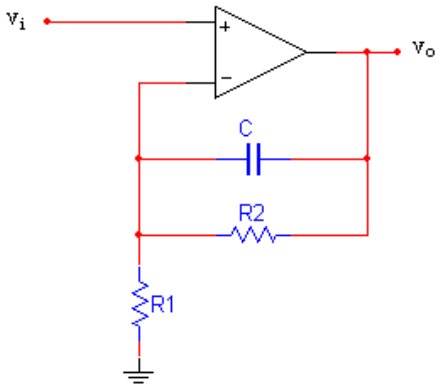
1) Con riferimento al circuito in fig.



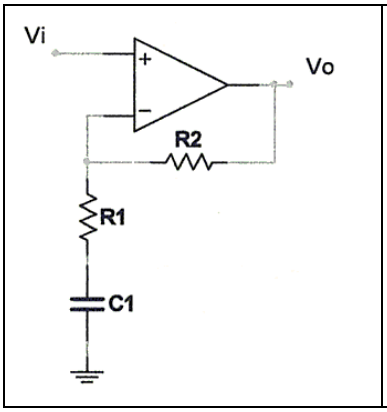
$R_1 = 2{,}5\Omega$

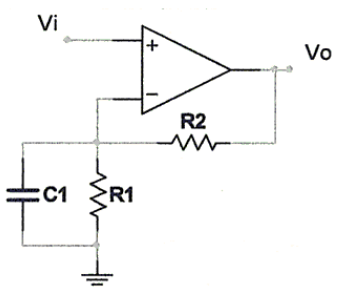
$R_2 = 10\Omega$

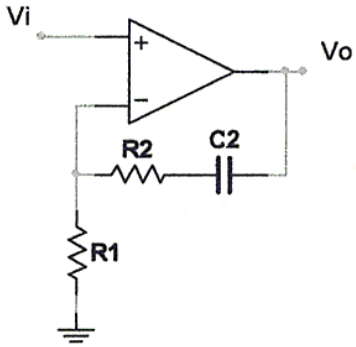
$C = 0{,}5F$

a) determinare la funzione di trasferimento
b) determinare gli eventuali poli e zeri
c) scrivere l'espressione del modulo e della fase della funzione di trasferimento
d) determinare il guadagno a frequenze molto basse ($f \to 0$) e molto alte ($f \to \infty$)
e) rappresentare il diagramma di Bode del modulo della f.d.t.

2) Determinare la funzione di trasferimento dei seguenti circuiti e rappresentare i relativi diagrammi di Bode (con i valori numerici assegnati)







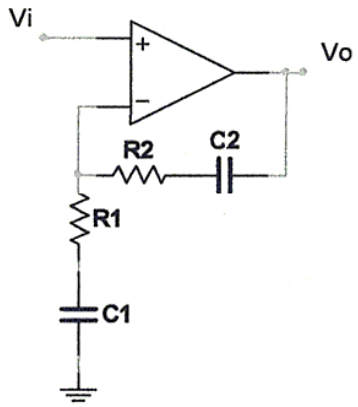
Dati

$R_1 = 1\Omega$

$R_2 = 4\Omega$

$C_1 = 1F$

$C_2 = 1F$



Dati 1

$R_1 = 1\Omega$

$R_2 = 1\Omega$

$C_1 = 1{,}25F$

$C_2 = 5F$

Dati 2

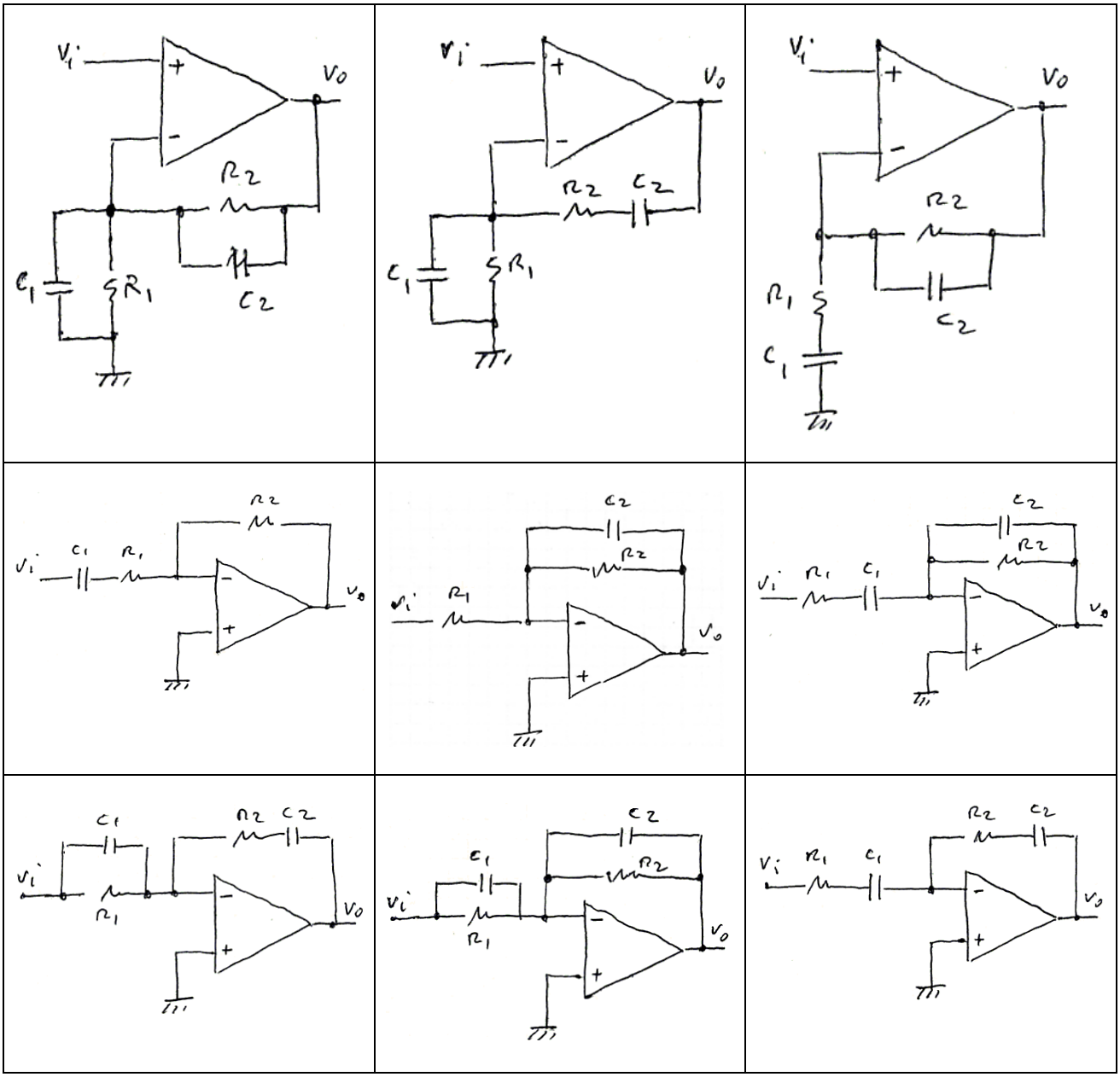
$R_1 = 1\Omega$

$R_2 = 1\Omega$

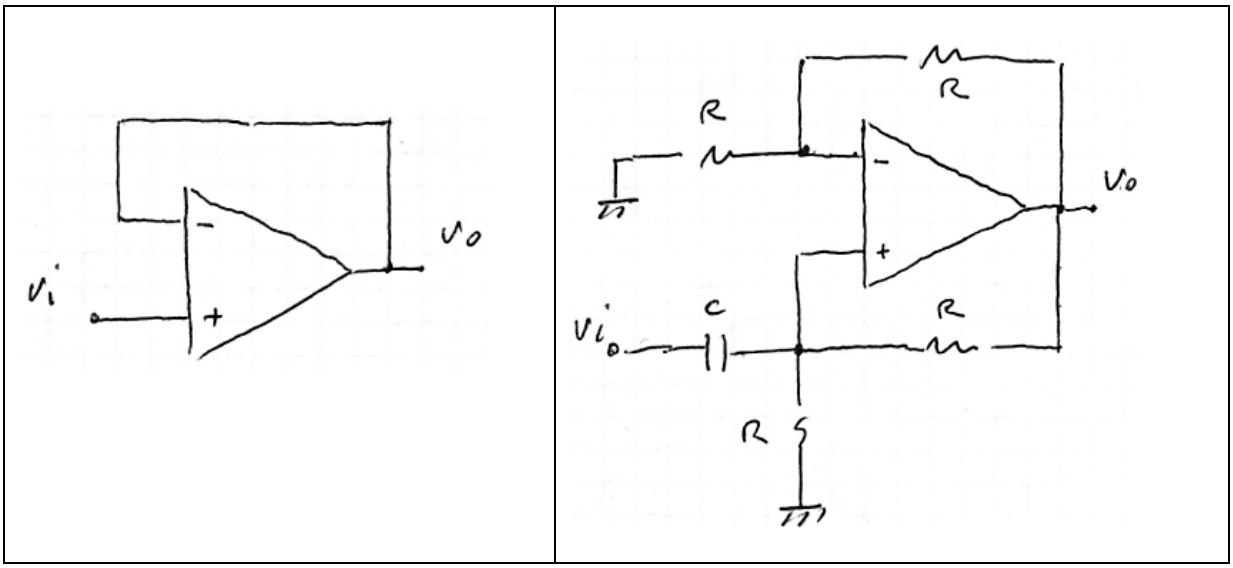
$C_1 = 5F$

$C_2 = 1{,}25F$

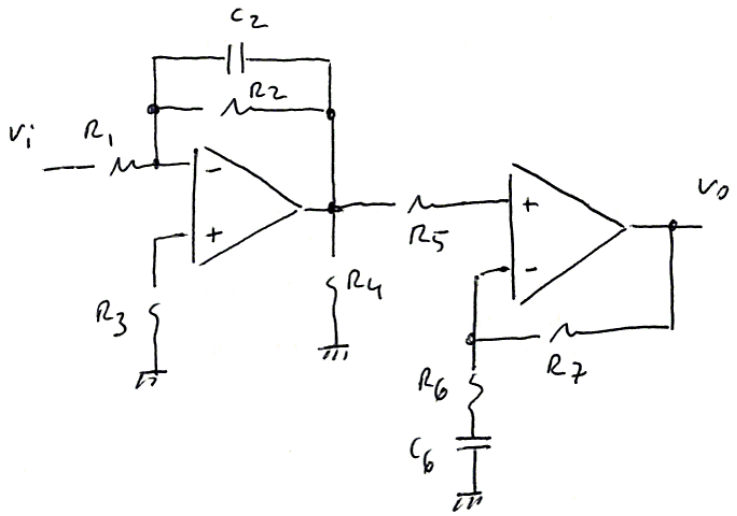
3) Determinare la funzione di trasferimento dei seguenti circuiti e rappresentare i relativi diagrammi di Bode (senza valori numerici)



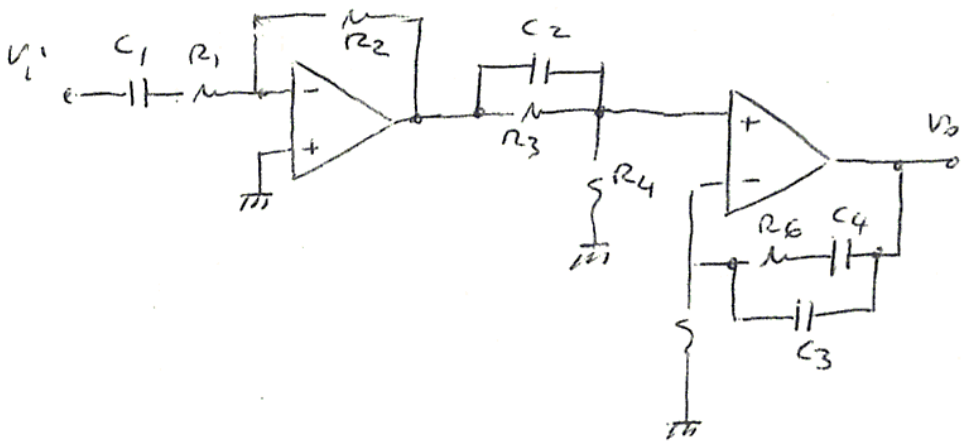
4) Determinare la funzione di trasferimento dei seguenti circuiti



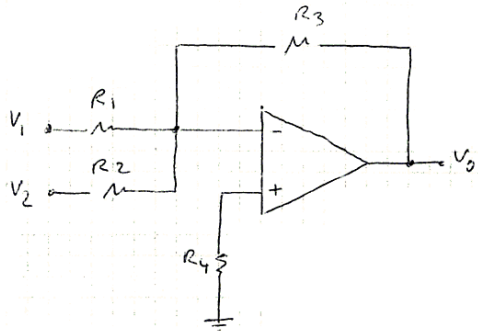
5) Determinare la funzione di trasferimento dei seguenti circuiti e rappresentare i relativi diagrammi di Bode



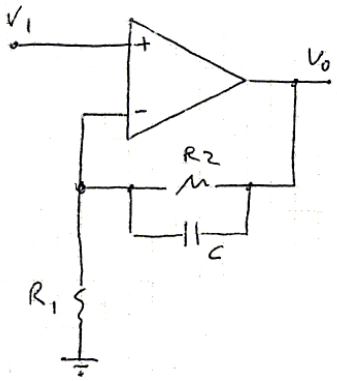
$R_1 = 2\,\Omega$
$R_2 = 10\,\Omega$
$C_2 = 25\,mF$
$R_3 = 1\,\Omega$
$R_4 = 10\,\Omega$
$R_5 = 10\,\Omega$
$R_6 = 50\,\Omega$
$C_6 = 10\,mF$
$R_7 = 150\,\Omega$



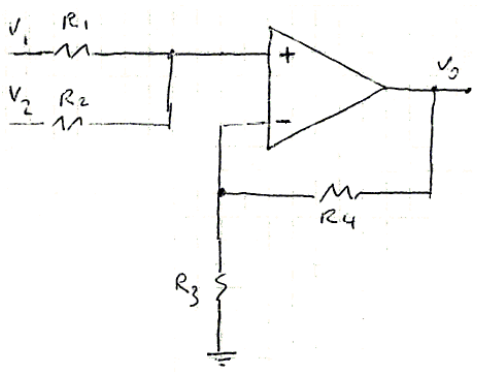
6) Determinare la relazione ingresso/i – uscita dei seguenti circuiti



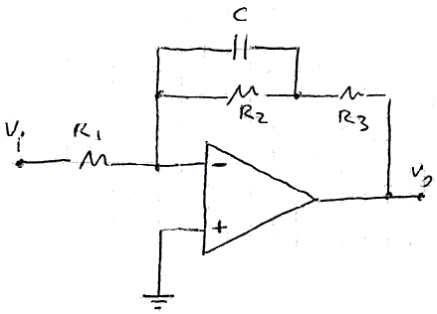
$R_1 = 1K\Omega$
$R_2 = 2K\Omega$
$R_3 = 3K\Omega$
$R_4 = 4K\Omega$



$R_1 = 1\Omega$
$R_2 = 4\Omega$
$C = 0,25\,F$

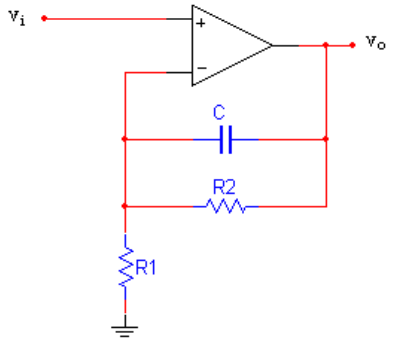


$R_1 = 1K\Omega$
$R_2 = 2K\Omega$
$R_3 = 1K\Omega$
$R_4 = 2K\Omega$



$R_1 = 1\Omega$
$R_2 = 4\Omega$
$R_3 = 1\Omega$
$C = 0,25\,F$

1)



$R_1 = 2{,}5\Omega$

$R_2 = 10\Omega$

$C = 0{,}5F$

1a) $F(s) = \dfrac{v_0}{v_i} = \dfrac{Z_1 + Z_2}{Z_1}$ con $\begin{cases} Z_2 = \dfrac{R_2 \cdot \dfrac{1}{sC}}{R_2 + \dfrac{1}{sC}} = \dfrac{R_2}{1 + sR_2C} \\ Z_1 = R_1 \end{cases}$

$$F(s) = \frac{R_1 + \dfrac{R_2}{1 + sR_2C}}{R_1} = \frac{R_1 + R_2}{R_1} \cdot \frac{1 + sR_pC}{1 + sR_2C} \qquad \text{con} \qquad R_p = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Sostituendo

$$F(s) = 5 \cdot \frac{1 + s}{1 + 5s} \qquad F(s) = \frac{N(s)}{D(s)}$$

1b) zeri: $N(s) = 0$ $\quad 1 + s = 0$ $\quad s = -1$ $\quad (z_1)$

poli: $D(s) = 0$ $\quad 1 + 5s = 0$ $\quad s = -0{,}2$ $\quad (p_1)$

1c) $s \to j\omega$ $\qquad F(j\omega) = 5 \cdot \dfrac{1 + j\omega}{1 + j5\omega}$

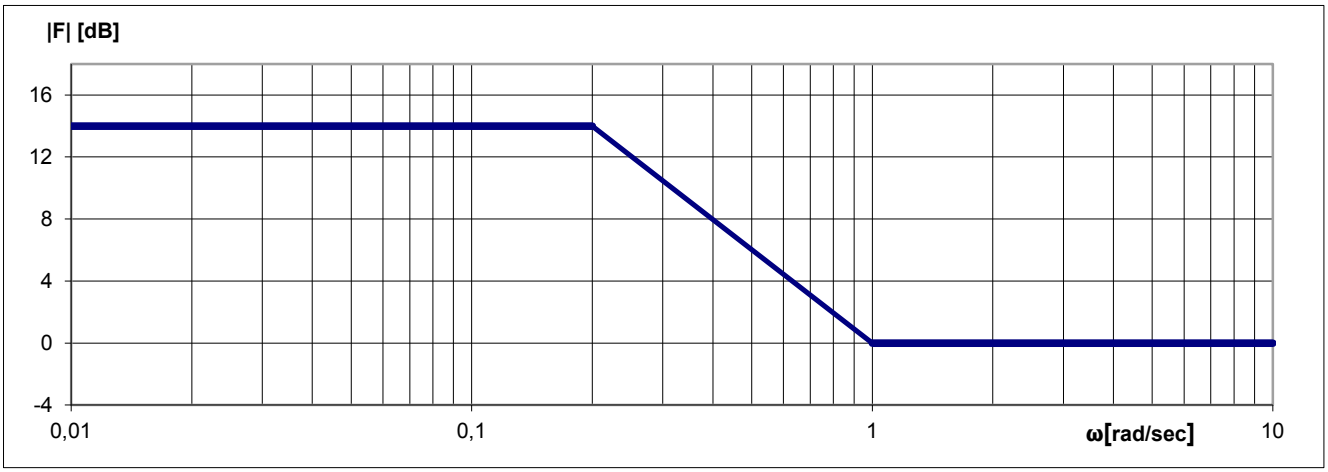
$$|F(j\omega)| = 5 \cdot \frac{\sqrt{1 + \omega^2}}{\sqrt{1 + (5\omega)^2}}$$

$$\varphi[F(j\omega)] = \arctan(\omega) - \arctan(5\omega)$$

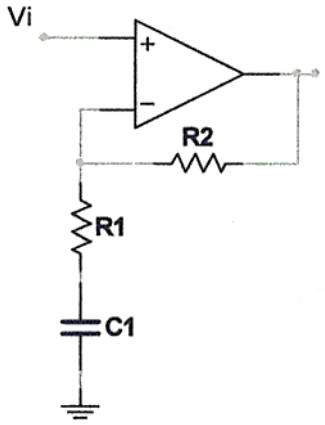
1d) alle freq. molto basse il cond. è un circuito aperto $\rightarrow$ $|F(j\omega)|_0 = 5 \quad (14\ dB)$

alle freq. molto alte il cond. è un corto circuito $\rightarrow$ $v_0 = v^- = v_i \rightarrow |F(j\omega)|_\infty = 1 \quad (0\ dB)$

1e)



2)



$R_1 = 1\Omega$

$R_2 = 4\Omega$

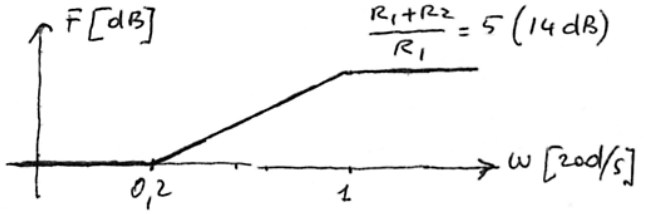
$C_1 = 1F$

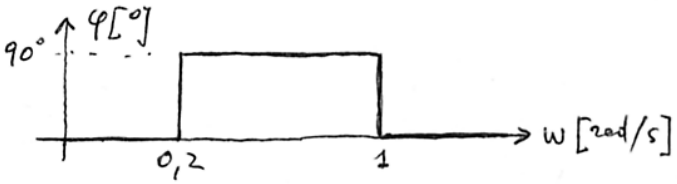
$$F(s) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1} \qquad Z_1 = \frac{1 + sR_1C_1}{sC_1} \qquad Z_2 = R_2$$

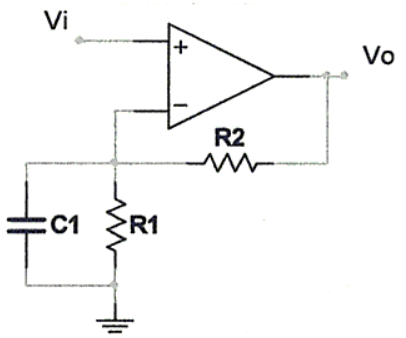
$$F(s) = \frac{\frac{1+sR_1C_1}{sC_1} + R_2}{\frac{1+sR_1C_1}{sC_1}} = \frac{1 + s(R_1+R_2)C_1}{1 + sR_1C_1}$$

SOSTIT.

$$F(s) = \frac{1+5s}{1+s} = \frac{1+\frac{s}{0,2}}{1+s}$$







$R_1 = 1\Omega$

$R_2 = 4\Omega$

$C_1 = 1F$

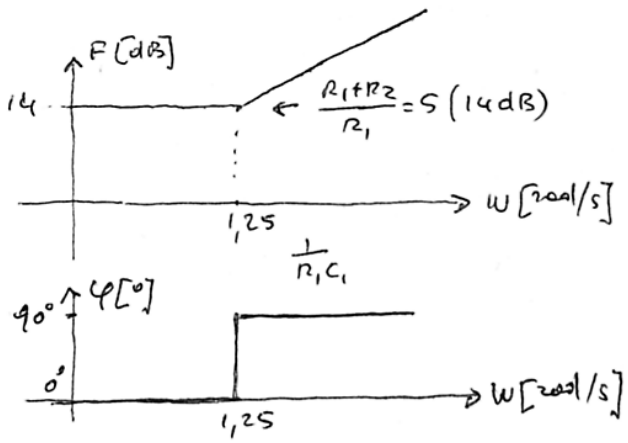
$$F(s) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1} \qquad Z_1 = \frac{R_1}{1 + sR_1C_1} \qquad Z_2 = R_2$$

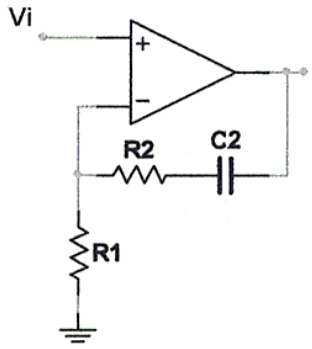
$$F(s) = \frac{\frac{R_1}{1+sR_1C_1} + R_2}{\frac{R_1}{1+sR_1C_1}} = \frac{R_1 + R_2 + sR_1R_2C_1}{R_1} =$$

$$= \frac{R_1+R_2}{R_1}\left(1 + sR_pC_1\right) \quad \text{con } R_p = R_1 \| R_2 = \frac{R_1R_2}{R_1+R_2} \quad R_p = \frac{4}{5} = 0{,}8\,\Omega$$

SOSTIT.

$$F(s) = 5\left(1 + 0{,}85\right) = 5\left(1 + \frac{s}{1{,}25}\right)$$





$R_1 = 1\Omega$

$R_2 = 4\Omega$

$C_2 = 1F$

$$F(S) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}$$

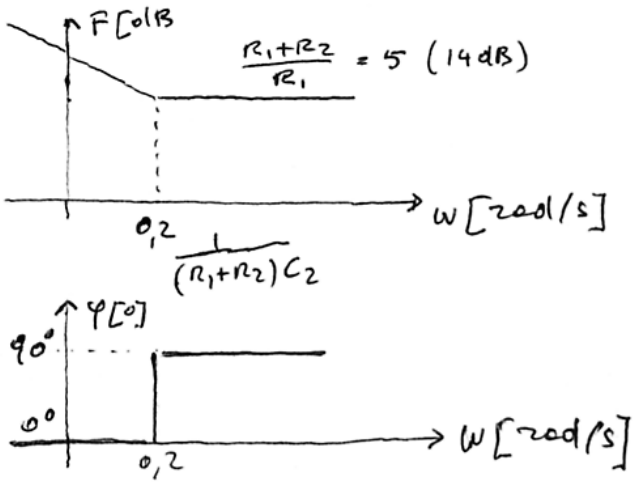
$$Z_1 = R_1$$

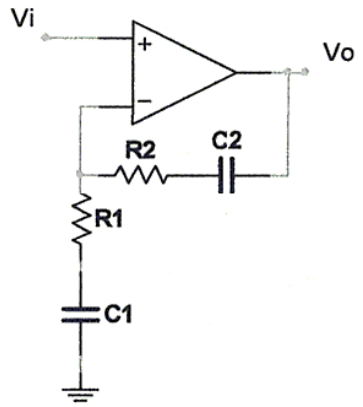
$$Z_2 = \frac{1 + S R_2 C_2}{S C_2}$$

$$F(S) = \frac{R_1 + \dfrac{1 + S R_2 C_2}{S C_2}}{R_1} = \frac{1 + S(R_1 + R_2) C_2}{S R_1 C_2}$$

SOSTIT.

$$F(S) = \frac{1 + 5S}{S}$$





	Dati 1	Dati 2
	$R_1 = 1\Omega$	$R_1 = 1\Omega$
	$R_2 = 1\Omega$	$R_2 = 1\Omega$
	$C_1 = 1{,}25F$	$C_1 = 5F$
	$C_2 = 5F$	$C_2 = 1{,}25F$

$$F(s) = \frac{U_o}{U_i} = \frac{Z_1 + Z_2}{Z_1} \qquad Z_1 = \frac{1 + sR_1C_1}{sC_1}$$

$$Z_2 = \frac{1 + sR_2C_2}{sC_2}$$

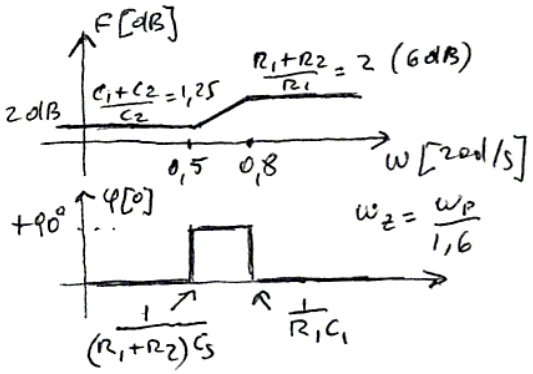
$$F(s) = \frac{\frac{1+sR_1C_1}{sC_1} + \frac{1+sR_2C_2}{sC_2}}{\frac{1+sR_1C_1}{sC_1}} = \frac{s^2[R_1C_1C_2 + R_2C_1C_2] + s(C_1+C_2)}{sC_2(1+sR_1C_1)} =$$

$$= \frac{C_1+C_2}{C_2} \cdot \frac{1 + s(R_1+R_2)C_s}{1+sR_1C_1} \qquad \text{con} \quad C_s = \frac{C_1C_2}{C_1+C_2} = 1F$$

SOSTITUENDO (DATI 1)

$$F(s) = 1{,}25 \frac{1+2s}{1+1{,}25s} = 1{,}25 \frac{1+\frac{s}{0{,}5}}{1+\frac{s}{0{,}8}}$$

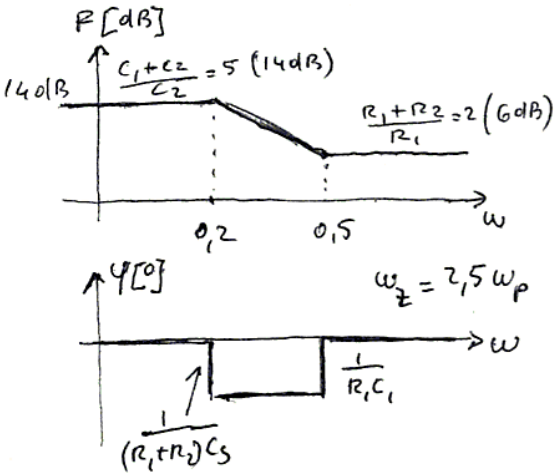
$\omega \to \infty \quad F(\omega) = \frac{C_1+C_2}{C_2} \cdot \frac{C_1C_2}{C_1+C_2} \cdot \frac{R_1+R_2}{C_1R_1} = \frac{R_1+R_2}{R_1} = 2$



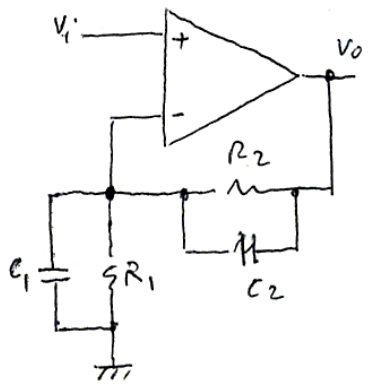
SOSTITUENDO (DATI 2)

$$F(s) = 5 \frac{1+2s}{1+5s} = 5 \frac{1+\frac{s}{0{,}5}}{1+\frac{s}{0{,}2}}$$

$\omega \to \infty \quad F(\omega) = 2$



3)



$$F(s) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1} \qquad Z_1 = \frac{R_1}{1+sR_1C_1} \qquad Z_2 = \frac{R_2}{1+sR_2C_2}$$

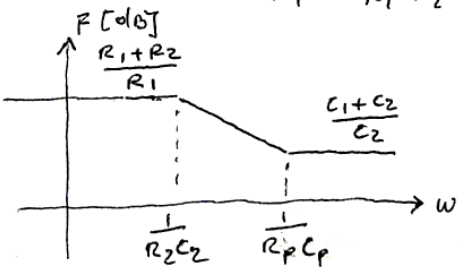
$$F(s) = \frac{\frac{R_1}{1+sR_1C_1} + \frac{R_2}{1+sR_2C_2}}{\frac{R_1}{1+sR_1C_1}} =$$

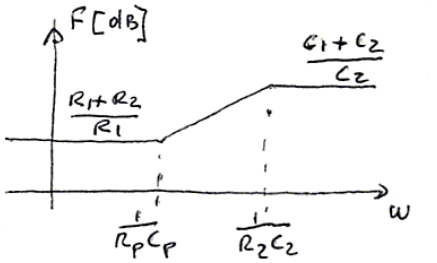
$$= \frac{R_1 + R_2 + s\left(R_1R_2C_1 + R_1R_2C_2\right)}{R_1\left(1+sR_2C_2\right)} =$$

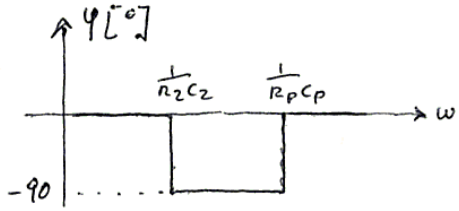
$$= \frac{R_1+R_2}{R_1} \cdot \frac{1 + sR_p(C_1+C_2)}{1+sR_2C_2} \qquad R_p = \frac{R_1R_2}{R_1+R_2} \qquad C_p = C_1+C_2$$

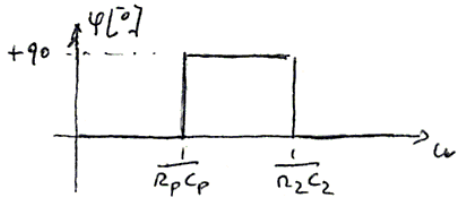
$\omega \to 0 \qquad F(\omega) = \frac{R_1+R_2}{R_1}$

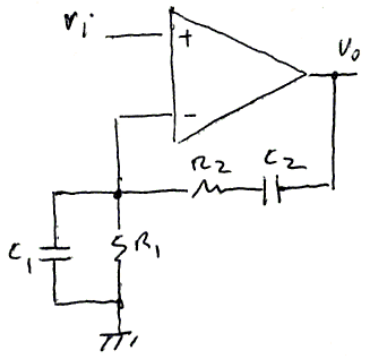
$\omega \to \infty \qquad F(\omega) = \frac{R_1+R_2}{R_1} \cdot \frac{R_1R_2}{R_1+R_2} \cdot \frac{C_1+C_2}{R_2C_2} = \frac{C_1+C_2}{C_2}$











$$F(s) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}$$

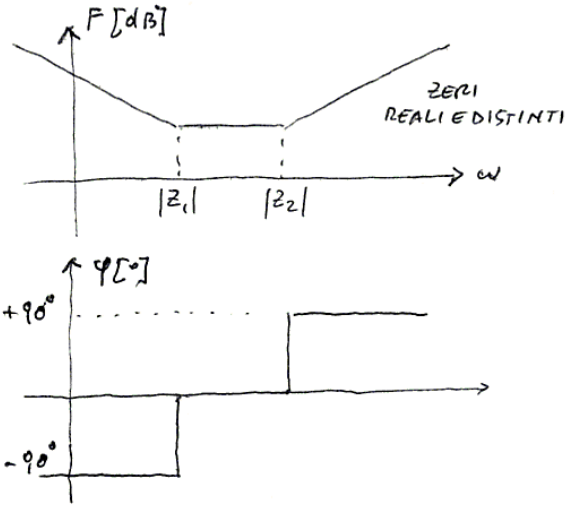
$$Z_1 = \frac{R_1}{1 + sR_1C_1}$$

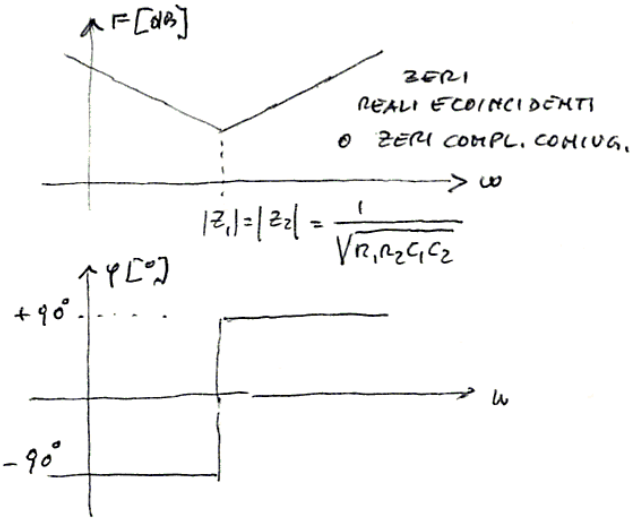
$$Z_2 = \frac{1 + sR_2C_2}{sC_2}$$

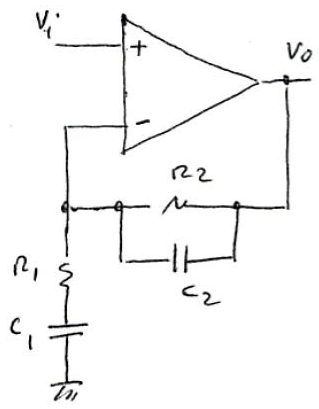
$$F(s) = \frac{\dfrac{R_1}{1+sR_1C_1} + \dfrac{1+sR_2C_2}{sC_2}}{\dfrac{R_1}{1+sR_1C_1}} =$$

$$= \frac{s^2 R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2 + R_1C_2) + 1}{sR_1C_2} =$$

$$= \frac{R_1R_2C_1C_2\,(s - z_1)(s - z_2)}{R_1C_2 \quad s} = \frac{R_2C_1\,(s - z_1)(s - z_2)}{s}$$







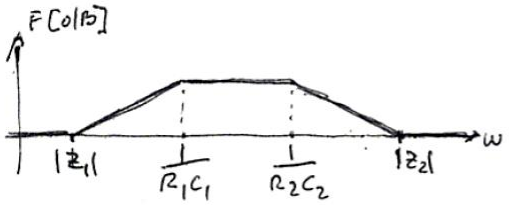
$$F(s) = \frac{V_o}{V_i} = \frac{Z_1 + Z_2}{Z_1}$$

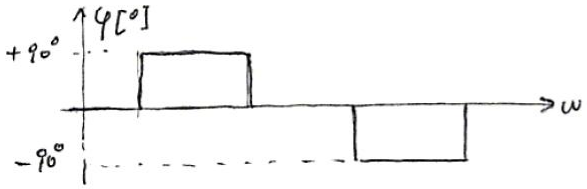
$$Z_1 = \frac{1 + sR_1C_1}{sC_1}$$

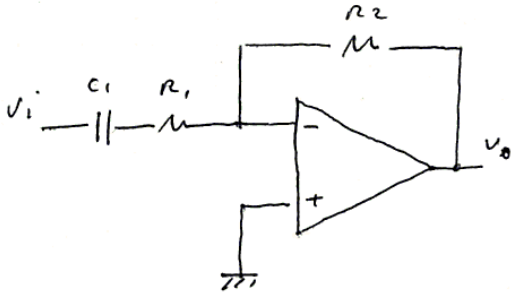
$$Z_2 = \frac{R_2}{1 + sR_2C_2}$$

$$F(s) = \frac{\dfrac{1 + sR_1C_1}{sC_1} + \dfrac{R_2}{1 + sR_2C_2}}{\dfrac{1 + sR_1C_1}{sC_1}} =$$

$$= \frac{s^2 R_1R_2C_1C_2 + s(R_1C_1 + R_2C_2 + R_2C_1) + 1}{(1 + sR_1C_1)(1 + sR_2C_2)}$$





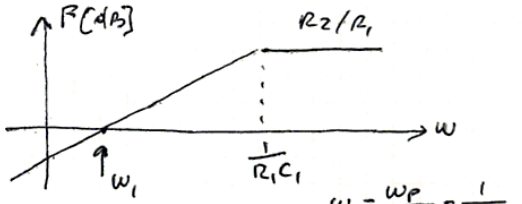


$$F(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

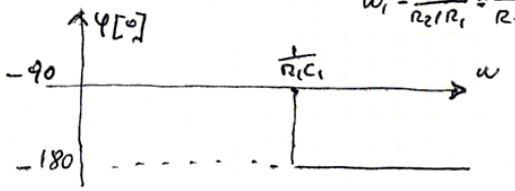
$$Z_1 = \frac{1+sR_1C_1}{sC_1}$$

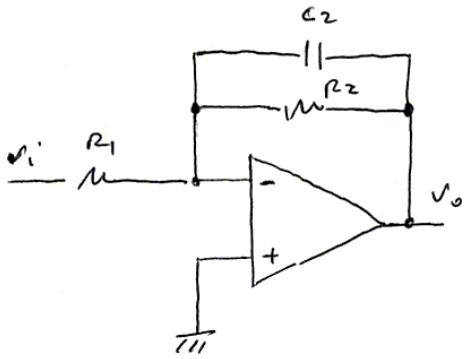
$$Z_2 = R_2$$

$$F(s) = -\frac{R_2}{\frac{1+sR_1C_1}{sC_1}} = -\frac{sR_2C_1}{1+sR_1C_1}$$



$$\omega_1 = \frac{\omega_p}{R_2/R_1} = \frac{1}{R_2C_1}$$



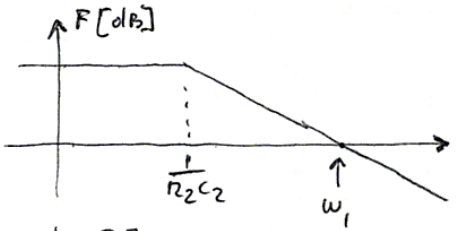


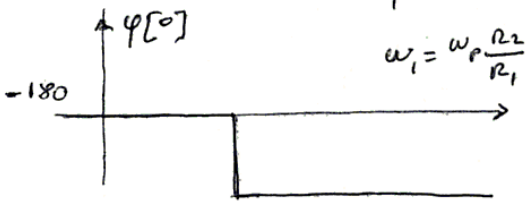
$$F(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

$$Z_1 = R_1$$

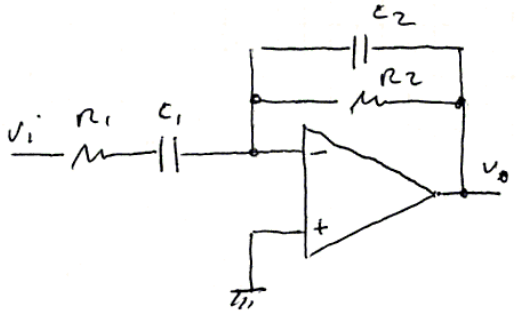
$$Z_2 = \frac{R_2}{1+sR_2C_2}$$

$$F(s) = -\frac{\frac{R_2}{1+sR_2C_2}}{R_1} = -\frac{R_2}{R_1}\cdot\frac{1}{1+sR_2C_2}$$





$$\omega_1 = \omega_p\frac{R_2}{R_1} = \frac{1}{R_1C_2}$$

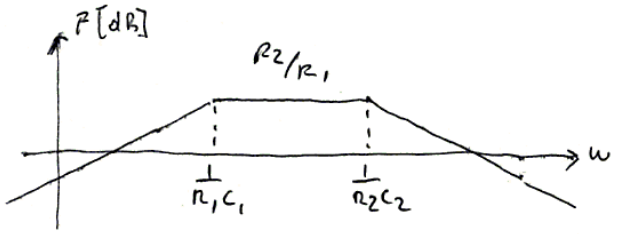


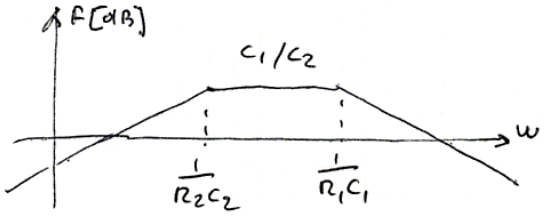
$$F(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

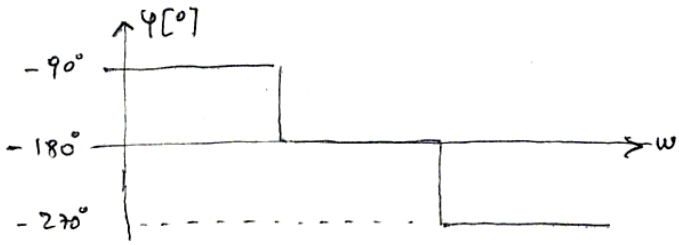
$$Z_1 = \frac{1+sR_1C_1}{sC_1}$$

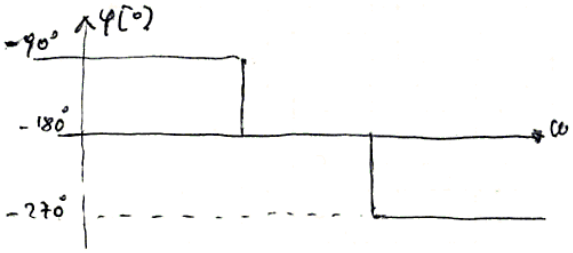
$$Z_2 = \frac{R_2}{1+sR_2C_2}$$

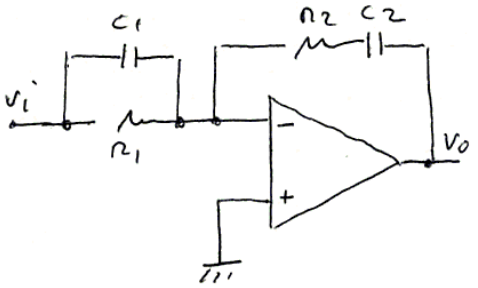
$$F(s) = -\frac{\frac{R_2}{1+sR_2C_2}}{\frac{1+sR_1C_1}{sC_1}} = -\frac{sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)}$$





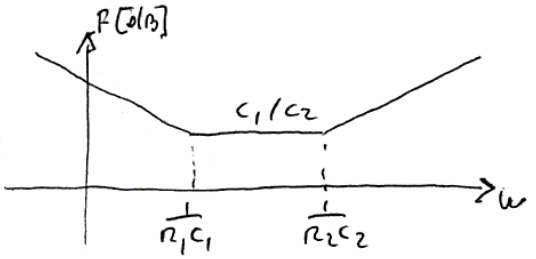


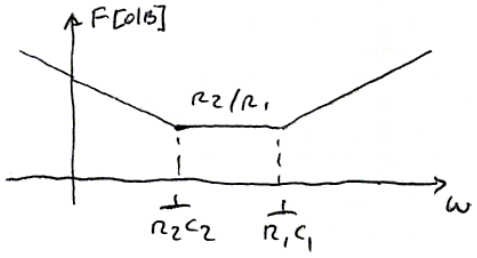


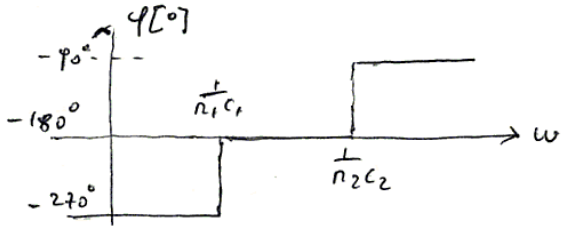


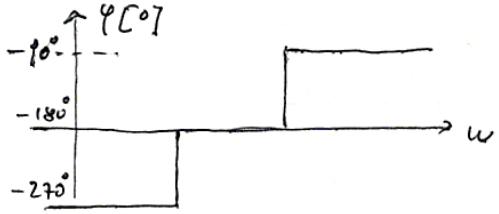
$$F(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1} \qquad Z_1 = \frac{R_1}{1+sR_1C_1} \qquad Z_2 = \frac{1+sR_2C_2}{sC_2}$$

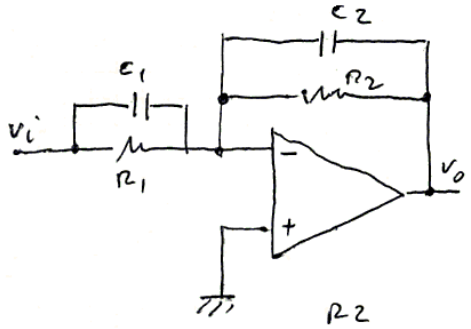
$$F(s) = -\frac{\frac{1+sR_2C_2}{sC_2}}{\frac{R_1}{1+sR_1C_1}} = -\frac{(1+sR_2C_2)(1+sR_1C_1)}{sR_1C_2}$$









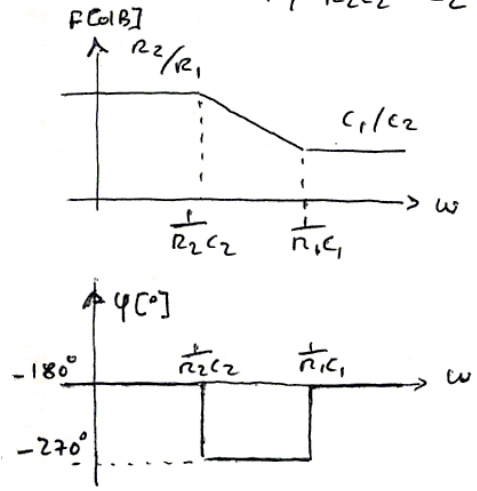


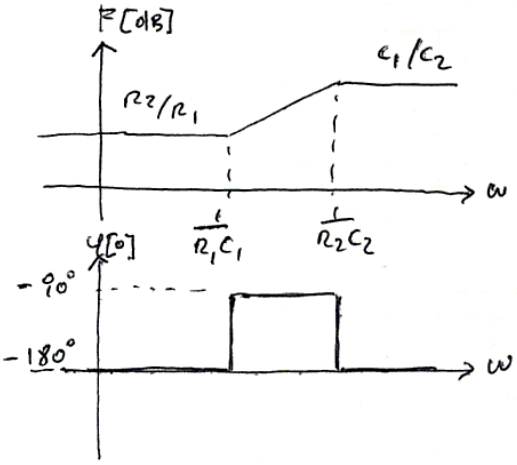
$$F(s) = \frac{V_0}{V_i} = -\frac{Z_2}{Z_1} \qquad Z_1 = \frac{R_1}{1+sR_1C_1} \qquad Z_2 = \frac{R_2}{1+sR_2C_2}$$

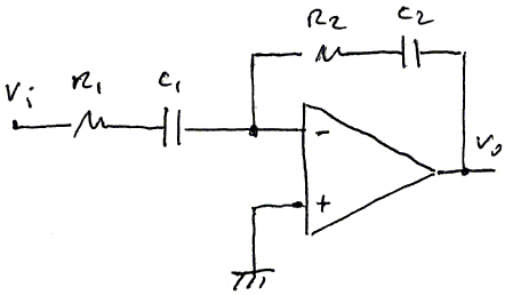
$$F(s) = -\frac{\frac{R_2}{1+sR_2C_2}}{\frac{R_1}{1+sR_1C_1}} = -\frac{R_2}{R_1}\,\frac{1+sR_1C_1}{1+sR_2C_2}$$

$\omega \to 0 \quad F(\omega) = \frac{R_2}{R_1}$

$\omega \to \infty \quad F(\omega) = \frac{R_2}{R_1}\cdot\frac{R_1C_1}{R_2C_2} = \frac{C_1}{C_2}$







$$F(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}$$

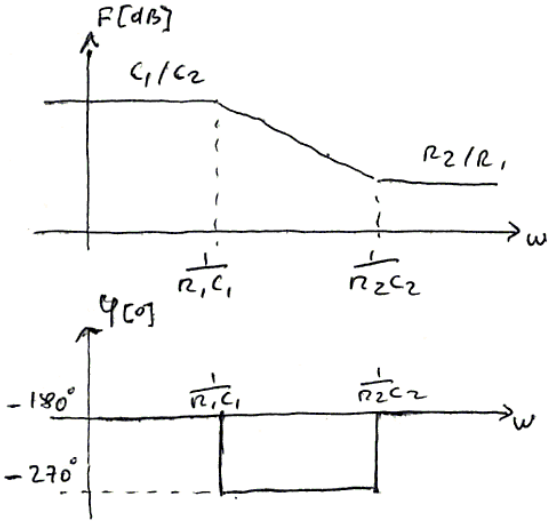
$$Z_1 = \frac{1+sR_1C_1}{sC_1}$$

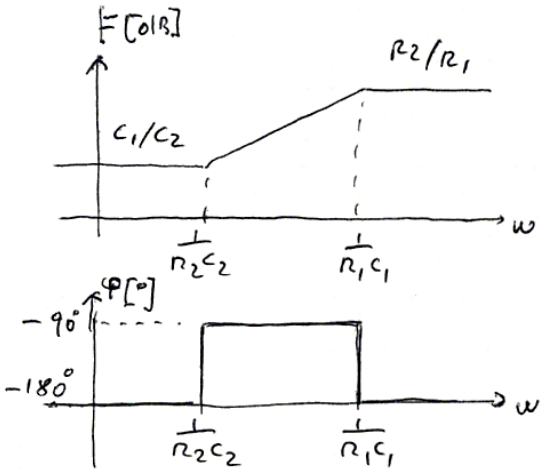
$$Z_2 = \frac{1+sR_2C_2}{sC_2}$$

$$F(s) = -\frac{\frac{1+sR_2C_2}{sC_2}}{\frac{1+sR_1C_1}{sC_1}} = -\frac{C_1}{C_2}\,\frac{1+sR_2C_2}{1+sR_1C_1}$$

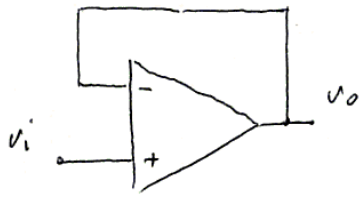
$\omega \to 0 \qquad F(\omega) = \frac{C_1}{C_2}$

$\omega \to \infty \qquad F(\omega) = \frac{C_1}{C_2} \cdot \frac{R_2C_2}{R_1C_1} = \frac{R_2}{R_1}$

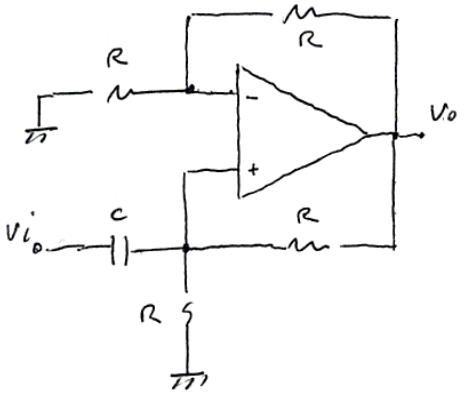


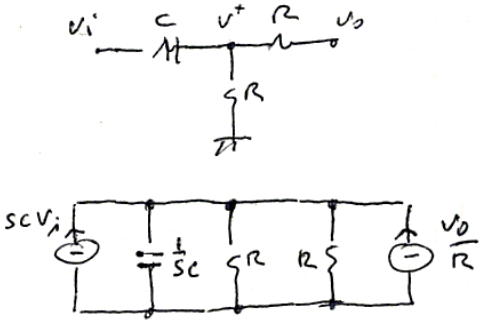


4)



$V_o = V^- = V^+ = V_i$ $\qquad F = \frac{V_o}{V_i} = 1$





$V^- = V_o/2$

$$V^+ = \left(sCV_i + \frac{V_o}{R}\right) \frac{\frac{R}{2}\cdot\frac{1}{sC}}{\frac{R}{2}+\frac{1}{sC}} = \left(sCV_i + \frac{V_o}{R}\right)\cdot\frac{R}{2+sRC}$$

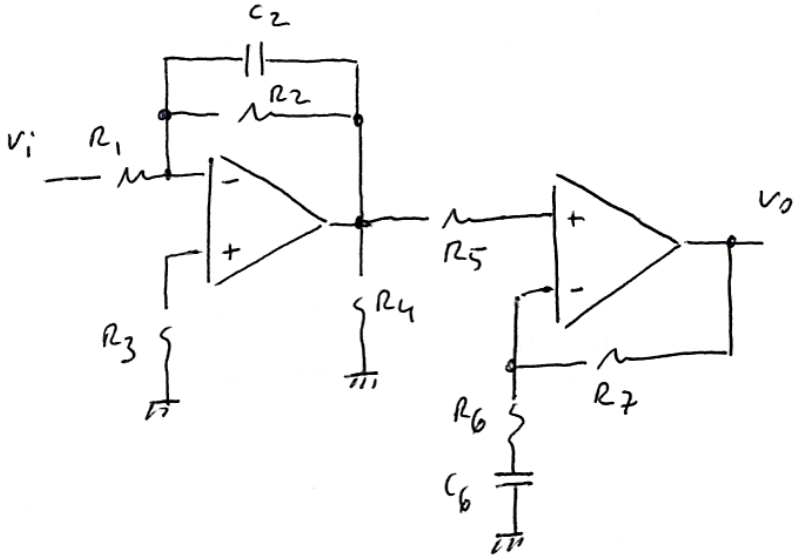
$V^+ = V^- \Rightarrow$

$$V_i \frac{sRC}{2+sRC} + \frac{V_o}{2+sRC} = \frac{V_o}{2} \quad \rightarrow \quad \frac{V_o}{2} - \frac{V_o}{2+sRC} = \frac{V_i\, sRC}{2+sRC}$$

$$2V_o + sRCV_o - 2V_o = 2V_i\, sRC$$

$$V_o = 2V_i \qquad F = \frac{V_o}{V_i} = 2$$

5)



$R_1 = 2\Omega$
$R_2 = 10\Omega$
$C_2 = 25mF$
$R_3 = 1\Omega$
$R_4 = 10\Omega$
$R_5 = 10\Omega$
$R_6 = 50\Omega$
$C_6 = 10mF$
$R_7 = 150\Omega$

$$F(s) = \frac{V_o}{V_i} = \frac{V_o}{V^+} \cdot \frac{V^+}{V_{o1}} \cdot \frac{V_{o1}}{V_i}$$

$$= \frac{Z_6 + Z_7}{Z_6} \cdot 1 \cdot \left(-\frac{Z_2}{Z_1}\right)$$

$$\frac{V^+}{V_{o1}} = 1$$

$$Z_6 = \frac{1 + sR_6C_6}{sC_6}$$

$$Z_7 = R_7$$

$$Z_2 = \frac{R_2}{1 + sR_2C_2}$$

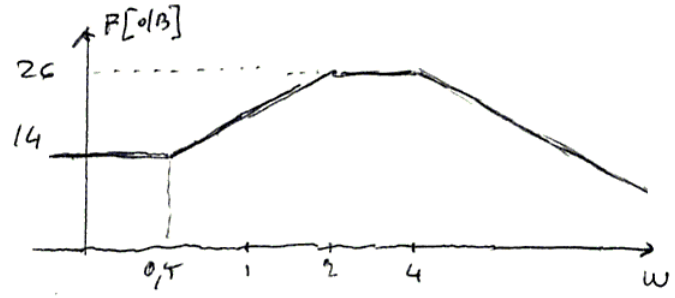
$$Z_1 = R_1$$

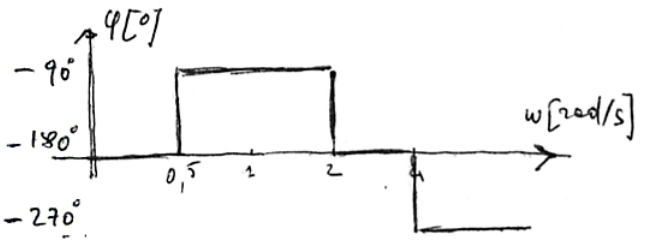
$$F(s) = -\frac{R_2}{R_1} \cdot \frac{1}{1 + sR_2C_2} \cdot \frac{1 + s(R_6 + R_7)C_6}{1 + sR_6C_6}$$

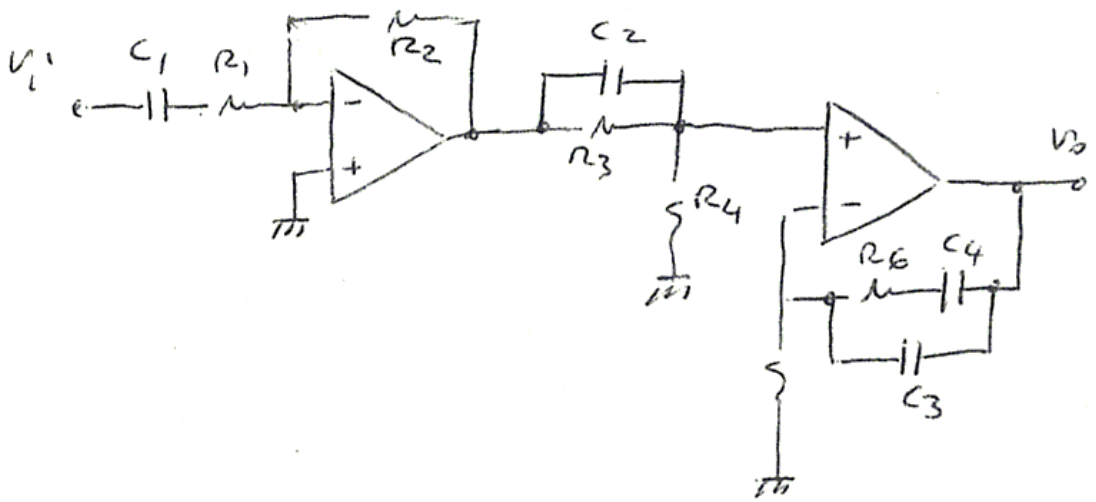
SOSTITUENDO

$$F(s) = -5 \frac{1}{1 + \frac{1}{4}s} \cdot \frac{1 + 2s}{1 + \frac{1}{2}s} =$$

$$= -5 \frac{1}{1 + \frac{s}{4}} \cdot \frac{1 + \frac{s}{0,5}}{1 + \frac{s}{2}}$$







$$F(s) = \frac{V_o}{V_i} = \frac{V_o}{V^+} \cdot \frac{V^+}{V_{o1}} \cdot \frac{V_{o1}}{V_i}$$

CONF. NON INV. $\quad \frac{V_o}{V^+} = \frac{Z_5 + Z_6}{Z_5}$

$Z_5 = R_5$

$$Z_6 = \frac{\left(R_6 + \frac{1}{sC_4}\right) \cdot \frac{1}{sC_3}}{R_6 + \frac{1}{sC_4} + \frac{1}{sC_3}} = \frac{\frac{1+sR_6C_4}{sC_4} \cdot \frac{1}{sC_3}}{\frac{1+sR_6C_4}{sC_4} + \frac{1}{sC_3}} \quad *$$

PART. TENS. $\quad \frac{V^+}{V_{o1}} = \frac{Z_4}{Z_3 + Z_4}$

$Z_4 = R_4$

$$Z_3 = \frac{R_3}{1 + sR_3C_2}$$

CONF. INV. $\quad \frac{V_{o1}}{V_i} = -\frac{Z_2}{Z_1}$

$Z_2 = R_2$

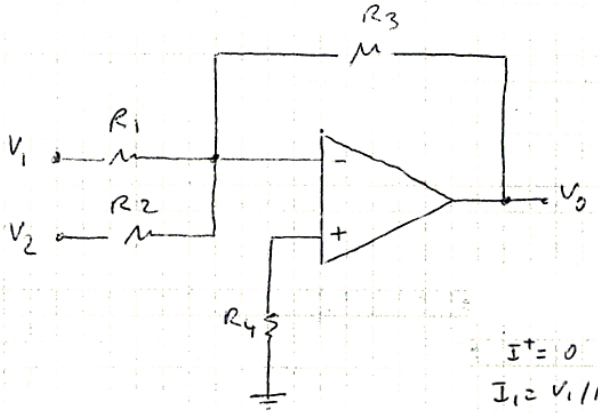
$$Z_1 = \frac{1 + sR_1C_1}{sC_1}$$

$$* \quad Z_6 = \frac{1 + sR_6C_4}{s^2R_6C_3C_4 + s(C_3 + C_4)} = \frac{1}{s} \cdot \frac{1 + sR_6C_4}{(C_3 + C_4)\left[1 + s\frac{C_3C_4}{C_3 + C_4}R_6\right]} =$$

$$= \frac{1}{C_3 + C_4} \cdot \frac{1 + sR_6C_4}{s\left(1 + sC_sR_6\right)} \qquad \text{con } C_s = \frac{C_3C_4}{C_3 + C_4}$$

6)

RELAZIONE INGRESSO / USCITA



$R_1 = 1K\Omega$

$R_2 = 2K\Omega$

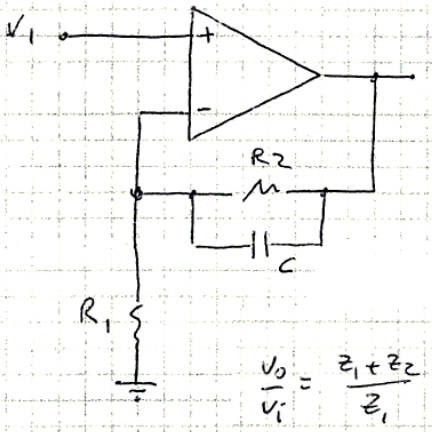
$R_3 = 3K\Omega$

$R_4 = 4K\Omega$

$I^+ = 0 \quad V^+ = 0 \quad V^- = 0$

$I_1 = V_1 / R_1 \quad I_2 = \frac{V_2}{R_2} \quad I_3 = I_1 + I_2 = \frac{V_1}{R_1} + \frac{V_2}{R_2}$

$V_0 = - R_3 I_3 = - R_3 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right) = - 3 \left(V_1 + \frac{V_2}{2} \right)$



$R_1 = 1\Omega$

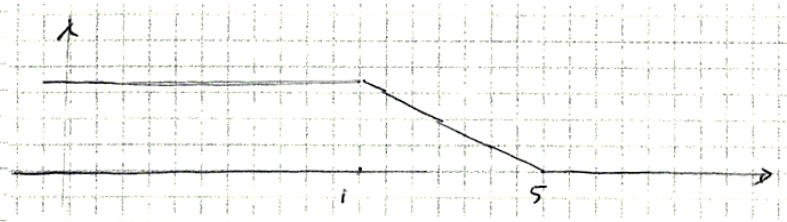
$R_2 = 4\Omega$

$C = 0,25\,F$

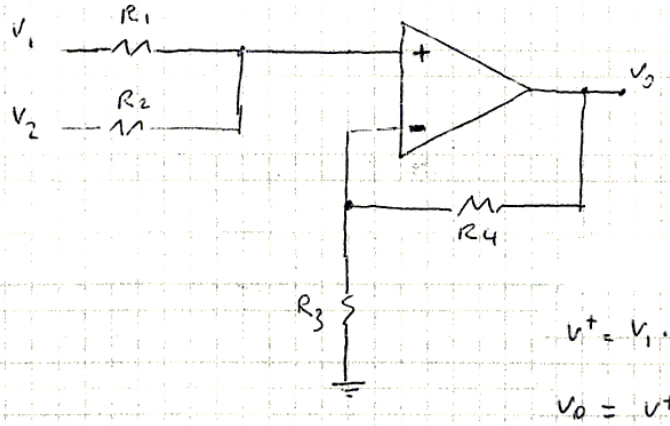
$$\frac{V_0}{V_i} = \frac{Z_1 + Z_2}{Z_1} = \frac{R_1 + \frac{R_2}{1+sR_2C}}{R_1} = \frac{\frac{R_1 + R_2 + sR_1R_2C}{1+sR_2C}}{R_1} = \frac{R_1+R_2}{R_1} \cdot \frac{1+sR_pC}{1+sR_2C}$$

$$\frac{V_0}{V_i} = 5 \; \frac{1 + s/5}{1+s}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{4}{5}$$



RELAZIONE INGRESSO/I - USCITA



$R_1 = 1K\Omega$

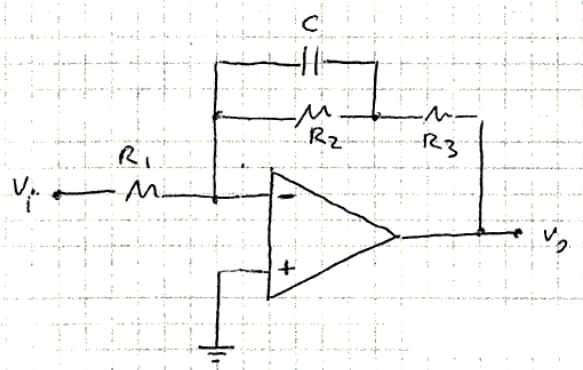
$R_2 = 2K\Omega$

$R_3 = 1K\Omega$

$R_4 = 2K\Omega$

$$V^+ = V_1 \cdot \frac{R_2}{R_1+R_2} + V_2 \cdot \frac{R_1}{R_1+R_2} = V_1 \cdot \frac{2}{3} + V_2 \cdot \frac{1}{3}$$

$$V_0 = V^+ \cdot \frac{R_3+R_4}{R_3} = V^+ \cdot \frac{3}{1} = 2V_1 + V_2$$



$R_1 = 1\Omega$

$R_2 = 4\Omega$

$R_3 = 1\Omega$

$C = 0{,}25\ F$

$$Z_2 = R_3 + \frac{R_2}{1+sR_2C} = \frac{R_2+R_3+sR_2R_3C}{1+sR_2C} = (R_2+R_3)\frac{1+sR_pC}{1+sR_2C}$$

$$Z_1 = R_1$$

$$R_p = \frac{R_2R_3}{R_2+R_3} = \frac{4}{5}$$

$$\frac{V_0}{V_i} = -\frac{Z_2}{Z_1} = -\frac{(R_2+R_3)}{R_1} \cdot \frac{1+sR_pC}{1+sR_2C} = -5 \cdot \frac{1+s/5}{1+s}$$