$ $s=-\frac{1}{2}=-0{,}5$

RISPOSTA IN FREQUENZA

$\overline{F}_1(j\omega) = 1 + j2\omega$

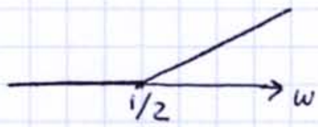
MODULO

$F_1(\omega) = \sqrt{1+(2\omega)^2}$

ZERO REALE NEL SEMIPIANO SX

MODULO APPROSSIMATO

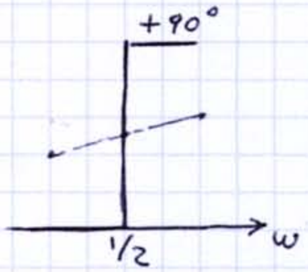
- PER $\omega < 1/2$ $F_1(\omega) = 1 \rightarrow 0\ dB$
- PER $\omega > 1/2$ $F_1(\omega) = 2\omega \rightarrow$ PENDENZA $+20\ dB/dec$ $(+1)$

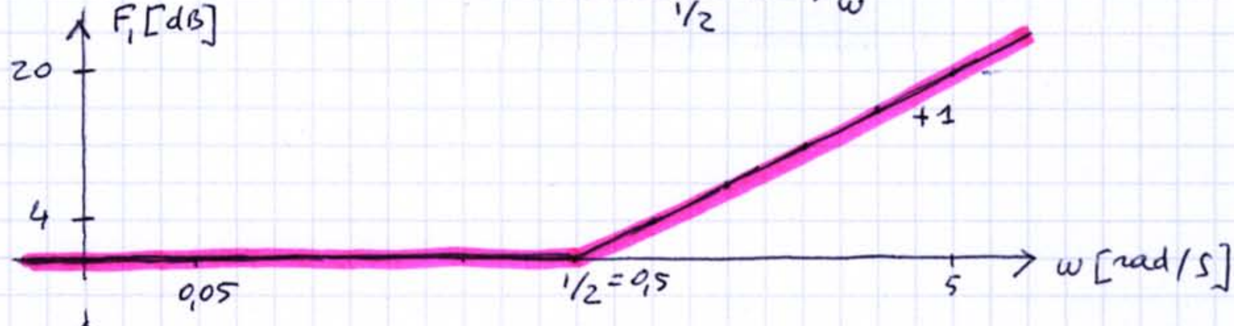


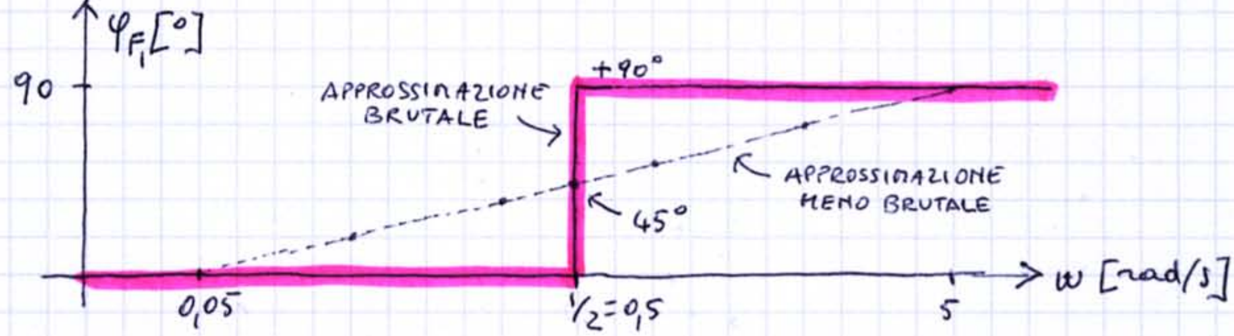
FASE

$\varphi_{F_1}(\omega) = \text{arctg}\, 2\omega$

FASE APPROSSIMATA



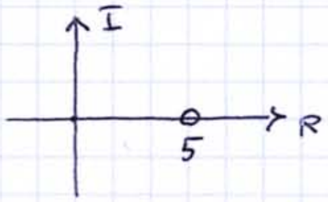




$F_2(s) = 1 - 0{,}2s$ ZERO $1 - 0{,}2s = 0$ $s = \frac{1}{0{,}2} = 5$

RISPOSTA IN FREQUENZA

$\overline{F}_2(j\omega) = 1 - j\,0{,}2\omega$



ZERO NEL SEMIPIANO DX

MODULO

$F_2(\omega) = \sqrt{1 + (0{,}2\omega)^2}$

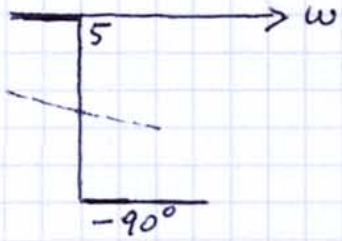
MODULO APPROSSIMATO

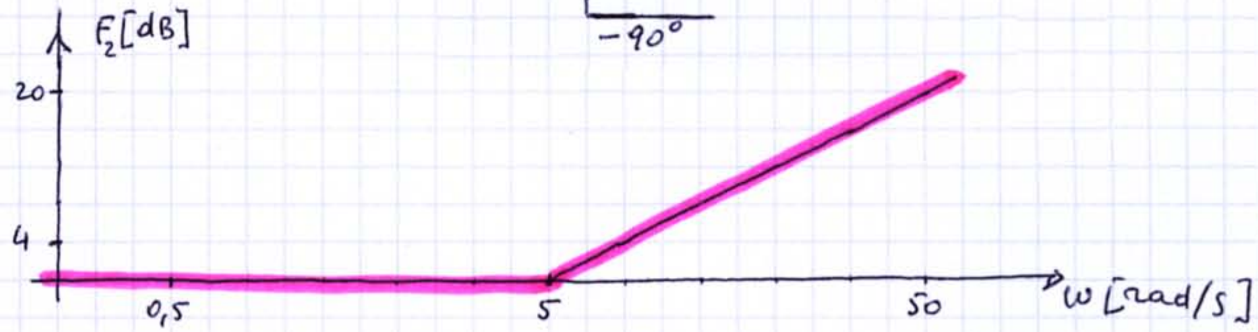
- PER $\omega < \frac{1}{0{,}2} = 5$ $F_2(\omega) = 1 \rightarrow 0\,dB$
- PER $\omega > 5$ $F_2(\omega) = 0{,}2\omega \rightarrow$ PENDENZA $+20\,dB/dec$

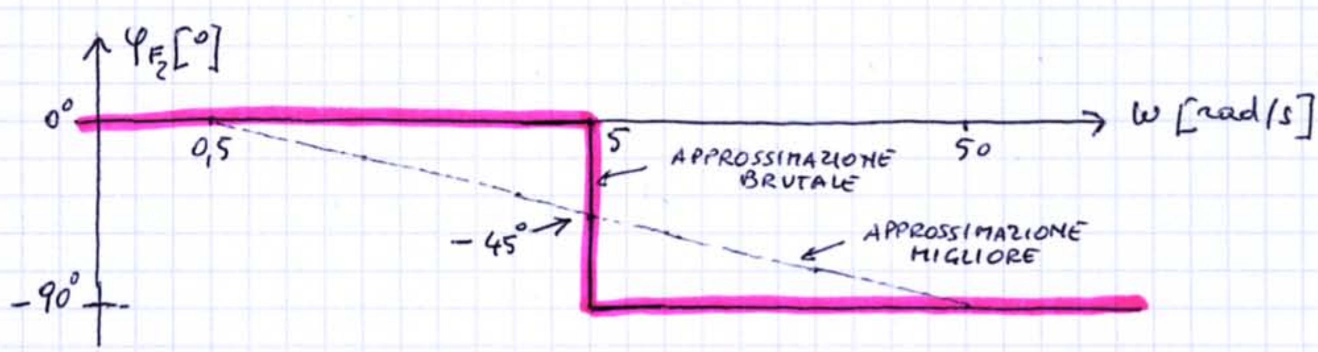
FASE

$\varphi_{F_2}(\omega) = \mathrm{arctg}(-0{,}2\omega) = -\,\mathrm{arctg}\,0{,}2\omega$

FASE APPROSSIMATA







$$F_3(s) = \frac{1}{1 + 0{,}8s}$$

POLO $1 + 0{,}8s = 0 \quad s = -\frac{1}{0{,}8} = -1{,}25$

POLO NEL SEMIPIANO SX

RISPOSTA IN FREQUENZA

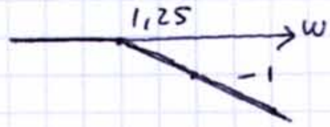
$$\overline{F}_3(j\omega) = \frac{1}{1 + j0{,}8\omega}$$

MODULO

$$F_3(\omega) = \frac{1}{\sqrt{1 + (0{,}8\omega)^2}}$$

MODULO APPROSSIMATO

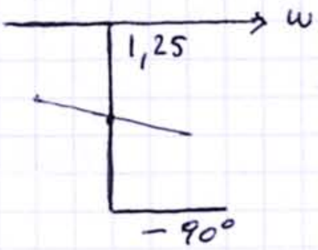
- PER $\omega < \frac{1}{0{,}8} = 1{,}25 \quad F_3(\omega) = 1 \rightarrow 0\ dB$
- PER $\omega > 1{,}25 \quad F_3(\omega) = \frac{1}{0{,}8\omega} \rightarrow$ PENDENZA $-20\ dB/dec$

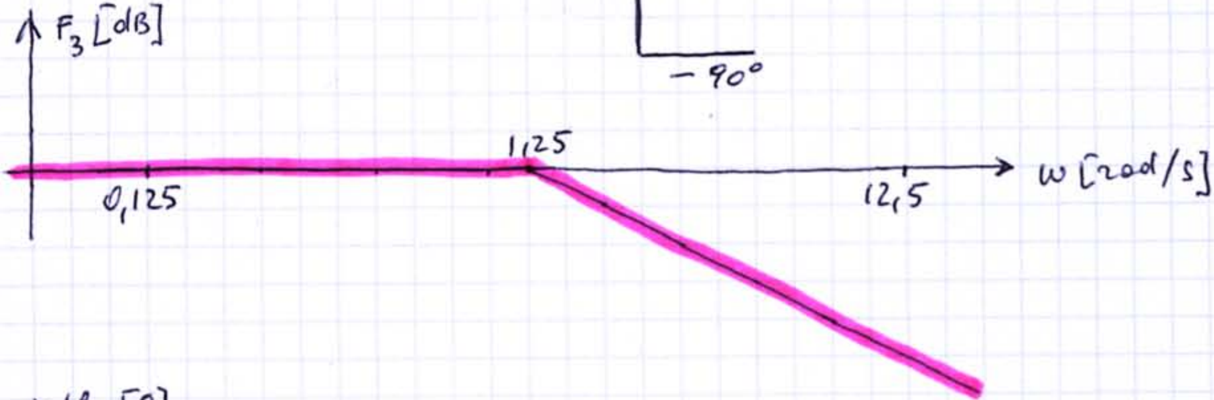


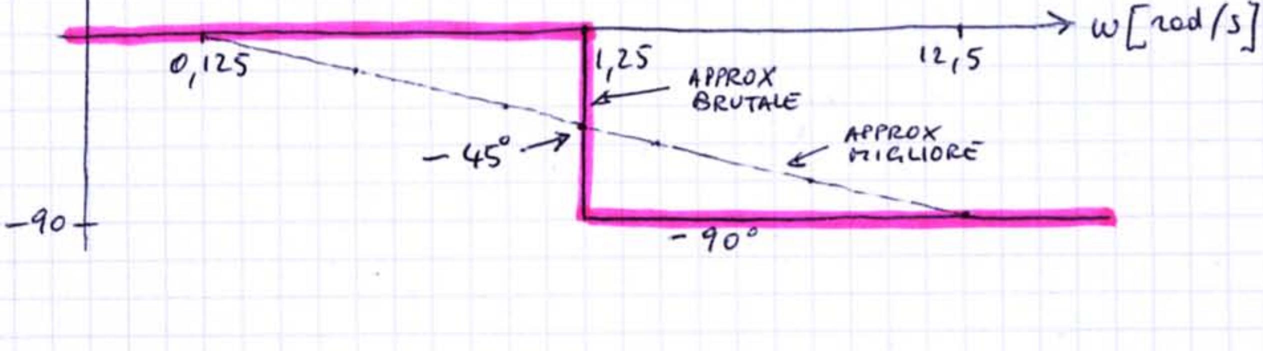
FASE

$$\varphi_{F_3}(\omega) = -\operatorname{arctg} 0{,}8\omega$$

FASE APPROSSIMATA

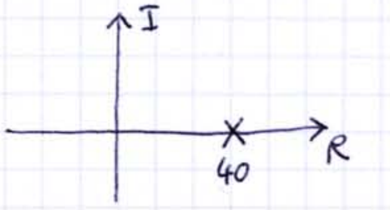






$$F_4(s) = \frac{1}{1 - \frac{s}{40}}$$

POLO $1 - \frac{s}{40} = 0 \quad s = +40$



POLO NEL SEMIPIANO DX

RISPOSTA IN FREQUENZA

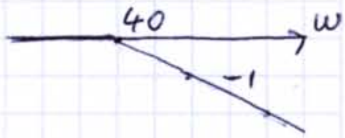
$$\overline{F_4}(j\omega) = \frac{1}{1 - j\frac{\omega}{40}}$$

MODULO

$$F_4(\omega) = \frac{1}{\sqrt{1 + \left(\frac{\omega}{40}\right)^2}}$$

MODULO APPROSSIMATO

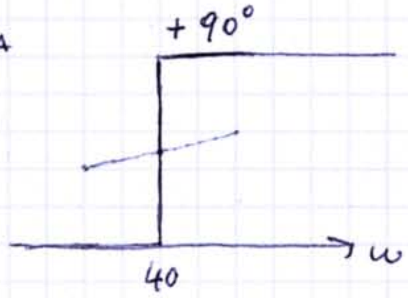
- PER $\omega < 40$ $\quad F_4(\omega) = 1 \rightarrow 0\,dB$
- PER $\omega > 40$ $\quad F_4(\omega) = \frac{1}{\frac{\omega}{40}} = \frac{40}{\omega} \rightarrow$ PENDENZA $-20\,dB/dec$

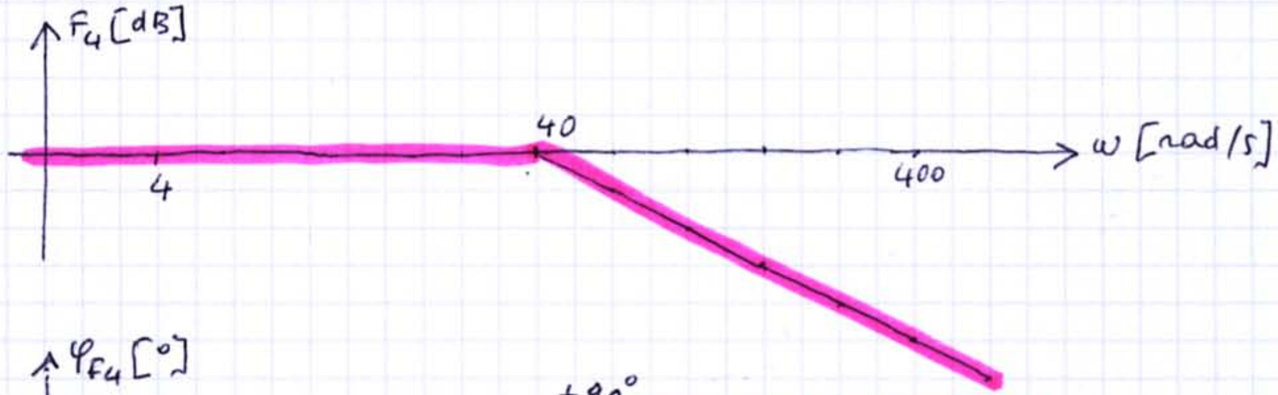


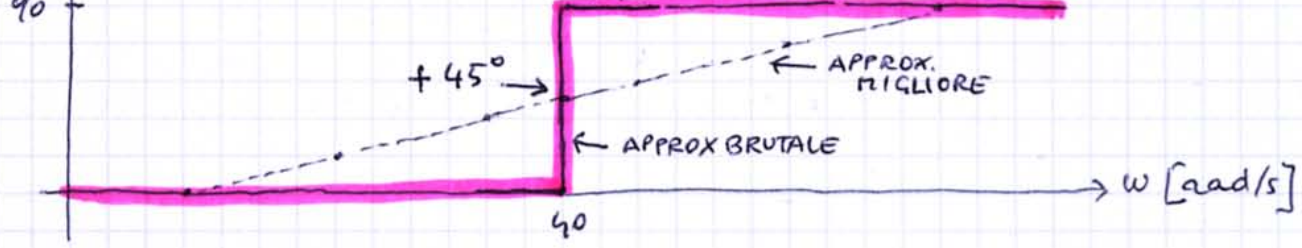
FASE

$$\varphi_{F_4}(\omega) = -\,\mathrm{arctg}\left(-\frac{\omega}{40}\right) = +\,\mathrm{arctg}\,\frac{\omega}{40}$$

FASE APPROSSIMATA





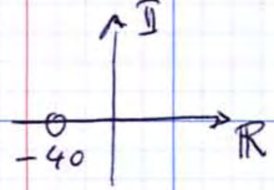


$$F(s) = 5 + \frac{s}{8} = 5\left(1 + \frac{s}{5 \cdot 8}\right) = 5\left(1 + \frac{s}{40}\right) \qquad \begin{cases} F_A = 5 \\ F_B = 1 + \frac{s}{40} \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = 5\left(1 + j\frac{\omega}{40}\right)$$

ZERO $1 + \frac{s}{40} = 0 \quad s = -40 \ (z)$



ZERO REALE NEL SEMIPIANO SX

MODULO

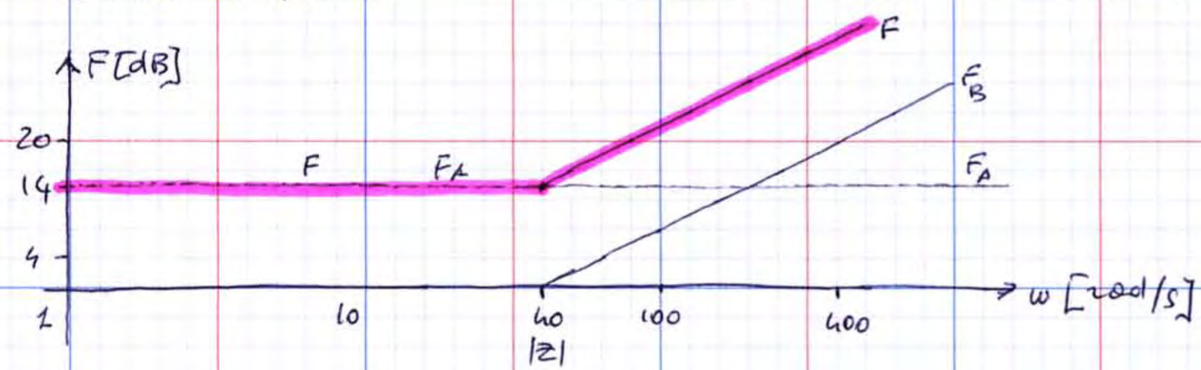
$$F(\omega) = 5\sqrt{1 + \left(\frac{\omega}{40}\right)^2}$$

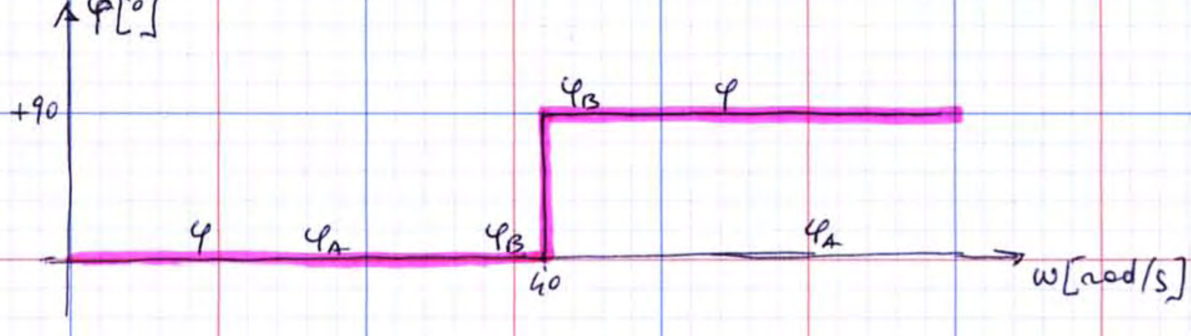
FASE

$$\varphi_F(\omega) = \operatorname{arctg} \frac{\omega}{40}$$

$F_{A\,dB} = 20\log_{10} 5 = 20\log_{10} 10 - 20\log_{10} 2 = 20\,dB - 6\,dB = 14\,dB$

DIAGRAMMI DI BODE



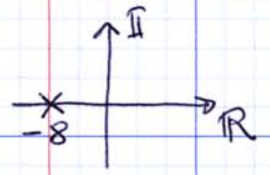


$$F(s) = \frac{20}{5 + \frac{s}{1,6}} = \frac{20}{5\left(1 + \frac{s}{5 \cdot 1,6}\right)} = 4 \cdot \frac{1}{1 + \frac{s}{8}} = F_A \cdot F_B \qquad \begin{cases} F_A = 4 \\ F_B = \dfrac{1}{1 + \frac{s}{8}} \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = 4 \cdot \frac{1}{1 + j\frac{\omega}{8}}$$

POLO $1 + \frac{s}{8} = 0 \quad s = -8 \; P$



MODULO

$$F(\omega) = 4 \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega}{8}\right)^2}}$$

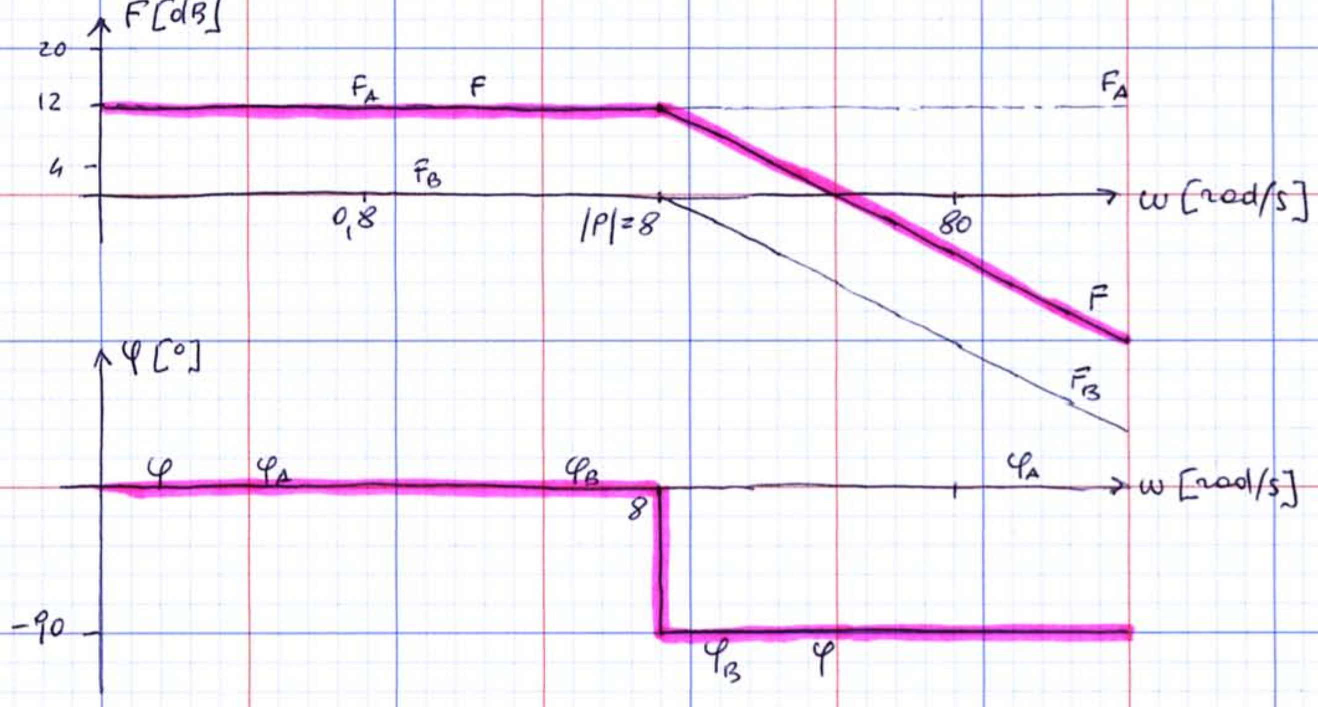
POLO REALE NEL SEMIPIANO SX

FASE

$$\varphi_F(\omega) = -\operatorname{arctg} \frac{\omega}{8}$$

DIAGRAMMI DI BODE

$$F_A \, dB = 20 \lg_{10} 4 = 2 \cdot 20 \lg_{10} 2 = 12 \, dB$$



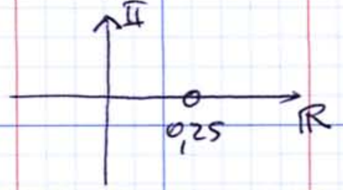
$$F(s) = s - 0{,}25 = -0{,}25\left(1 - \frac{s}{0{,}25}\right)$$

$$\begin{cases} F_A = -0{,}25 \\ F_B = 1 - \dfrac{s}{0{,}25} \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = -0{,}25\left(1 - j\frac{\omega}{0{,}25}\right)$$

ZERO $\quad 1 - \dfrac{s}{0{,}25} = 0 \quad s = 0{,}25\ (z)$



ZERO REALE NEL SEMIPIANO DX

MODULO

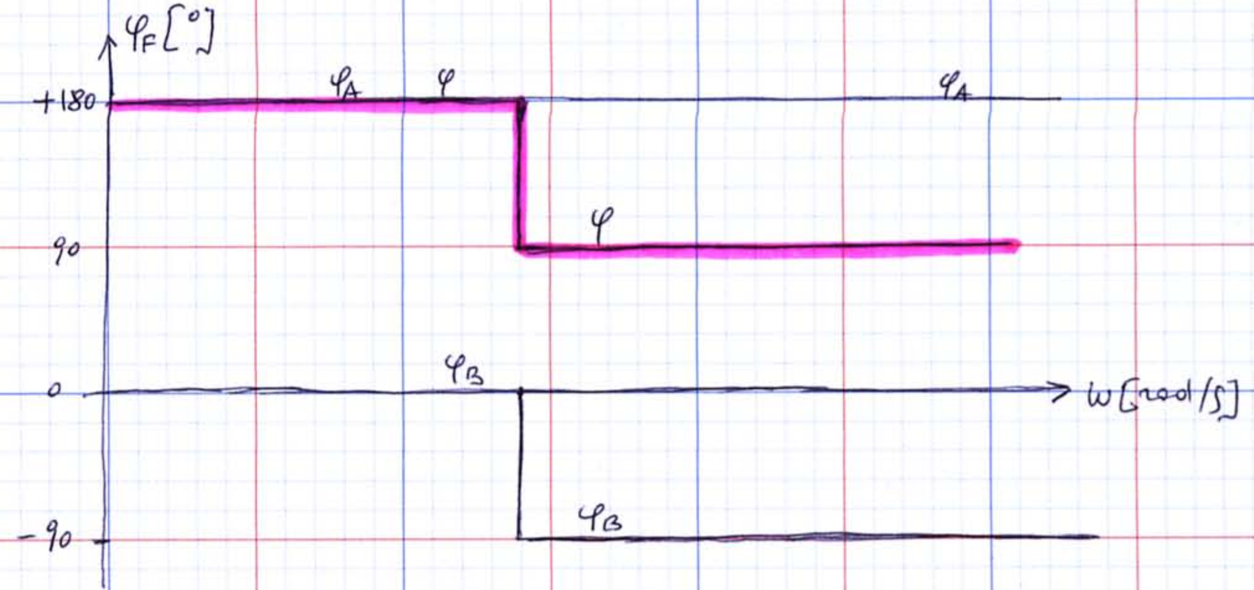
$$F(\omega) = 0{,}25\sqrt{1 + \left(\frac{\omega}{0{,}25}\right)^2}$$

FASE

$$\varphi_F(\omega) = 180^\circ + \mathrm{arctg}\left(-\frac{\omega}{0{,}25}\right) = 180^\circ - \mathrm{arctg}\,\frac{\omega}{0{,}25}$$

$$F_{AdB} = 20\log_{10} 0{,}25 = 20\log_{10}\frac{4}{10} = 2\cdot 20\log_{10} 2 - 20\log_{10} 10 =$$
$$= 2\cdot 6\,dB - 20\,dB = -8\,dB$$

DIAGRAMMI DI BODE



$$F(s) = \frac{1}{0{,}05s - \frac{1}{20}} = \frac{1}{-\frac{1}{20}\left(1 - \frac{0{,}05s}{1/20}\right)} = -20\,\frac{1}{1-s} \quad \begin{cases} F_A = -20 \\ F_B = \frac{1}{1-s} \end{cases}$$

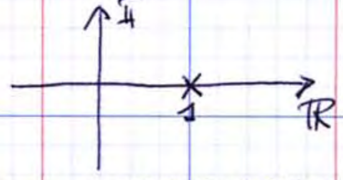
RISPOSTA IN FREQUENZA

$$F(j\omega) = -20\left(\frac{1}{1-j\omega}\right)$$

POLO $1 - s = 0$ $\quad s = 1$ (P)

MODULO

$$F(\omega) = 20\,\frac{1}{\sqrt{1+\omega^2}}$$



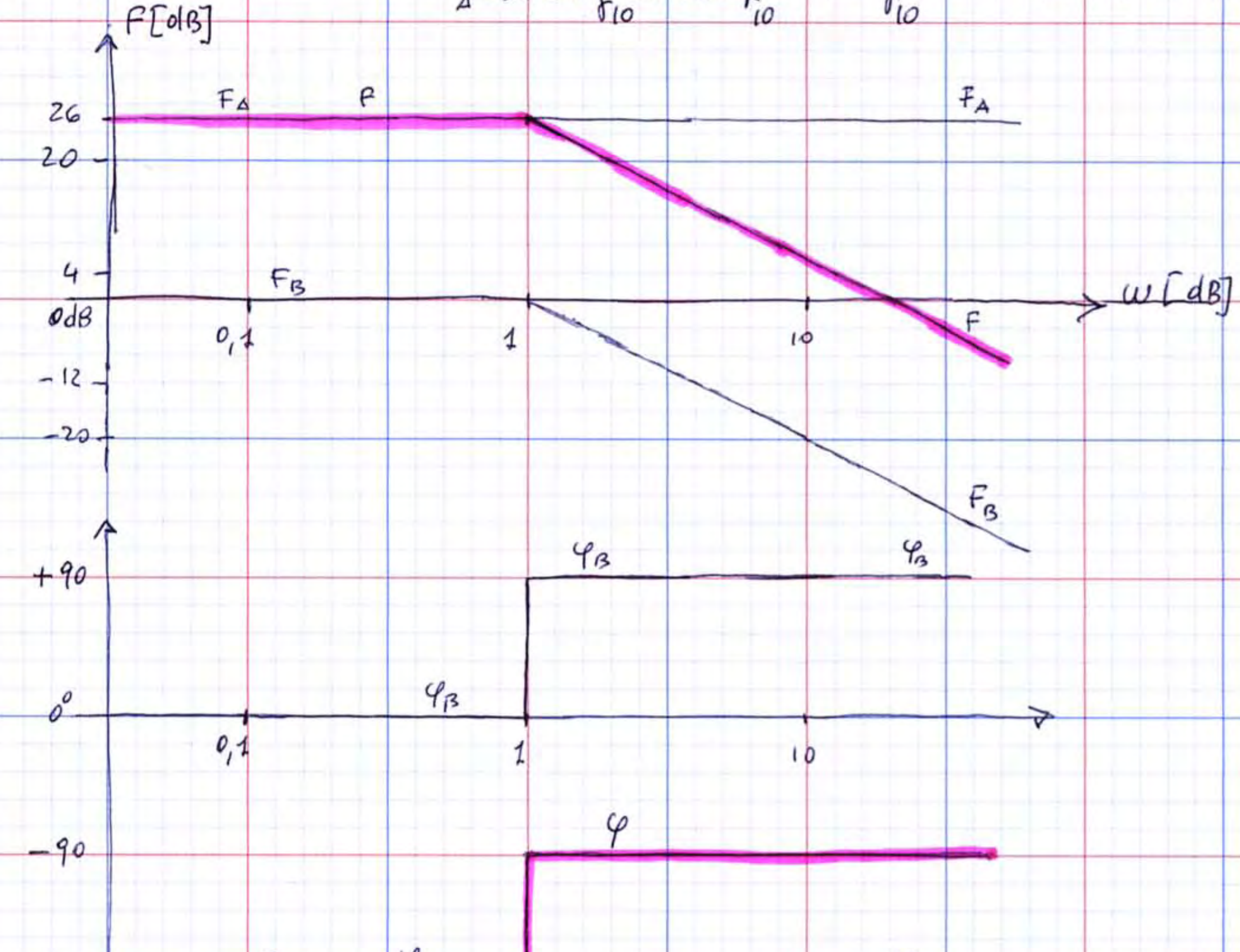
POLO REALE NEL SEMIPIANO DX

FASE

$$\varphi_F(\omega) = \pm 180° - \operatorname{arctg}(-\omega) = -180° + \operatorname{arctg}\omega$$

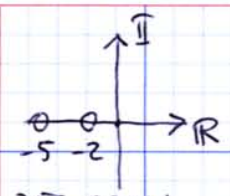
DIAGRAMMI DI BODE

$$F_A\,dB = 20\log_{10} 20 = 20\log_{10} 10 + 20\log_{10} 2 = 20dB + 6dB = 26dB$$



$F(s) = s^2 + 7s + 10$

ZERI $\quad s^2 + 7s + 10 = 0 \quad s = \frac{-7 \pm \sqrt{49-40}}{2} = \frac{-7 \pm 3}{2} = \begin{cases} -2 & z_1 \\ -5 & z_2 \end{cases}$



$F(s) = s^2 + 7s + 10 = (s+5)(s+2) = 10\left(1+\frac{s}{5}\right)\left(1+\frac{s}{2}\right)$

$\begin{cases} F_1 = 10 \\ F_2 = 1 + \frac{s}{5} \\ F_3 = 1 + \frac{s}{2} \end{cases}$

ZER REALI DISTINTI NEL SEMIPIANO SX

RISPOSTA IN FREQUENZA

$F(j\omega) = 10\left(1 + j\frac{\omega}{5}\right)\left(1 + j\frac{\omega}{2}\right)$

MODULO

$F(\omega) = 10\sqrt{1+\left(\frac{\omega}{5}\right)^2}\sqrt{1+\left(\frac{\omega}{2}\right)^2} = F_1 \cdot F_2 \cdot F_3$

$F_1 \qquad 20\lg_{10} 10 = 20\,dB$

$F_2 \begin{cases} \cdot \text{ PER } \omega < 5 & F_2 = 1 \quad F_{2dB} = 0 \\ \cdot \text{ PER } \omega > 5 & F_2 = \frac{\omega}{5} \quad F_{2dB} = 20\lg_{10}\frac{\omega}{5} \end{cases}$ PENDENZA +20 dB/DEC

$F_3 \begin{cases} \cdot \text{ PER } \omega < 2 & F_3 = 1 \quad F_{3dB} = 0 \\ \cdot \text{ PER } \omega > 2 & F_3 = \frac{\omega}{2} \quad F_{3dB} = 20\lg_{10}\frac{\omega}{2} \end{cases}$ PENDENZA +20 dB/dec

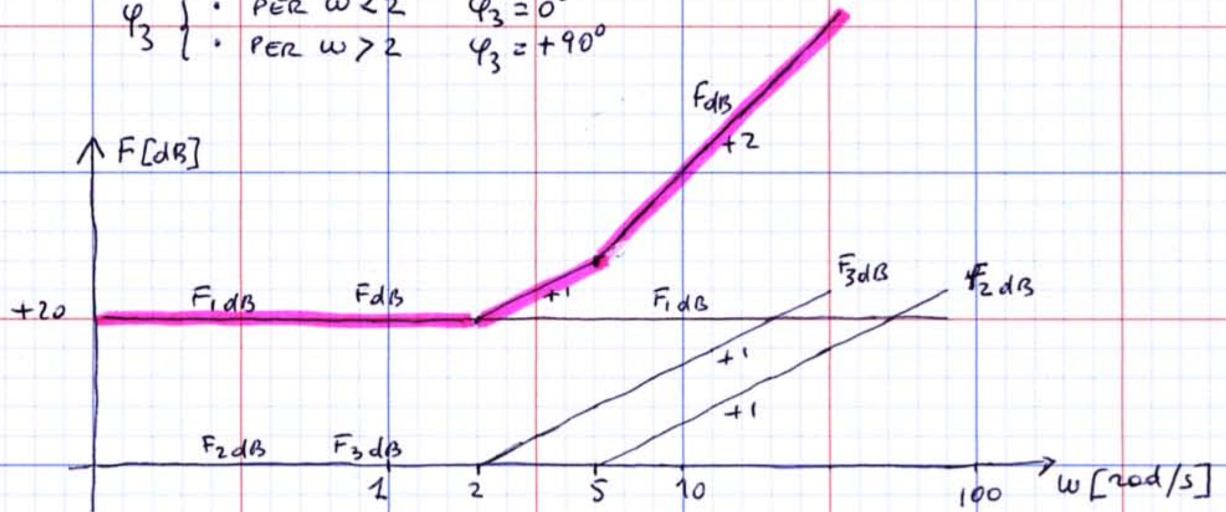
FASE

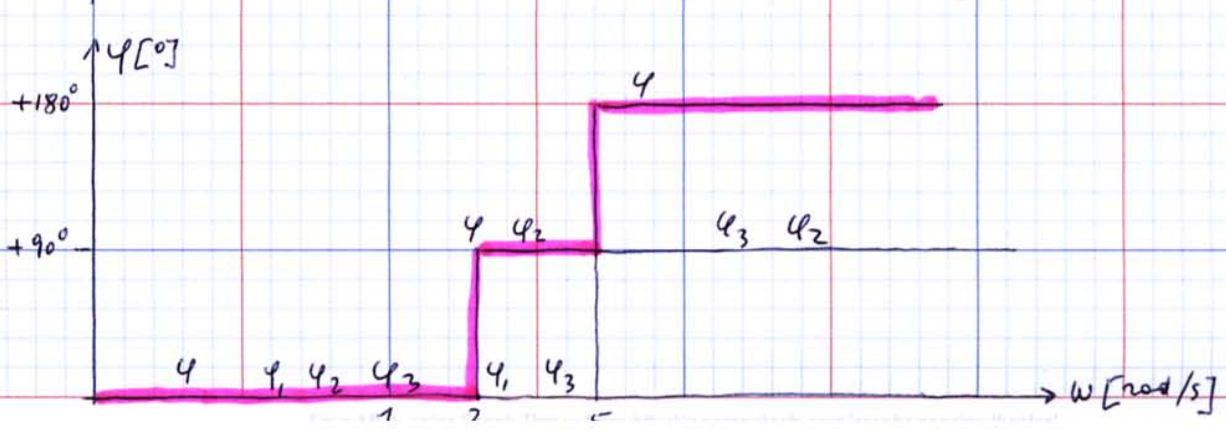
$\varphi_F = 0^\circ + \operatorname{arctg}\frac{\omega}{5} + \operatorname{arctg}\frac{\omega}{2} = \varphi_1 + \varphi_2 + \varphi_3$

$\varphi_1 = 0^\circ \quad \forall\omega$

$\varphi_2 \begin{cases} \cdot \text{ PER } \omega < 5 & \varphi_2 = 0^\circ \\ \cdot \text{ PER } \omega > 5 & \varphi_2 = +90^\circ \end{cases}$

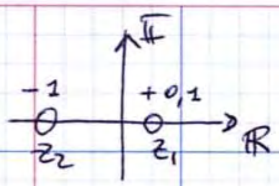
$\varphi_3 \begin{cases} \cdot \text{ PER } \omega < 2 & \varphi_3 = 0^\circ \\ \cdot \text{ PER } \omega > 2 & \varphi_3 = +90^\circ \end{cases}$





$F(s) = s^2 + 0{,}9s - 0{,}1$

ZERI $\quad s^2 + 0{,}9s - 0{,}1 = 0 \quad s = \begin{cases} +0{,}1 & z_1 \\ -1 & z_2 \end{cases}$



$F(s) = (s+1)(s-0{,}1) = \underbrace{-0{,}1}_{F_1(s)}\,\underbrace{(1+s)}_{F_2(s)}\,\underbrace{\left(1-\frac{s}{0{,}1}\right)}_{F_3(s)}$

ZERI REALI E DISTINTI NEL SEMIPIANO SX E DX

RISPOSTA IN FREQUENZA

$F(j\omega) = -0{,}1\,(1+j\omega)\left(1-j\frac{\omega}{0{,}1}\right)$

MODULO

$F(\omega) = 0{,}1\sqrt{1+\omega^2}\,\sqrt{1+\left(\frac{\omega}{0{,}1}\right)^2} = F_1 \cdot F_2 \cdot F_3$

$F_1 \quad 20\log_{10} 0{,}1 = -20\,dB \quad \forall\omega$

$F_2 \begin{cases} \cdot \text{ PER } \omega < 1 & F_2 = 1 \rightarrow 0\,dB \\ \cdot \text{ PER } \omega > 1 & F_2 = \omega \rightarrow +20\,dB/dec \end{cases}$

$F_3 \begin{cases} \cdot \text{ PER } \omega < 0{,}1 & F_3 = 1 \rightarrow 0\,dB \\ \cdot \text{ PER } \omega > 0{,}1 & F_3 = \frac{\omega}{0{,}1} \rightarrow +20\,dB/dec \end{cases}$

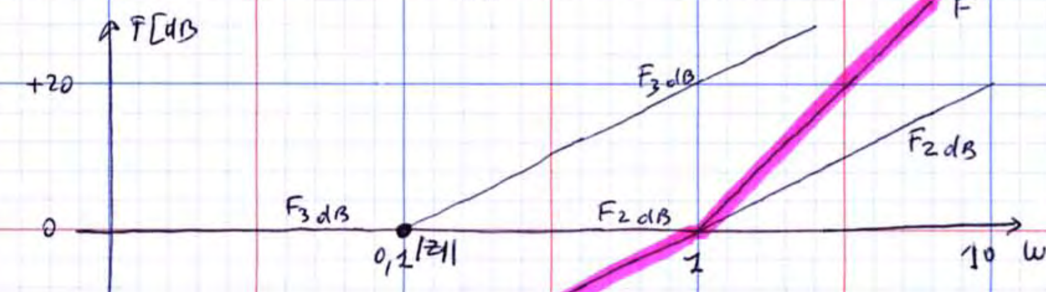
FASE

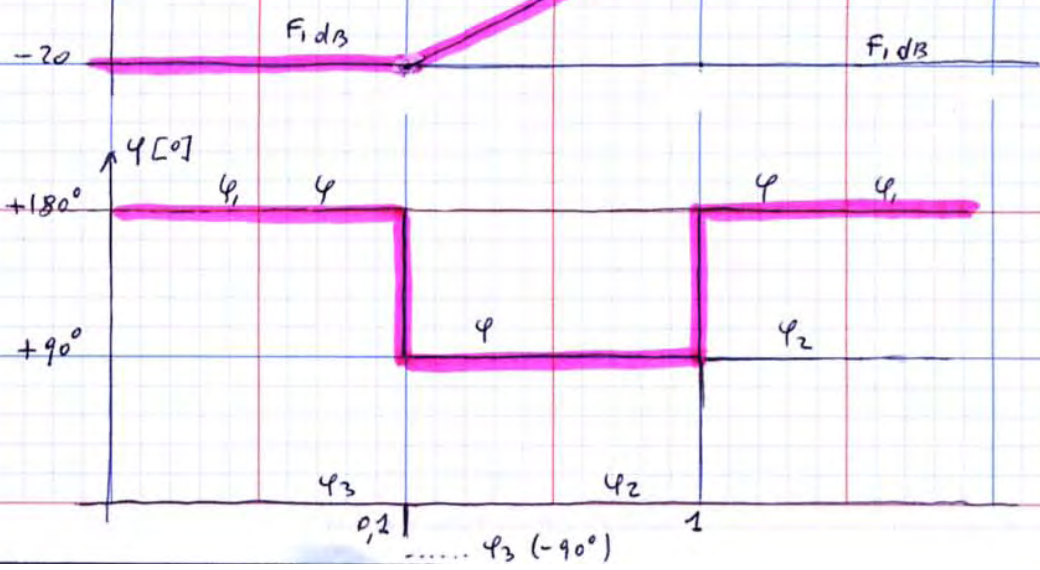
$\varphi(\omega) = \pm 180^\circ + \text{arctg}\,\omega + \text{arctg}\,\frac{\omega}{0{,}1}$

$\varphi_1 = \pm 180^\circ \quad \forall\omega$

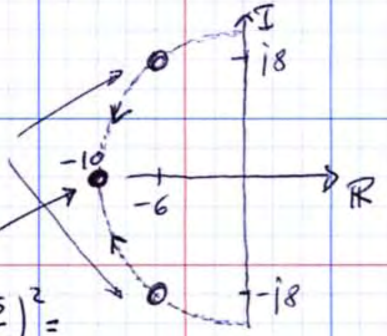
$\varphi_2 \begin{cases} \cdot \text{ PER } \omega < 1 & \varphi_2 = 0^\circ \\ \cdot \text{ PER } \omega > 1 & \varphi_2 = +90^\circ \end{cases}$

$\varphi_3 \begin{cases} \cdot \text{ PER } \omega < 0{,}1 & \varphi_3 = 0^\circ \\ \cdot \text{ PER } \omega > 0{,}1 & \varphi_3 = -90^\circ \end{cases}$! NEL SEMIPIANO DX





$$F(s) = \frac{s^2+12s+100}{400}$$



ZERI $s^2+12s+100=0$ $\quad s = -6 \pm \sqrt{6^2-100} = -6 \pm j8$

APPROSSIMO FUNZIONE PER ZERI REALI COINCIDENTI $s_{1,2} = -10$

$$F(s) = \frac{1}{400}\left(s^2+12s+100\right) \simeq \frac{1}{400}(s+10)^2 = \frac{10^2}{400}\left(1+\frac{s}{10}\right)^2 =$$

$$= \frac{1}{4}\left(1+\frac{s}{10}\right)^2 = F_1 \cdot F_2 \qquad \begin{cases} F_1 = 0{,}25 \\ F_2 = \left(1+\frac{s}{10}\right)^2 \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = 0{,}25\left(1+j\frac{\omega}{10}\right)^2$$

MODULO

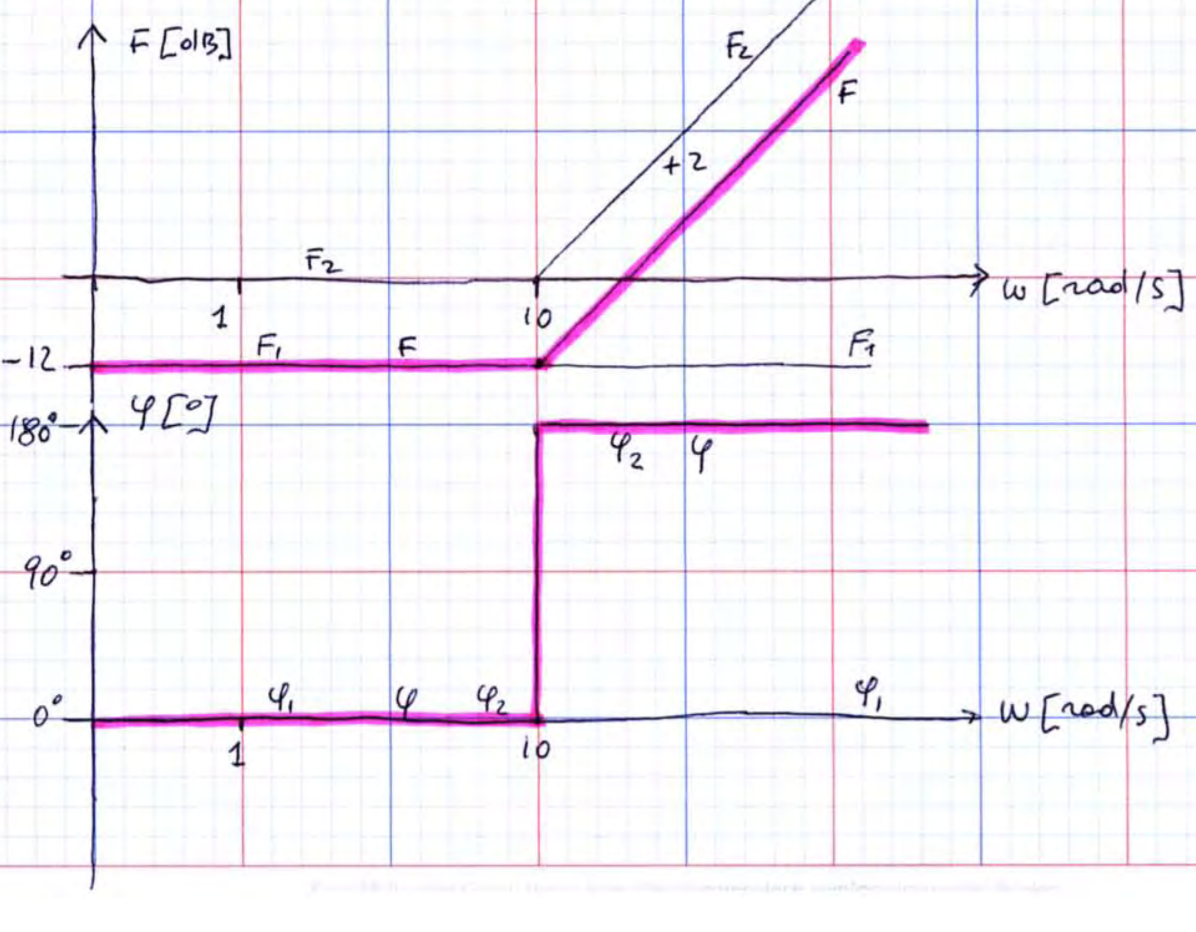
$$F(\omega) = 0{,}25 \cdot \sqrt{1+\left(\frac{\omega}{10}\right)^2} \cdot \sqrt{1+\left(\frac{\omega}{10}\right)^2} = \underset{\uparrow F_1}{0{,}25} \cdot \underset{\uparrow F_2}{\left[1+\left(\frac{\omega}{10}\right)^2\right]}$$

$$F_{1dB} = 20\log_{10}\frac{1}{4} = -20\log_{10}4 = -2\cdot 20\log_{10}2 = -12\,dB \qquad (20\log_{10}2 = 6dB)$$

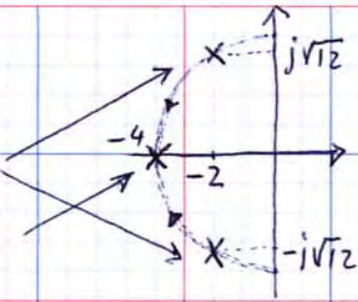
MODULO APPROSSIMATO DI F_2

- PER $\omega < 10$ $\quad F_2 = 1 \rightarrow 0dB$
- PER $\omega > 10$ $\quad F_2 = \left(\frac{\omega}{10}\right)^2 \rightarrow F_{2dB} = 20\log_{10}\left(\frac{\omega}{10}\right)^2 = 40\log_{10}\frac{\omega}{10}$ $\quad$ PENDENZA $+40\,dB/DEC.$ $(+2)$

DIAGRAMMI DI BODE



$$F(s) = \frac{400}{s^2+4s+16}$$



POLI $s^2+4s+16=0 \quad s=-2\pm\sqrt{4-16} = -2\pm j\sqrt{12}$

APPROSSIMO FUNZIONE PER POLI REALI COINCIDENTI $s_{12}=-4$

$$F(s) = \frac{400}{s^2+4s+16} \simeq \frac{400}{16\left(1+\frac{s}{4}\right)^2} = 25 \cdot \frac{1}{\left(1+\frac{s}{4}\right)^2} = F_1 \cdot F_2 \qquad \begin{cases} F_1 = 25 \\ F_2 = \dfrac{1}{\left(1+\frac{s}{4}\right)^2} \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = 25 \cdot \frac{1}{\left(1+j\frac{\omega}{4}\right)^2}$$

MODULO

$$F(\omega) = \overset{F_1}{25} \cdot \frac{1}{\sqrt{1+\left(\frac{\omega}{4}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\omega}{4}\right)^2}} = 25 \cdot \overset{F_2}{\frac{1}{\left[1+\left(\frac{\omega}{4}\right)^2\right]}}$$

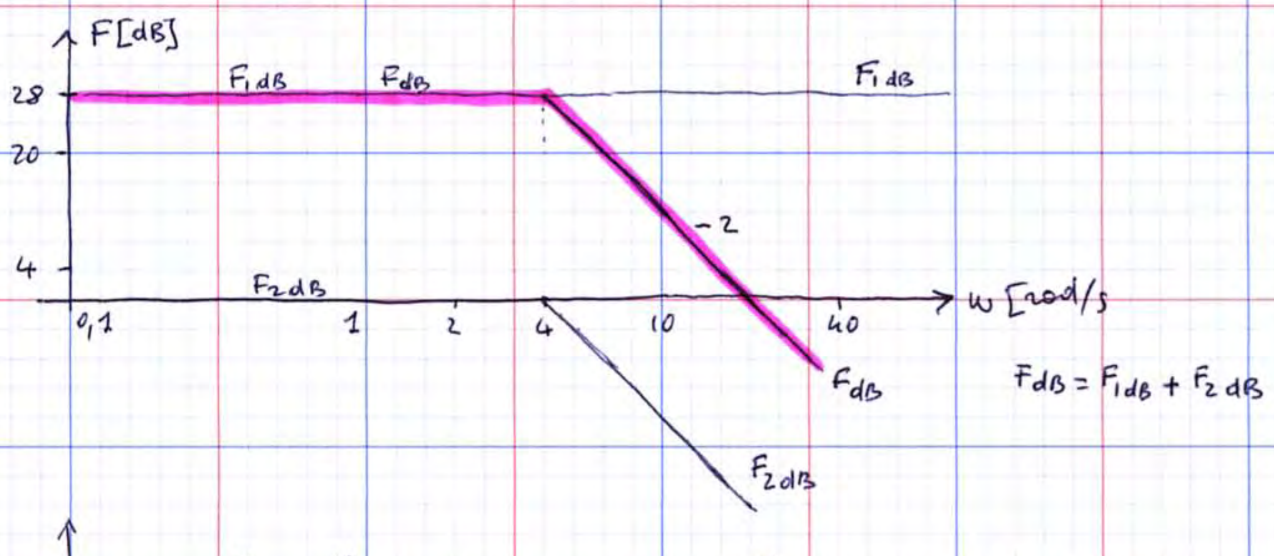
$$\log_{10} 4 = \log_{10} 2^2 = 2\log_{10} 2$$

$$F_1\,dB = 20\log_{10} 25 = 20\log_{10}\frac{100}{4} = 20\log_{10} 100 - 20\log_{10} 4 = 40\,dB - \underbrace{20\cdot 2\log_{10} 2}_{2\cdot\underbrace{20\log_{10} 2}_{\simeq 6dB}} = 40\,dB - 12\,dB = 28\,dB$$

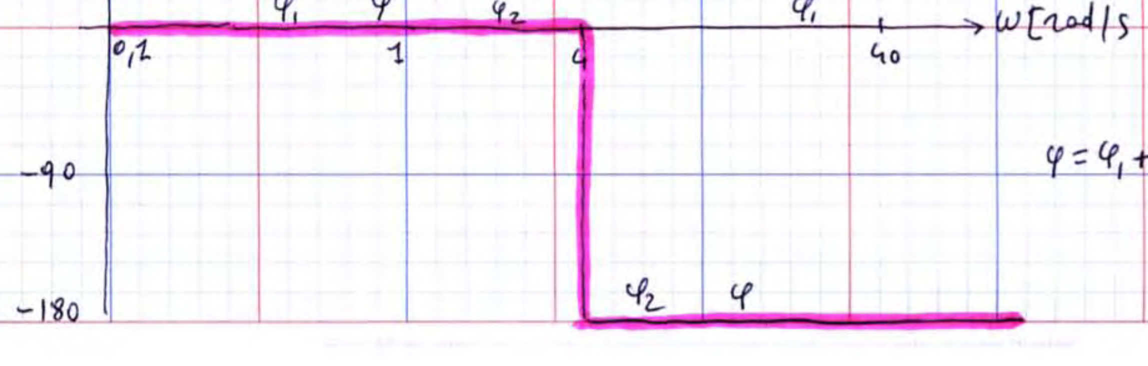
MODULO APPROSSIMATO DI F_2

- PER $\omega < 4 \quad F_2 = 1 \quad \to 0\,dB$
- PER $\omega > 4 \quad F_2 = \dfrac{1}{\left(\frac{\omega}{4}\right)^2} \to F_{2dB} = 20\log_{10}\dfrac{1}{\left(\frac{\omega}{4}\right)^2} = -40\log_{10}\dfrac{\omega}{4}$ PENDENZA -40 dB/DEC (-2)

DIAGRAMMI DI BODE



$F_{dB} = F_{1dB} + F_{2dB}$



$\varphi = \varphi_1 + \varphi_2$

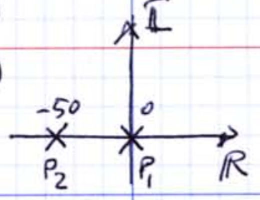
$$F(s) = 10\,\frac{1}{s\,(1+0{,}02s)}$$

$$\begin{cases} F_A = 10 \\ F_B = \dfrac{1}{s} \\ F_C = \dfrac{1}{1+0{,}02s} \end{cases}$$

RISPOSTA IN FREQUENZA

$$F(j\omega) = \frac{10}{j\omega\,(1+j0{,}02\omega)}$$

POLI $\begin{cases} s = 0 & \rightarrow \quad (P_1) \\ 1+0{,}02s = 0 \quad s = -50 & (P_2) \end{cases}$



MODULO

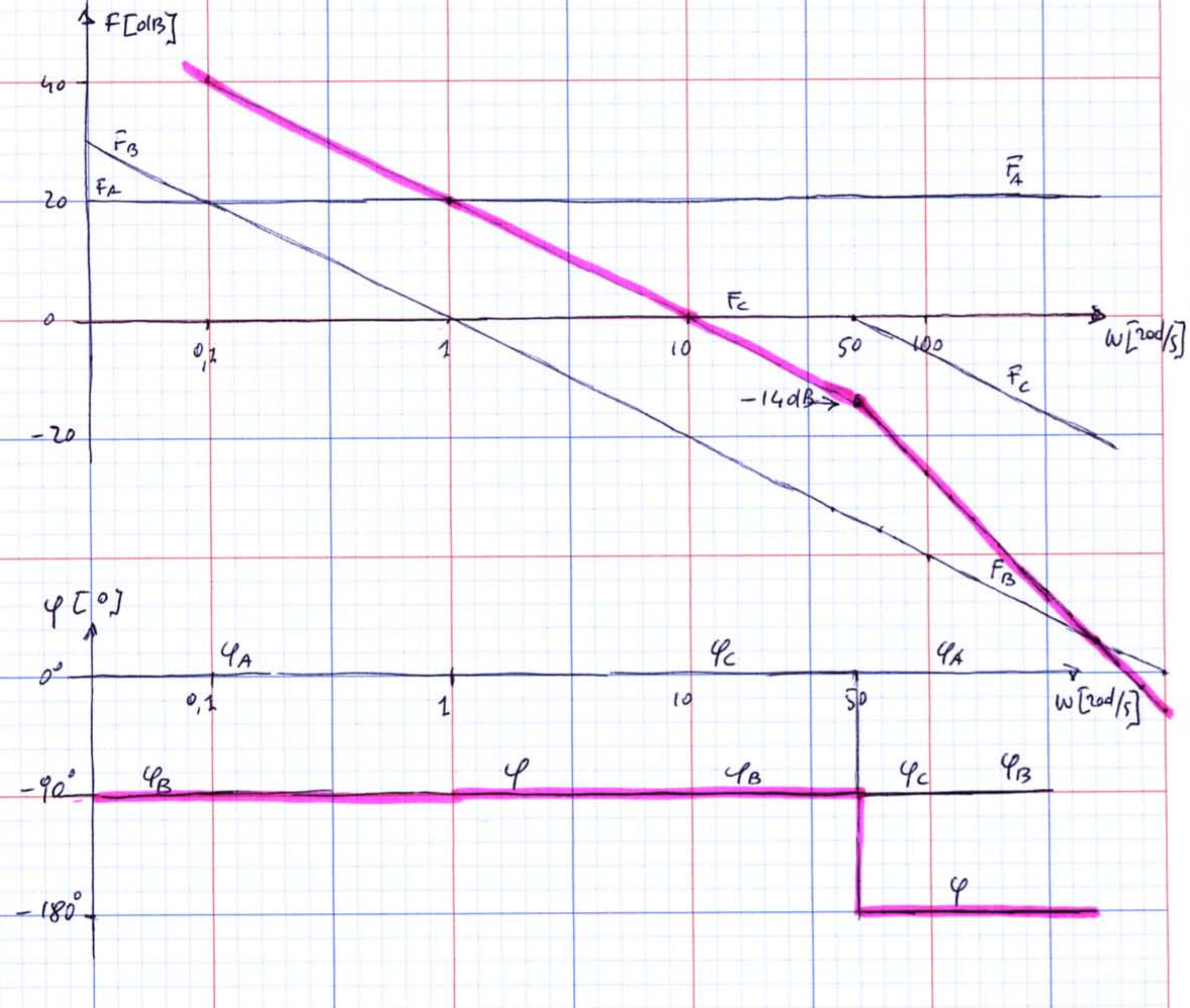
$$F(\omega) = \frac{\overset{F_A}{10}}{\underset{F_B}{\omega}\,\underset{F_C}{\sqrt{1+(0{,}02\omega)^2}}}$$

$$20\log_{10} F_A = 20\log_{10} 10 = 20\,dB$$

FASE

$$\varphi_F(\omega) = -90^\circ - \operatorname{arctg} 0{,}02\omega$$

DIAGRAMMI DI BODE



$$F(s) = 50\,\frac{1+s}{(1+5s)(1+10s)} = F_1 \cdot F_2 \cdot F_3 \cdot F_4$$

$F_1 = 50$

$F_2 = 1+s$

$F_3 = \frac{1}{1+5s}$

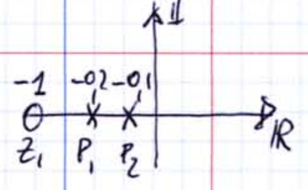
$F_4 = \frac{1}{1+10s}$

POLI E ZERI

ZERI $\quad 1+s=0 \quad s=-1 \; (z_1)$

POLI $\quad 1+5s=0 \quad s=-0{,}2 \; (p_1)$

$\quad\quad 1+10s=0 \quad s=-0{,}1 \; (p_2)$



RISPOSTA IN FREQUENZA

$$F(j\omega) = 50\,\frac{1+j\omega}{(1+j5\omega)(1+j10\omega)}$$

MODULO

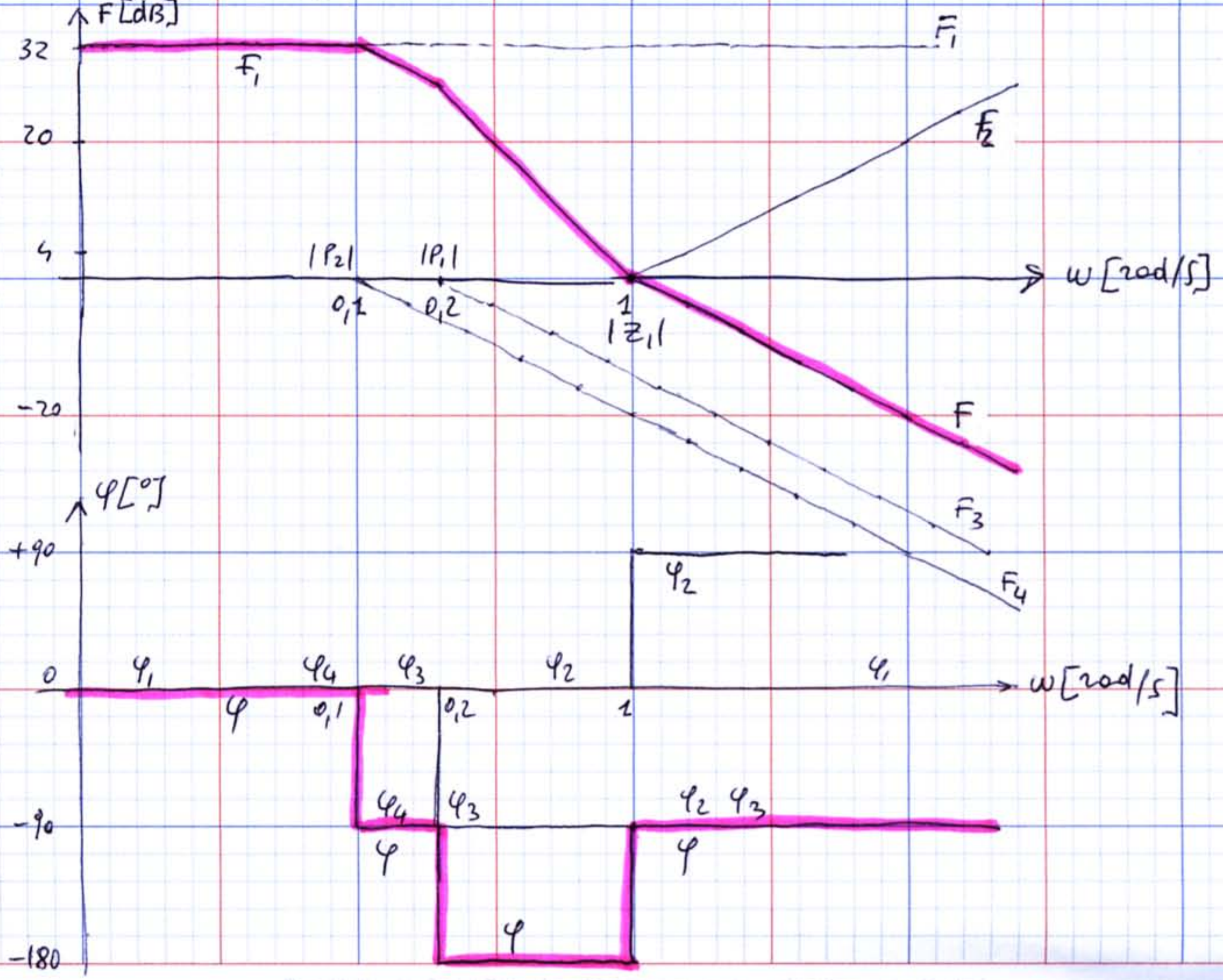
$$F(\omega) = \underset{F_1}{50}\,\frac{\overset{F_2}{\sqrt{1+\omega^2}}}{\underset{F_3}{\sqrt{1+(5\omega)^2}}\;\underset{F_4}{\sqrt{1+(10\omega)^2}}}$$

FASE

$$\varphi(\omega) = 0^\circ + \underset{\varphi_2}{\mathrm{arctg}\,\omega} - \underset{\varphi_3}{\mathrm{arctg}\,5\omega} - \underset{\varphi_4}{\mathrm{arctg}\,10\omega}$$

$$20\log_{10} 50 = 20\log_{10}\frac{100}{2} = 20\log_{10}100 - 20\log_{10}2 = 40\,dB - 6\,dB = 34\,dB$$

DIAGRAMMI DI BODE



$$F(s) = \frac{20s\,(1-0{,}2s)}{(1+2s)(1+0{,}8s)} = F_1 \cdot F_2 \cdot F_3 \cdot F_4 \cdot F_5$$

$F_1 = 20$ $\quad F_4 = \dfrac{1}{1+2s}$

$F_2 = s$

$F_3 = 1-0{,}2s$ $\quad F_5 = \dfrac{1}{1+0{,}8s}$

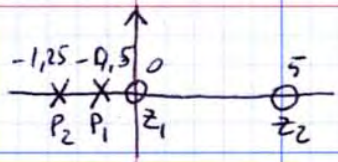
POLI / ZERI

ZERI $N(s)=0$ $\quad s=0$ $\quad s=0 \quad z_1$

$1-0{,}2s=0 \quad s=5 \quad z_2$

POLI $D(s)=0$ $\quad 1+2s=0 \quad s=-0{,}5 \quad p_1$

$1+0{,}8s=0 \quad s=-1{,}25 \quad p_2$



RISPOSTA IN FREQUENZA

$$\bar{F}(j\omega) = \frac{j20\omega\,(1-j0{,}2\omega)}{(1+j2\omega)(1+j0{,}8\omega)}$$

MODULO

$$F(\omega) = \frac{20\,\omega\sqrt{1+(0{,}2\omega)^2}}{\sqrt{1+(2\omega)^2}\,\sqrt{1+(0{,}8\omega)^2}}$$

$F_1 = 20$ $\quad F_4 = \dfrac{1}{\sqrt{1+(2\omega)^2}}$

$F_2 = \omega$

$F_3 = \sqrt{1+(0{,}2\omega)^2}$ $\quad F_5 = \dfrac{1}{\sqrt{1+(0{,}8\omega)^2}}$

- $F_1 = 20 \quad F_{1dB} = 20\lg_{10} 20 = 20\lg_{10} 10\cdot 2 = 20\lg_{10} 10 + 20\lg_{10} 2 = 20dB + 6dB = 26dB$
- $F_2 = \omega \quad F_2 dB = 20\lg_{10}\omega \quad \forall\omega$ PENDENZA +20 dB/dec (+1)
- $F_3 \begin{cases} \cdot \text{ PER } \omega < 5 & F_3 = 1 \quad F_3 dB = 20\lg_{10} 1 = 0\,dB \\ \cdot \text{ PER } \omega > 5 & F_3 = 0{,}2\omega \quad F_3 dB = +20\lg 0{,}2\omega \text{ PENDENZA } +20\,dB/dec.\ (+1) \end{cases}$
- $F_4 \begin{cases} \cdot \text{ PER } \omega < 0{,}5 & F_4 = 1 \quad F_4 dB = 20\lg_{10} 1 = 0\,dB \\ \cdot \text{ PER } \omega > 0{,}5 & F_4 = \frac{1}{2\omega} \quad F_4 dB = -20\lg_{10} 2\omega \text{ PENDENZA } -20\,dB/dec\ (-1) \end{cases}$
- $F_5 \begin{cases} \cdot \text{ PER } \omega < 1{,}25 & F_5 = 1 \quad F_{5dB} = 20\lg 1 = 0\,dB \\ \cdot \text{ PER } \omega > 1{,}25 & F_5 = \frac{1}{0{,}8\omega} \quad F_{5dB} = -20\lg_{10} 0{,}8\omega \text{ PENDENZA } -20\,dB/dec\ (-1) \end{cases}$

FASE

$$\varphi(\omega) = 0^\circ + 90^\circ - \operatorname{arctg} 0{,}2\omega - \operatorname{arctg} 2\omega - \operatorname{arctg} 0{,}8\omega$$

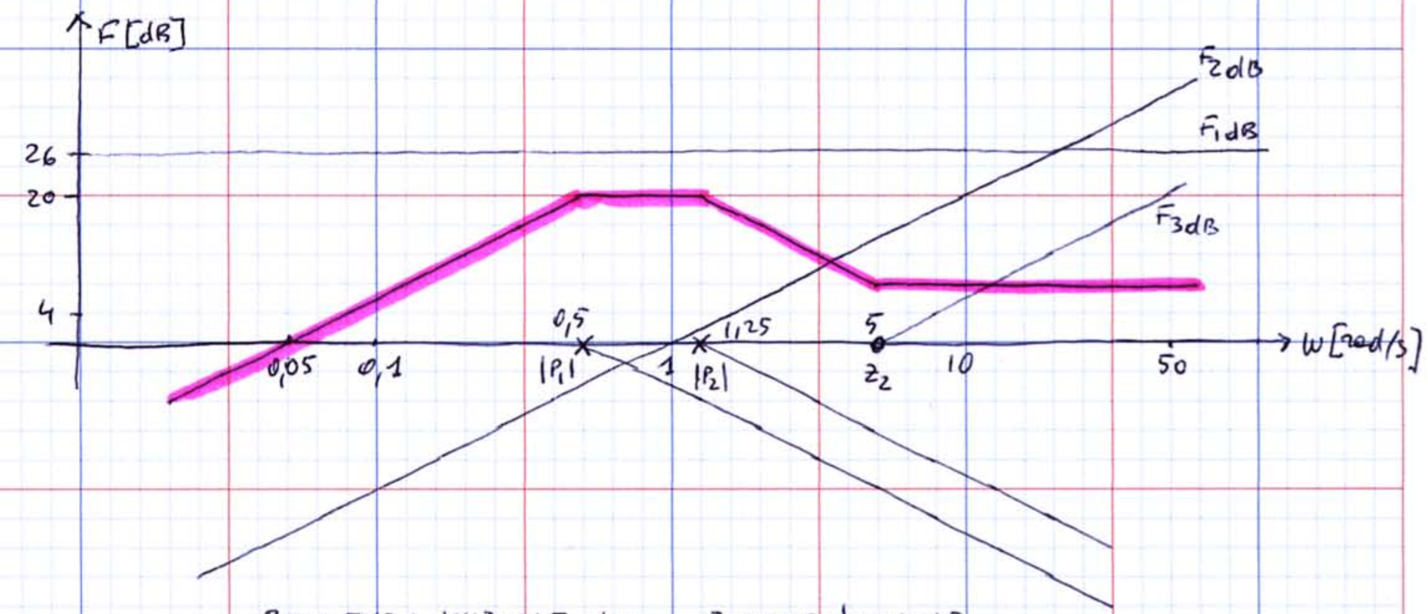
- $\varphi_1 = 0^\circ$
- $\varphi_2 = +90^\circ$
- $\varphi_3 = -\operatorname{arctg} 0{,}2\omega \begin{cases} \cdot \text{ PER } \omega < 5 & \varphi_3 = 0^\circ \\ \cdot \text{ PER } \omega > 5 & \varphi_3 = -90^\circ \end{cases}$
- $\varphi_4 = -\operatorname{arctg} 2\omega \begin{cases} \cdot \text{ PER } \omega < 0{,}5 & \varphi_4 = 0^\circ \\ \cdot \text{ PER } \omega > 0{,}5 & \varphi_4 = -90^\circ \end{cases}$
- $\varphi_5 = -\operatorname{arctg} 0{,}8\omega \begin{cases} \cdot \text{ PER } \omega < 1{,}25 & \varphi_5 = 0^\circ \\ \cdot \text{ PER } \omega > 1{,}25 & \varphi_5 = -90^\circ \end{cases}$

DIAGRAMMI DI BODE

DOMINIO DI RAPPRESENTAZIONE

$$\omega_{min} = \frac{1}{10} \min\{|P|, |Z|\} = \frac{1}{10}\cdot 0{,}5 = 0{,}05 \text{ rad/s}$$

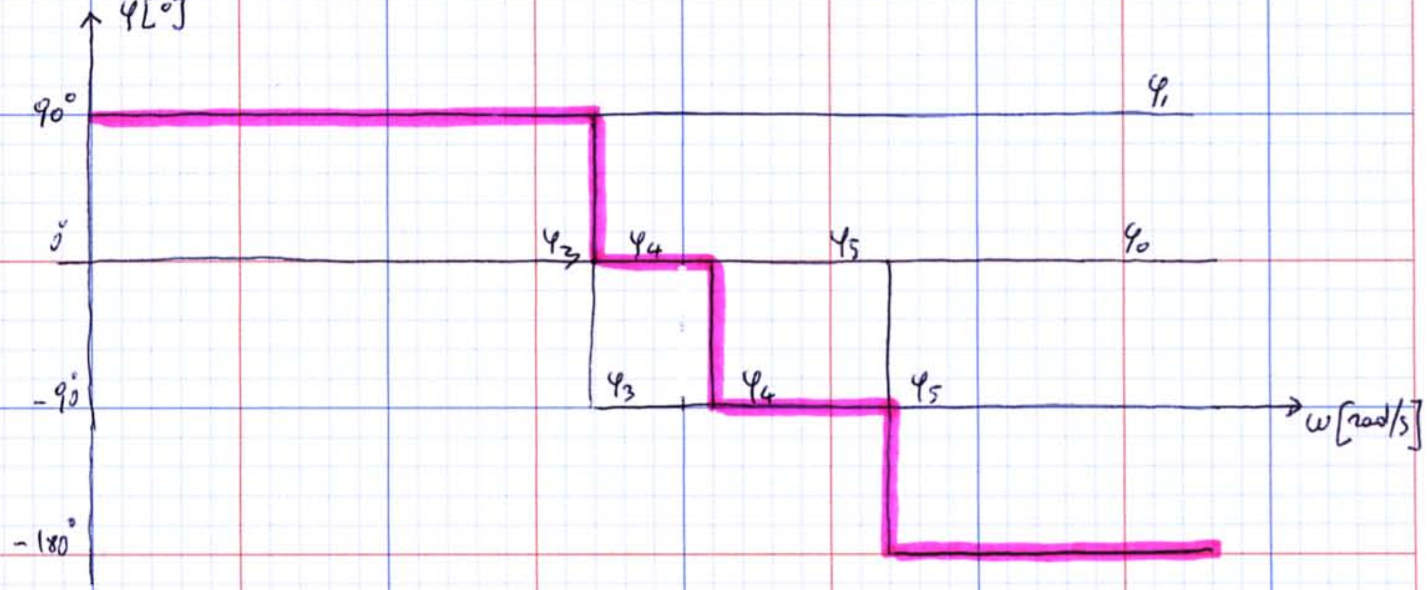
$$\omega_{max} = 10 \max\{|P|, |Z|\} = 10 \cdot 5 = 50 \text{ rad/s}$$



PENDENZA INIZIALE $+1$ ZERO NELL'ORIGINE

PENDENZA FINALE 0 N° ZERI = N° POLI

VALORE FINALE $F_\infty = \dfrac{20 \cdot 0{,}2}{2 \cdot 0{,}8} = 2{,}5 \rightarrow F_{\infty dB} = 20 \lg_{10} 2{,}5 = 8$ dB



FASE INIZIALE $+90°$ ZERO NELL'ORIGINE E GUADAGNO STATICO > 0

FASE FINALE $-180°$