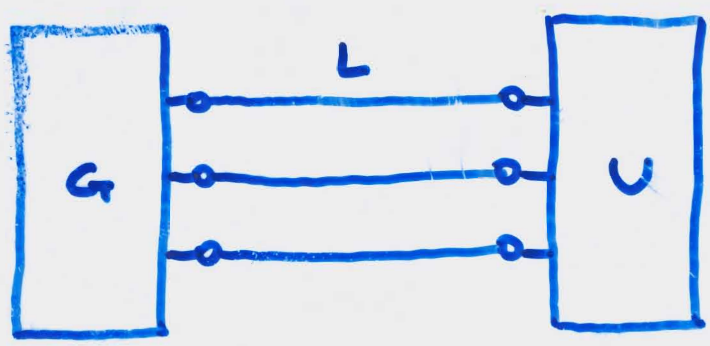
CIRCUITI TRIFASI

NELLE APPLICAZIONI DI POTENZA
È FREQUENTE TROVARE, IN REGIME P.A.S.

DISPOSITIVI A 3 MORSETTI,
LINEE A 3 FILI :
CIRCUITI TRIFASI



I CIRCUITI TRIFASI RAPPRESENTANO UNA SOLUZIONE RAZIONALE X GENERARE, TRASMETTERE, UTILIZZARE POTENZA ELETTRICA

G : GENERATORI (ALTERNATORI) COMPATTI

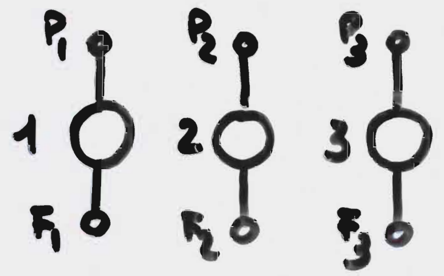
L : LINEE + VANTAGGIOSE CHE MONOFASI

U : UTILIZZATORI (MOTORI ASINCRONI) ROBUSTI
E AUTO AVVIANTI

GENERATORE TRIFASE IDEALE (DI V O I):
a

DISPOSITIVO A 3 MORSETTI

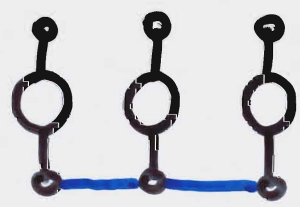
OTTENUTO DA 3 GENER. MONOFASI IDEALI



$a_1 + a_2 + a_3 = 0$

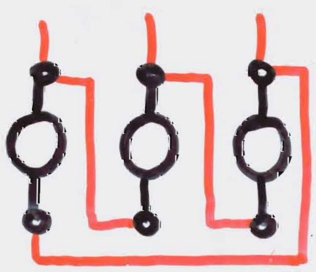
CONNESSI A

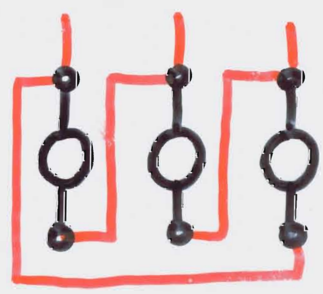
STELLA (Y)



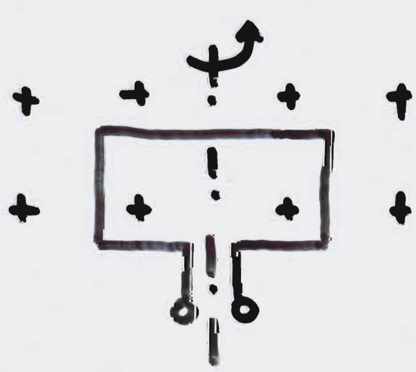
OPPURE A

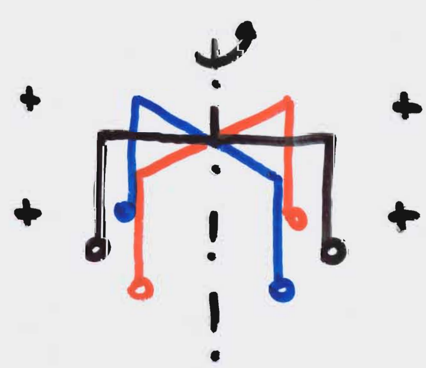
TRIANGOLO (Δ)





AD ES.





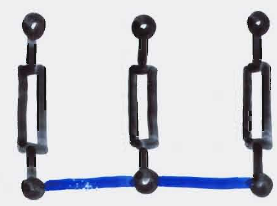
GEN. DI F.E.M. MONOFASE

GEN. DI F.E.M. TRIFASE

UTILIZZATORE TRIFASE PASSIVO

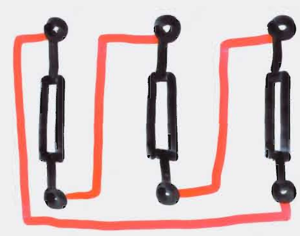
DISPOSITIVO A 3 MORSETTI
OTTENUTO DA 3 UTILIZZ. MONOFASI PASSIVI
CONNESSI A

STELLA (Y)

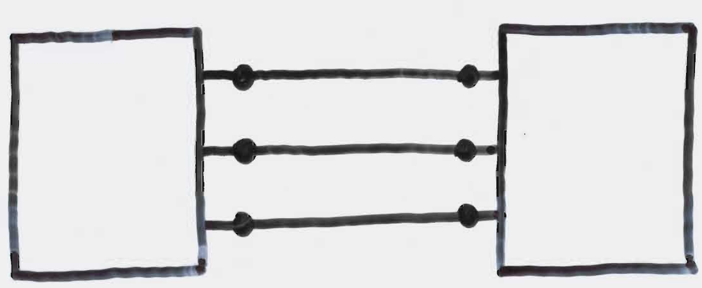


OPPURE A

TRIANGOLO (D)

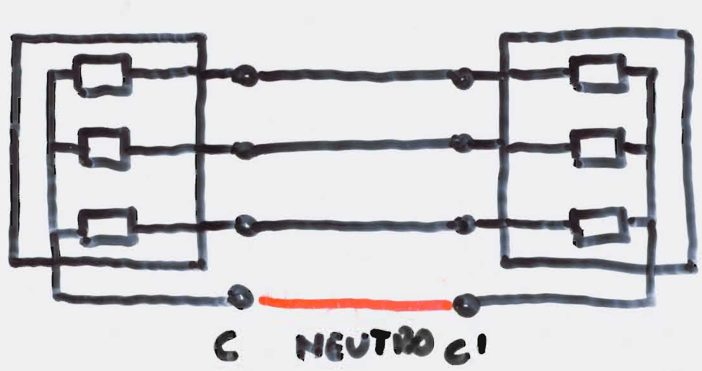


DALLA CONNESSIONE DI DISPOSITIVI A 3 MORSETTI CON LINEE A 3 FILI → CIRCUITO TRIFASE



C. TRIFASE ELEMENTARE

ESISTONO PURE DISPOSITIVI, LINEE, CIRCUITI TRIFASI A 4 FILI



C, C': CENTRI STELLA

SISTEMI TRIFASI

SISTEMA TRIFASE È

UN SISTEMA DI 3 GRANDEZZE (TENS. O CORR)

PERIODICHE ALTERNATE DI $= f$

$a_1(t)$, $a_2(t)$, $a_3(t)$ TALE CHE

$$a_1(t) + a_2(t) + a_3(t) = 0$$

UN SISTEMA TRIFASE SI DICE

SIMMETRICO (SE DI TENS.), **EQUILIBRATO** (SE DI CORR)

SE È COSTITUITO DA 3 GRANDEZZE P. A. S.

DI = FREQUENZA, = VALORE EFFICACE, = SFASAMENTO

RECIPROCO ($2\pi/3$)

IL SISTEMA È A <u>SENSO CICLICO DIRETTO</u>

SE a_2 È IN RITARDO SU a_1 E COSÌ VIA

È A <u>SENSO CICLICO INVERSO</u>

SE a_2 È IN ANTICIPO SU a_1 E COSÌ VIA

RAPPRESENTAZIONE DI UN SISTEMA TRIFASE

AD ES. SIST. TR. SIMM. O EQUIL.
S. CICLICO DIRETTO

- R. ANALITICA
  IN DOMINIO DI t

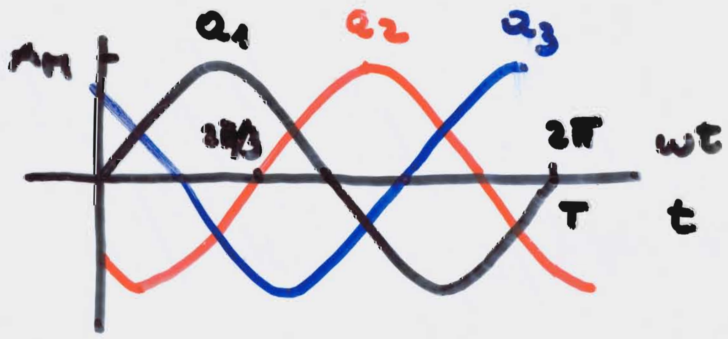
$$a_1(t) = A_M \cos(\omega t + \varphi)$$

$$a_2(t) = A_M \cos\left(\omega t + \varphi - \frac{2\pi}{3}\right)$$

$$a_3(t) = A_M \cos\left(\omega t + \varphi - \frac{4\pi}{3}\right)$$

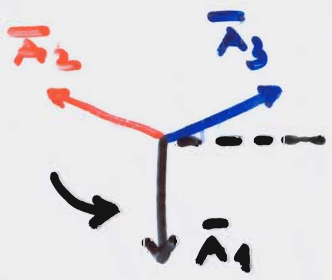
VERIFICA: $a_1 + a_2 + a_3 = 0$

- R. GRAFICA
  IN DOMINIO DI t

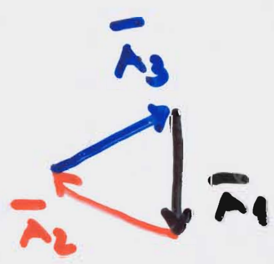


- R. GRAFICA FASORIALE

$a_1 \rightarrow \bar{A}_1$; $a_2 \rightarrow \bar{A}_2$, $a_3 \rightarrow \bar{A}_3$

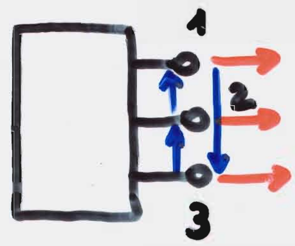


VETTORI SPICCATI DA STESSO PUNTO



VETTORI SPICCATI UNO DOPO L'ALTRO

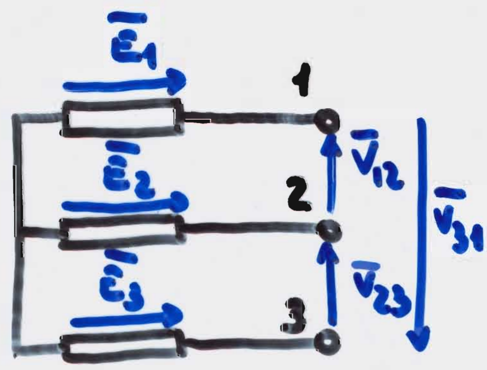
SI CONSIDERINO TENSIONI E CORRENTI AI MORSETTI DI UN DISPOSITIVO TRIFASE LINEARE IN REGIME P.A.S.



$\bar{V}_{12}, \bar{V}_{23}, \bar{V}_{31}$: TENS. DI LINEA

$\bar{I}_1, \bar{I}_2, \bar{I}_3$: CORR. DI LINEA

SE IL DISPOSITIVO È COLLEGATO A Y POSSIAMO INDIVIDUARE TENSIONI DI FASE ($\bar{E}_1, \bar{E}_2, \bar{E}_3$)



$$\bar{V}_{12} = \bar{E}_1 - \bar{E}_2$$

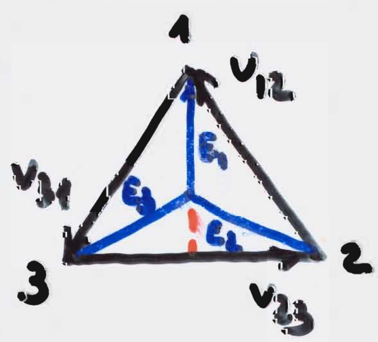
$$\bar{V}_{23} = \bar{E}_2 - \bar{E}_3$$

$$\bar{V}_{31} = \bar{E}_3 - \bar{E}_1$$

SE V SIMM E I EQUIL. $\longrightarrow$ E SIMMETRICO

$$E_1 = E_2 = E_3 = E$$

LE E POSSONO ESSERE RICAVATE DALLE V ANALITICAM. O VETTORIALMENTE

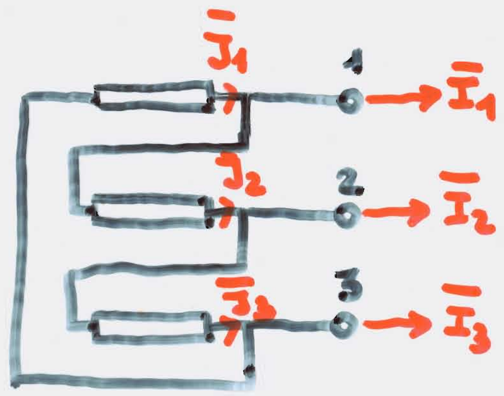


RISULTA

$$V = 2E\cos\frac{\pi}{6} = 2E\frac{\sqrt{3}}{2} = \sqrt{3}\,E$$

SE IL DISPOSITIVO È COLLEGATO A Δ

POSSIAMO INDIVIDUARE CORRENTI DI FASE ($\bar{J}_1, \bar{J}_2, \bar{J}_3$)

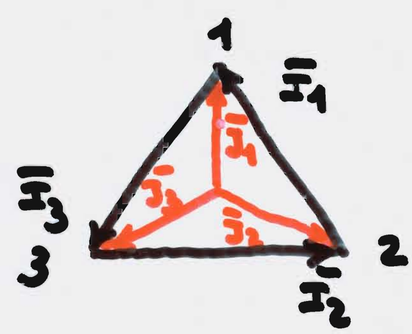


$$\bar{I}_1 = \bar{J}_1 - \bar{J}_2$$
$$\bar{I}_2 = \bar{J}_2 - \bar{J}_3$$
$$\bar{I}_3 = \bar{J}_3 - \bar{J}_1$$

SE I È EQUIL. E V È SIMM $\rightarrow$ J È EQUILIBRATO

$$J_1 = J_2 = J_3 = J$$

LE J POSSONO ESSERE RICAVATE DALLE I ANALITICAM O VETTORIALMENTE



RISULTA

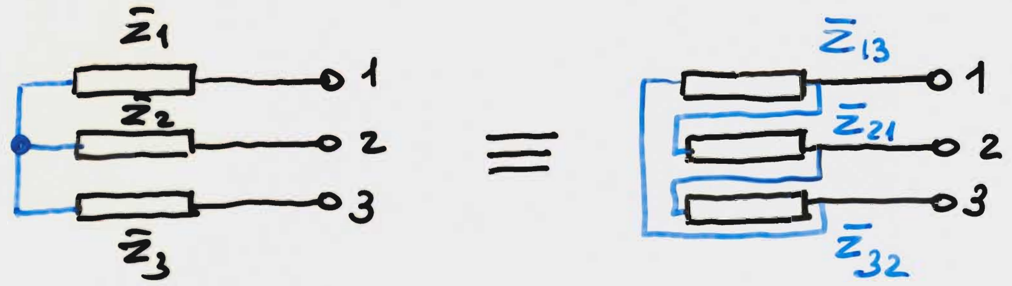
$$I = \sqrt{3}\, J$$

OSS. – NEL COLLEG. A Y SI POSSONO ANCHE INTRODURRE CORRENTI DI FASE $J = I$

– NEL COLLEG. A Δ SI POSSONO ANCHE INTRODURRE TENSIONI DI FASE $E = V$

IMPEDENZA TRIFASE

EQUIVALENZA Y-D



1-2 $\quad \bar{Z}_1 + \bar{Z}_2 = \dfrac{\bar{Z}_{21}(\bar{Z}_{13}+\bar{Z}_{32})}{\bar{Z}_{21}+\bar{Z}_{13}+\bar{Z}_{32}} = \dfrac{\bar{Z}_{21}\bar{Z}_{13}}{\Sigma} + \dfrac{\bar{Z}_{21}\bar{Z}_{32}}{\Sigma}$

2-3 $\quad \bar{Z}_2 + \bar{Z}_3 = \dfrac{\bar{Z}_{32}(\bar{Z}_{21}+\bar{Z}_{13})}{\bar{Z}_{21}+\bar{Z}_{13}+\bar{Z}_{32}} = \dfrac{\bar{Z}_{32}\bar{Z}_{21}}{\Sigma} + \dfrac{\bar{Z}_{32}\bar{Z}_{13}}{\Sigma}$

3-1 $\quad \bar{Z}_3 + \bar{Z}_1 = \dfrac{\bar{Z}_{13}(\bar{Z}_{32}+\bar{Z}_{21})}{\bar{Z}_{21}+\bar{Z}_{13}+\bar{Z}_{32}} = \dfrac{\bar{Z}_{13}\bar{Z}_{32}}{\Sigma} + \dfrac{\bar{Z}_{13}\bar{Z}_{21}}{\Sigma}$

$$\Sigma = \bar{Z}_{21}+\bar{Z}_{13}+\bar{Z}_{32}$$

PER ISPEZIONE SI HA

$$\bar{Z}_1 = \frac{\bar{Z}_{21}\bar{Z}_{13}}{\Sigma} \qquad \bar{Z}_2 = \frac{\bar{Z}_{21}\bar{Z}_{32}}{\Sigma} \qquad \bar{Z}_3 = \frac{\bar{Z}_{13}\bar{Z}_{32}}{\Sigma} \qquad (*)$$

SE $\bar{Z}_{21} = \bar{Z}_{13} = \bar{Z}_{32} = \bar{Z}$

$$\bar{Z}_1 = \frac{\bar{Z}\cdot\bar{Z}}{3\bar{Z}} = \frac{1}{3}\bar{Z} \quad ; \quad \bar{Z}_2 = \frac{1}{3}\bar{Z} \quad ; \quad \bar{Z}_3 = \frac{1}{3}\bar{Z}$$

VICEVERSA

$$\overline{Z}_{12} = \overline{Z}_1 + \overline{Z}_2 + \frac{\overline{Z}_1\,\overline{Z}_2}{\overline{Z}_3}$$

$$\overline{Z}_{23} = \overline{Z}_2 + \overline{Z}_3 + \frac{\overline{Z}_2\,\overline{Z}_3}{\overline{Z}_1}$$

$$\overline{Z}_{31} = \overline{Z}_3 + \overline{Z}_1 + \frac{\overline{Z}_3\,\overline{Z}_1}{\overline{Z}_2}$$

INFATTI, APPLICANDO LA (*)

$$\Sigma = 2\overline{Z}_1 + 2\overline{Z}_2 + 2\overline{Z}_3 + \frac{\overline{Z}_1\,\overline{Z}_2}{\overline{Z}_3} + \frac{\overline{Z}_2\,\overline{Z}_3}{\overline{Z}_1} + \frac{\overline{Z}_3\,\overline{Z}_1}{\overline{Z}_2}$$

$$= \frac{1}{\overline{Z}_1\,\overline{Z}_2\,\overline{Z}_3} \cdot \left[\overline{Z}_1\,\overline{Z}_2\,\overline{Z}_3\left(2\overline{Z}_1 + 2\overline{Z}_2 + 2\overline{Z}_3\right) + \overline{Z}_1^2\,\overline{Z}_2^2 + \overline{Z}_2^2\,\overline{Z}_3^2 + \overline{Z}_3^2\,\overline{Z}_1^2\right]$$

$$= \frac{1}{\overline{Z}_1\,\overline{Z}_2\,\overline{Z}_3}\left(\overline{Z}_1\,\overline{Z}_3 + \overline{Z}_2\,\overline{Z}_1 + \overline{Z}_3\,\overline{Z}_2\right)^2$$

$$\frac{\overline{Z}_{12}\,\overline{Z}_{13}}{\Sigma} = \frac{1}{\Sigma} \cdot \left(\overline{Z}_1 + \overline{Z}_2 + \frac{\overline{Z}_1\,\overline{Z}_2}{\overline{Z}_3}\right)\left(\overline{Z}_3 + \overline{Z}_1 + \frac{\overline{Z}_3\,\overline{Z}_1}{\overline{Z}_2}\right)$$

$$= \frac{1}{\Sigma} \cdot \frac{1}{\overline{Z}_2\,\overline{Z}_3} \cdot \left(\overline{Z}_1\,\overline{Z}_3 + \overline{Z}_2\,\overline{Z}_3 + \overline{Z}_1\,\overline{Z}_2\right)\left(\overline{Z}_2\,\overline{Z}_3 + \overline{Z}_1\,\overline{Z}_2 + \overline{Z}_3\,\overline{Z}_1\right)$$

$$= \frac{1}{\Sigma} \cdot \frac{1}{\overline{Z}_2\,\overline{Z}_3} \cdot \left(\overline{Z}_1\,\overline{Z}_3 + \overline{Z}_2\,\overline{Z}_1 + \overline{Z}_3\,\overline{Z}_2\right)^2$$

$$= \frac{\overline{Z}_1\,\overline{Z}_2\,\overline{Z}_3}{\left(\overline{Z}_1\,\overline{Z}_3 + \overline{Z}_2\,\overline{Z}_1 + \overline{Z}_3\,\overline{Z}_2\right)^2} \cdot \frac{1}{\overline{Z}_2\,\overline{Z}_3} \cdot \left(\overline{Z}_1\,\overline{Z}_3 + \overline{Z}_2\,\overline{Z}_1 + \overline{Z}_3\,\overline{Z}_2\right)^2$$

$$= \overline{Z}_1$$

SE $\overline{Z}_1 = \overline{Z}_2 = \overline{Z}_3 = \overline{Z}$

$\overline{Z}_{12} = \overline{Z} + \overline{Z} + \overline{Z} = 3\overline{Z}$; $\overline{Z}_{23} = 3\overline{Z}$; $\overline{Z}_{31} = 3\overline{Z}$

POTENZA DI UN DISPOSITIVO TRIFASE LINEARE

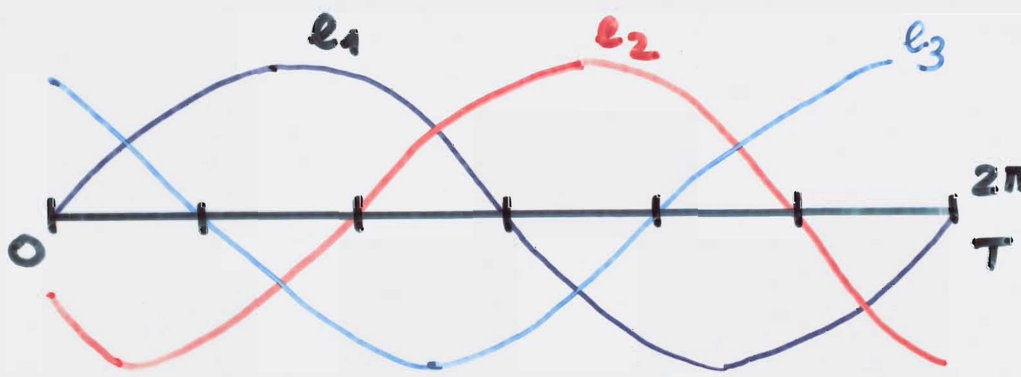
CONV. UTILIZZ.

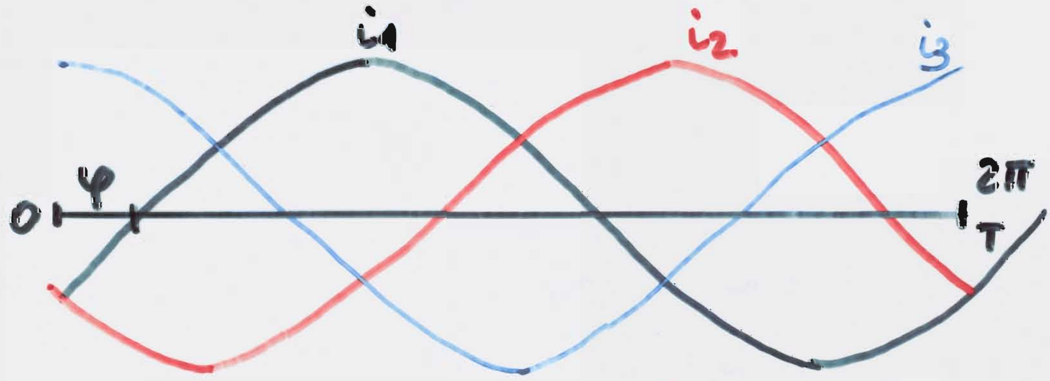
e_1, e_2, e_3 SIST. SIMM.

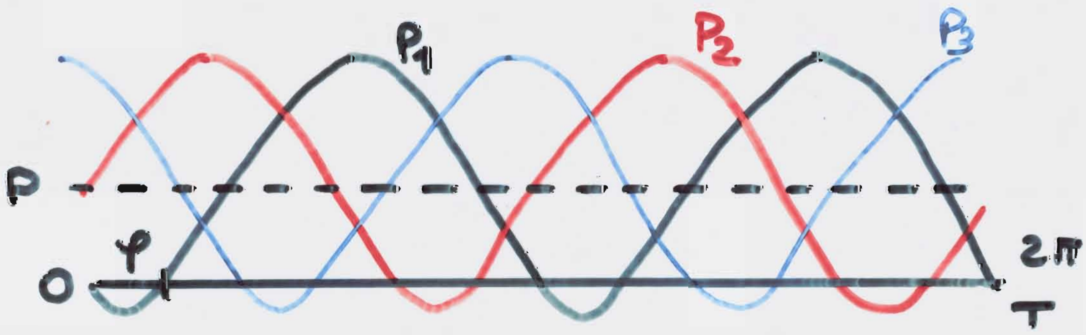
i_1, i_2, i_3 SIST. EQUIL.

PERIODO T

$p_1 = e_1 i_1$ $\quad p_2 = e_2 i_2$ $\quad p_3 = e_3 i_3$







$p_1(t), p_2(t), p_3(t)$
PER. SIN.
DI VAL. MEDIO
P
SFASATE DI
$2\pi/3$ TRA LORO

RISPETTO AL VAL. MEDIO P : $\quad p_1(t) + p_2(t) + p_3(t) = 0$

RISPETTO ALLO ZERO $\quad p_1(t) + p_2(t) + p_3(t) = 3P$

È COSTANTE NEL TEMPO !

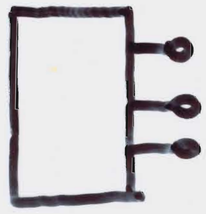
NEL DOMINIO DEI FASORI

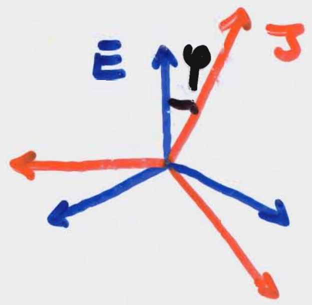
POTENZA ATTIVA, REATTIVA, APPARENTE

SIANO : V, E TENSIONI DI LINEA E DI FASE SIMM.

I, J CORRENTI DI LINEA E DI FASE EQUIL

CONV. DI UTILIZZ.





SI PENSI IL DISPOSITIVO OTTENUTO DA 3 DISP. MONOFASI

LE POTENZE TOTALI ASSORBITE

$$P = 3EJ\cos\varphi \quad ; \quad Q = 3EJ\sin\varphi \quad ; \quad A = 3EJ$$

φ = ANGOLO TRA E E J

SE IL DISPOSITIVO È COLLEGATO A Y

$$(V = \sqrt{3}E \quad ; \quad I = J)$$

$$P = \sqrt{3}\sqrt{3}EJ\cos\varphi = \sqrt{3}VI\cos\varphi \quad ; \quad Q = \sqrt{3}VI\sin\varphi \quad ; \quad A = \sqrt{3}VI$$

SE IL DISPOSITIVO È COLLEGATO A D

$$(V = E \quad ; \quad I = \sqrt{3}J)$$

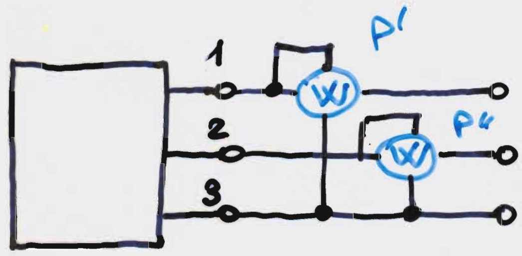
$$P = \sqrt{3}\sqrt{3}JE\cos\varphi = \sqrt{3}VI\cos\varphi \quad ; \quad Q = \sqrt{3}VI\sin\varphi \quad ; \quad A = \sqrt{3}VI$$

MISURA DELLA POTENZA ATTIVA
IN DISPOSITIVO TRIFASE QUALSIASI

SI PUÒ FARE CON 2 WATTMETRI ATTRAVERSATI DALLE CORRENTI DI 2 FASI E SOGGETTI ALLE TENSIONI DELLE 2 FASI RISPETTO AL RIFERIMENTO

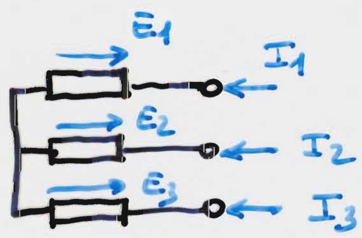
(SISTEMA ARON)

AD ES.



INFATTI

COLL A Y

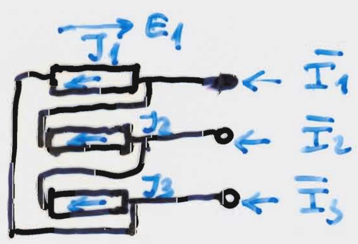


$$P' + P'' = \bar{V}_{13}\cdot\bar{I}_1 + \bar{V}_{23}\cdot\bar{I}_2 = (\bar{E}_1 - \bar{E}_3)\cdot\bar{I}_1 + (\bar{E}_2 - \bar{E}_3)\cdot\bar{I}_2$$

$$= \bar{E}_1\cdot\bar{I}_1 + \bar{E}_2\cdot\bar{I}_2 + \bar{E}_3\cdot(-\bar{I}_1 - \bar{I}_2) = \bar{E}_1\cdot\bar{I}_1 + \bar{E}_2\cdot\bar{I}_2 + \bar{E}_3\cdot\bar{I}_3$$

$$= P_1 + P_2 + P_3 = P$$

COLL A Δ



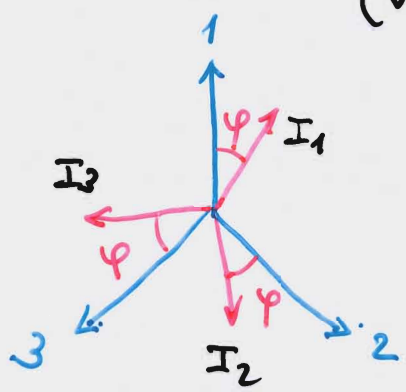
$$P' + P'' = \bar{V}_{13}\cdot\bar{I}_1 + \bar{V}_{23}\cdot\bar{I}_2 = \bar{E}_1\cdot(\bar{J}_1 - \bar{J}_2) - \bar{E}_3\cdot(\bar{J}_2 - \bar{J}_3)$$

$$= \bar{E}_1\cdot\bar{J}_1 + \bar{E}_3\cdot\bar{J}_3 + (-\bar{E}_1 - \bar{E}_3)\cdot\bar{J}_2$$

$$= \bar{E}_1\cdot\bar{J}_1 + \bar{E}_3\cdot\bar{J}_3 + \bar{E}_2\cdot\bar{J}_2 = P_1 + P_2 + P_3 = P$$

SE IL SISTEMA È SIMMETRICO (V) ED EQUILIBRATO (I)

CON A Y



(*) $P' = \mathrm{Re}(\overline{V}_{13} \cdot \overline{I}_1^*) = VI\cos(\varphi - 30°)$ $\lessgtr 0$ DIP DA φ

(**) $P'' = \mathrm{Re}(\overline{V}_{23} \cdot \overline{I}_2^*) = VI\cos(\varphi + 30°)$ $\lessgtr 0$ "

$$P' + P'' = VI\left(\frac{\sqrt{3}}{2}\cos\varphi + \frac{1}{2}\operatorname{sen}\varphi\right) +$$
$$VI\left(\frac{\sqrt{3}}{2}\cos\varphi - \frac{1}{2}\operatorname{sen}\varphi\right)$$
$$= \sqrt{3}\,VI\cos\varphi = P$$

$\varphi = 0$; $P' = P''$

$$P' - P'' = VI\operatorname{sen}\varphi = \frac{Q}{\sqrt{3}}$$

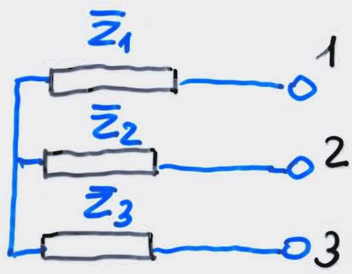
(*) fase di $\overline{V}_{13} = 60°$ fase di $\overline{I}_1 = 90° - \varphi$

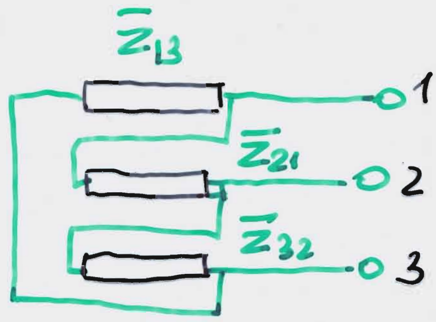
fase di $\overline{V}_{13}\,\overline{I}_1^* = 60° - (90° - \varphi) = \varphi - 30°$

(**) fase di $\overline{V}_{23} = 0°$ fase di $\overline{I}_2 = -30° - \varphi$

fase di $\overline{V}_{23}\,\overline{I}_2^* = \varphi + 30°$

TRASFORMAZIONE Δ → Y DI IMPEDENZE





$$\bar Z_{12} = \bar Z_{21}; \quad \bar Z_{32} = \bar Z_{23}; \quad \bar Z_{31} = \bar Z_{13}$$

$$\Sigma = \bar Z_{21} + \bar Z_{13} + \bar Z_{32}$$

EQUIVALENZA

1-2 $$\bar Z_1 + \bar Z_2 = \frac{\bar Z_{21}(\bar Z_{13} + \bar Z_{32})}{\bar Z_{21} + \bar Z_{13} + \bar Z_{32}} = \frac{\bar Z_{21}\bar Z_{13}}{\Sigma} + \frac{\bar Z_{21}\bar Z_{32}}{\Sigma}$$

2-3 $$\bar Z_2 + \bar Z_3 = \frac{\bar Z_{32}(\bar Z_{13} + \bar Z_{21})}{\Sigma} = \frac{\bar Z_{32}\bar Z_{13}}{\Sigma} + \frac{\bar Z_{32}\bar Z_{21}}{\Sigma}$$

3-1 $$\bar Z_1 + \bar Z_3 = \frac{\bar Z_{13}(\bar Z_{21} \;\; \bar Z_{32})}{\Sigma} = \frac{\bar Z_{13}\bar Z_{21}}{\Sigma} + \frac{\bar Z_{13}\bar Z_{32}}{\Sigma}$$

PER ISPEZIONE

$$\bar Z_1 = \frac{\bar Z_{21}\bar Z_{13}}{\Sigma}$$

$$\bar Z_2 = \frac{\bar Z_{21}\bar Z_{32}}{\Sigma}$$

$$\bar Z_3 = \frac{\bar Z_{13}\bar Z_{32}}{\Sigma}$$

SE $$\bar Z_{21} = \bar Z_{13} = \bar Z_{32} = \bar Z$$

$$\bar Z_1 = \frac{\bar Z^2}{3\bar Z} = \frac{1}{3}\bar Z; \quad \bar Z_2 = \frac{1}{3}\bar Z; \quad \bar Z_3 = \frac{1}{3}\bar Z$$

E VICEVERSA $Y \rightarrow D$

IO DICO CHE:

$$\bar{Z}_{12} = \bar{Z}_1 + \bar{Z}_2 + \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_3}$$

$$\bar{Z}_{23} = \bar{Z}_2 + \bar{Z}_3 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_1}$$

$$\bar{Z}_{31} = \bar{Z}_3 + \bar{Z}_1 + \frac{\bar{Z}_3 \bar{Z}_1}{\bar{Z}_2}$$

INFATTI

$$\Sigma = \bar{Z}_{12} + \bar{Z}_{23} + \bar{Z}_{31} = 2\bar{Z}_1 + 2\bar{Z}_2 + 2\bar{Z}_3 + \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_3} + \frac{\bar{Z}_3 \bar{Z}_2}{\bar{Z}_1} + \frac{\bar{Z}_3 \bar{Z}_1}{\bar{Z}_2}$$

$$= \frac{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3 (2\bar{Z}_1 + 2\bar{Z}_2 + 2\bar{Z}_3) + Z_1^2 Z_2^2 + Z_2^2 Z_3^2 + Z_3^2 Z_1^2}{\bar{Z}_1 \cdot \bar{Z}_2 \cdot \bar{Z}_3}$$

$$= \frac{2\bar{Z}_1^2 \bar{Z}_2 \bar{Z}_3 + 2\bar{Z}_1 \bar{Z}_2^2 \bar{Z}_3 + 2\bar{Z}_1 \bar{Z}_2 \bar{Z}_3^2 + \bar{Z}_1^2 Z_2^2 + Z_2^2 Z_3^2 + Z_3^2 Z_1^2}{\bar{Z}_1 \cdot \bar{Z}_2 \cdot \bar{Z}_3}$$

$$= \frac{(\bar{Z}_1 Z_3 + \bar{Z}_2 Z_3 + Z_3 Z_1)^2}{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3}$$

$$\frac{\bar{Z}_{21} \, Z_{13}}{\Sigma} = \frac{\left(\bar{Z}_1 + \bar{Z}_2 + \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_3}\right)\left(\bar{Z}_3 + \bar{Z}_1 + \frac{\bar{Z}_3 \bar{Z}_1}{\bar{Z}_2}\right)}{\Sigma}$$

$$= \frac{(\bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_2)(\bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_2 + \bar{Z}_3 \bar{Z}_1)}{\bar{Z}_2 \bar{Z}_3 \, \Sigma}$$

$$= \frac{(\bar{Z}_1 Z_3 + \bar{Z}_{23} + \bar{Z}_1 \bar{Z}_2)^2}{\bar{Z}_2 \bar{Z}_3 \, \Sigma}$$

$$= \frac{(\bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_2)^2}{\bar{Z}_2 \bar{Z}_3} \cdot \frac{\bar{Z}_1 \bar{Z}_2 \bar{Z}_3}{(\bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3 + \bar{Z}_1 \bar{Z}_2)^2} = \bar{Z}_1$$

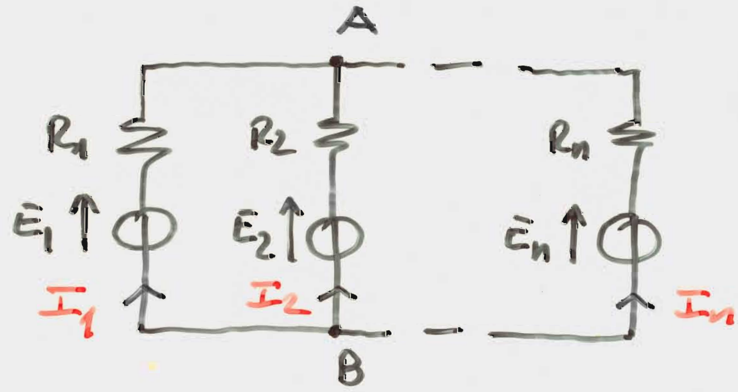
VEDI $D \rightarrow Y$

SE $\bar{Z}_1 = \bar{Z}_2 = \bar{Z}_3 = \bar{Z}$

$\bar{Z}_{12} = 2\bar{Z} + \bar{Z} = 3\bar{Z}$; $\bar{Z}_{23} = 3\bar{Z}$; $\bar{Z}_{31} = 3\bar{Z}$

TEOREMA DI MILLMAN

VALE X RETE LINEARE COSTITUITA DAL PARALLELO DI GEN. REALI DI TENSIONE



SI HA PER OL:

$$V_{AB} = E_1 - R_1 I_1 \rightarrow I_1 = \frac{E_1 - V_{AB}}{R_1}$$

$$V_{AB} = E_2 - R_2 I_2 \rightarrow I_2 = \frac{E_2 - V_{AB}}{R_2}$$

$$V_{AB} = E_n - R_n I_n \rightarrow I_n = \frac{E_n - V_{AB}}{R_n}$$

DALLA KCL AL NODO A

$$I_1 + I_2 + \quad + I_n = 0$$

$$\frac{E_1 - V_{AB}}{R_1} + \frac{E_2 - V_{AB}}{R_2} + \quad + \frac{E_n - V_{AB}}{R_n} = 0$$

$$\frac{E_1}{R_1} + \frac{E_2}{R_2} + \quad + \frac{E_n}{R_n} = V_{AB}\left(\frac{1}{R_1} + \frac{1}{R_2} + \quad + \frac{1}{R_n}\right)$$

$$V_{AB} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} + \quad + \frac{E_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \quad + \frac{1}{R_n}} = \frac{E_1 G_1 + E_2 G_2 + \quad + E_n G_n}{G_1 + G_2 + \quad + G_n}$$

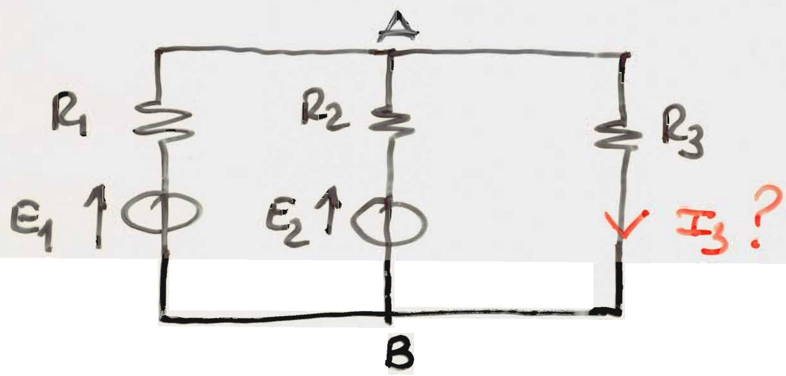
LA TENSIONE DEL PARALLELO DI n GENERATORI REALI DI TENSIONE È =
ALLA MEDIA DELLE FORZE ELETTROMOTRICI PESATA ATTRAVERSO LE CONDUTTANZE

OSS.

- SE 1 GEN È GEN DI CORRENTE → GEN. DI TENSIONE (TEOR. DI THEVENIN)

- PARTICOLARMENTE UTILE PER $n = 3$ (V. SISTEMA TRIFASE)

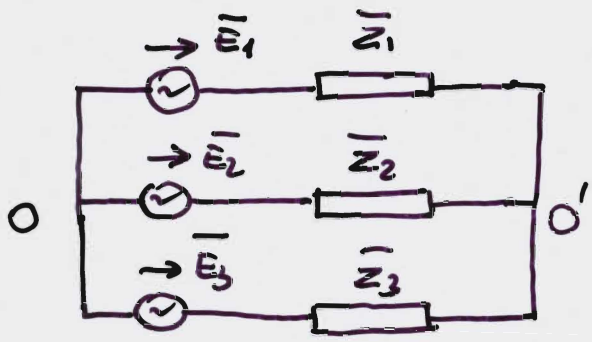
ESEMPIO



$$V_{AB} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_3 = \frac{V_{AB}}{R_3}$$

DAL TEOREMA DI MILLMAN



$\bar{Y}_1 = \frac{1}{\bar{Z}}$; $\bar{Y}_2 = \frac{1}{\bar{Z}_2}$; $\bar{Y}_3 = \frac{1}{\bar{Z}_3}$

$$\bar{V}_{OO'} = \frac{\bar{E}_1 \bar{Y}_1 + \bar{E}_2 \bar{Y}_2 + \bar{E}_3 \bar{Y}_3}{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3}$$

SE $\bar{Y}_1 = \bar{Y}_2 = \bar{Y}_3 = Y$ E $\bar{E}_1 + \bar{E}_2 + \bar{E}_3 = 0$

$$\bar{V}_{OO'} = \frac{\bar{Y}(\bar{E}_1 + \bar{E}_2 + \bar{E}_3)}{3\bar{Y}} = 0$$

COLLEGAMENTO

G(AY) — U(AY)

TEOREMA DI MILLMAN

$$\bar{V}_{O'O} = \frac{\bar{E}_1\bar{Y}_1 + \bar{E}_2\bar{Y}_2 + \bar{E}_3\bar{Y}_3}{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3}$$

$$\bar{I}_1 = \bar{Y}_1(\bar{E}_1 - \bar{V}_{O'O}) \; ; \quad \bar{I}_2 = \bar{Y}_2(\bar{E}_2 - \bar{V}_{O'O}) \; ; \quad \bar{I}_3 = \bar{Y}_3(\bar{E}_3 - \bar{V}_{O'O})$$

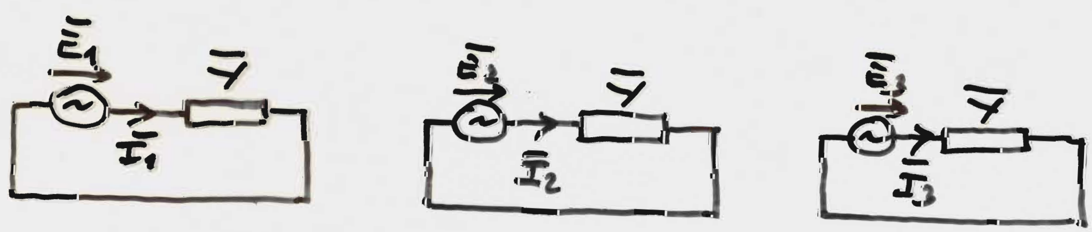
SE G : SIMM $\bar{E}_1 + \bar{E}_2 + \bar{E}_3 = 0$

E U : EQUIL $\bar{Y}_1 = \bar{Y}_2 = \bar{Y}_3 = \bar{Y}$

$$\bar{V}_{O'O} = \frac{\bar{Y}(\bar{E}_1 + \bar{E}_2 + \bar{E}_3)}{3\bar{Y}} = 0$$

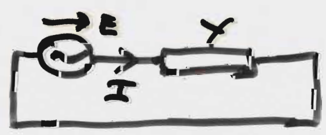
$$\bar{I}_1 = \bar{Y}\bar{E}_1 \; ; \quad \bar{I}_2 = \bar{Y}\bar{E}_2 \; ; \quad \bar{I}_3 = \bar{Y}\bar{E}_3$$

$\bar{I}_1, \bar{I}_2, \bar{I}_3$: SIST. EQUILIBRATO



3 CIRCUITI — FASORI : = AMPIEZZA ; SFAS. $2\pi/3$

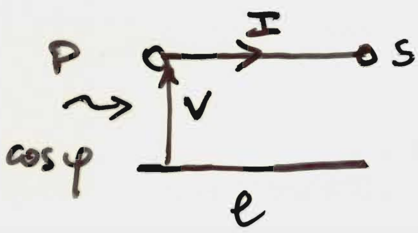
BASTA CONSIDERARE 1 SOLO CIRCUITO MONOFASE

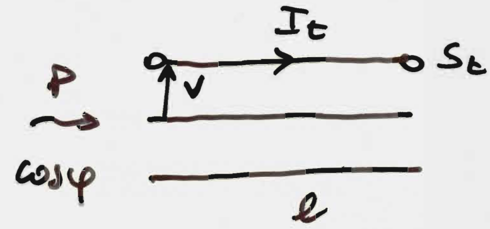


$I = YE$ $\quad E = |\bar{E}_1| = |\bar{E}_2| = |\bar{E}_3|$; $\quad I = |\bar{I}_1| = |\bar{I}_2| = |\bar{I}_3|$

CONFRONTO

LINEA 2 FILI ; LINEA 3 FILI





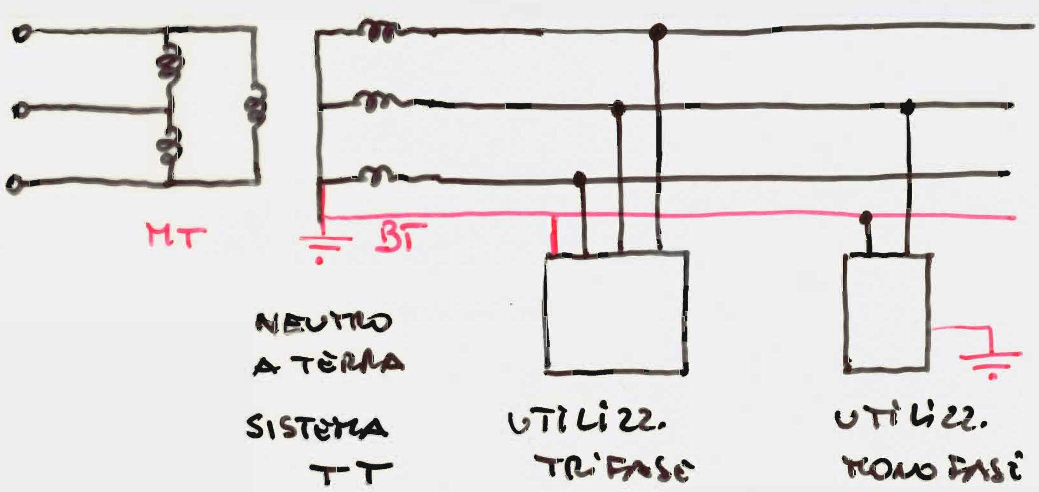
$$I = \frac{P}{V\cos\varphi} \qquad\qquad I_t = \frac{P}{\sqrt{3}\,V\cos\varphi}$$

$$S = \frac{I}{\sigma} \qquad\qquad S_t = \frac{I_t}{\sigma}$$

$$Vol_{con} = S\cdot 2\ell = \frac{P}{V\cos\varphi\,\sigma}\cdot 2\ell \qquad\qquad Vol_{cont} = S_t\cdot 3\ell = \frac{P}{\sqrt{3}\,V\cos\varphi\,\sigma}\cdot 3\ell$$

$$= \times 2 \quad > \quad = \times\sqrt{3}$$

DISTRIBUZIONE POTENZA ELETTRICA IN ITALIA

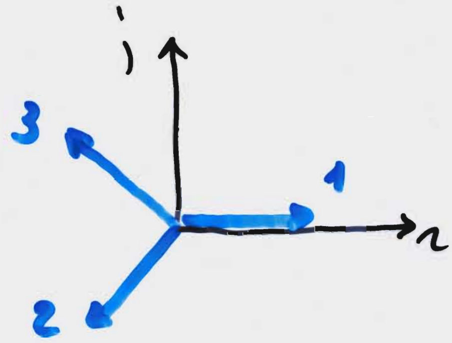


SEQUENZA DIRETTA

SISTEMA TRIFASE (SIMM/EQ.) CON SENSO C. DIRETTO

DI VETTORI DI MODULO 1

(RADICI CUBICHE DELL'UNITÀ)



$$\begin{cases} 1 \\ -\frac{1}{2} - j\frac{\sqrt{3}}{2} \\ -\frac{1}{2} + j\frac{\sqrt{3}}{2} \end{cases} = \begin{cases} 1 \\ e^{-j\frac{2\pi}{3}} \\ e^{+j\frac{2\pi}{3}} \end{cases}$$

INTRODUCENDO L'OPERATORE

$$\alpha = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \quad ; \quad \alpha^2 = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - j\frac{\sqrt{3}}{2} - \frac{3}{4} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$\alpha^3 = \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = 1 \quad ; \quad \alpha^0 = 1$$

SI HA CHE

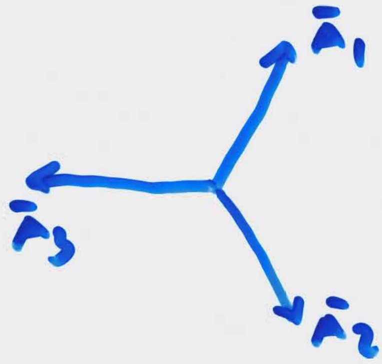
$$1 + \alpha + \alpha^2 = 0$$

LA TERNA DI VETTORI

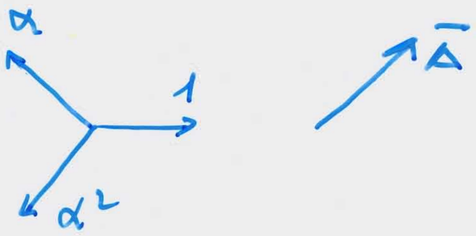
$$\begin{cases} 1 \\ e^{-j\frac{2\pi}{3}} \\ e^{j\frac{2\pi}{3}} \end{cases} = \begin{cases} 1 \\ \alpha^2 \\ \alpha \end{cases}$$ SI CHIAMA SEQUENZA DIRETTA

ESSA PUÒ GENERARE UN QUALSIASI

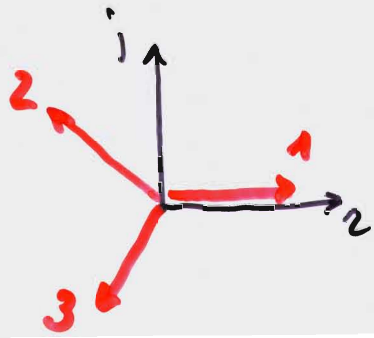
SIST. TRIF. SIMM(ETR) A SENSO CICLICO DIRETTO



$$\begin{cases} \bar{A}_1 \\ \bar{A}_2 \\ \bar{A}_3 \end{cases} = \begin{cases} 1\,\bar{A} \\ \alpha^2 \bar{A} \\ \alpha\,\bar{A} \end{cases}$$



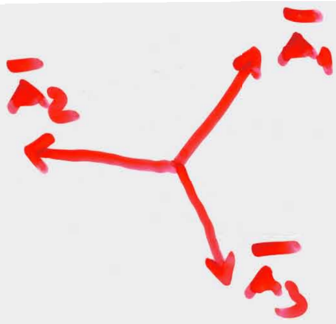
SEQUENZA INVERSA



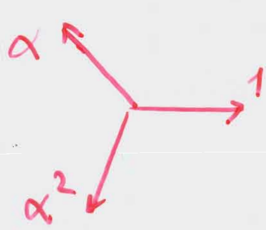
$$\begin{cases} 1 \\ -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{cases} = \begin{cases} 1 \\ e^{+j2\pi/3} \\ e^{+j\,2\pi/3} \end{cases} = \begin{cases} 1 \\ \alpha \\ \alpha^2 \end{cases}$$

SI CHIAMA SEQUENZA INVERSA

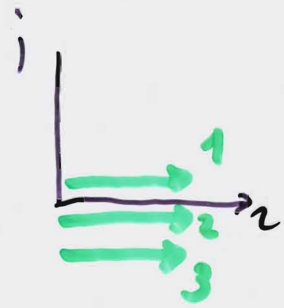
ESSA PUÒ GENERARE UN QUALSIASI SISTEMA TR. SIMM(EQ) A SENSO CICLICO INVERSO



$$\begin{cases} \bar{A}_1 \\ \bar{A}_2 \\ \bar{A}_3 \end{cases} = \begin{cases} 1\,\bar{A} \\ \alpha\,\bar{A} \\ \alpha^2\bar{A} \end{cases}$$



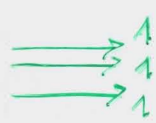
SEQUENZA OMOPOLARE (ZERO)

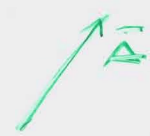


$\begin{cases}1\\1\\1\end{cases}$ SI CHIAMA SEQUENZA OMOPOLARE

ÈSSA PUÒ GENERARE UNA QUALSIASI
TERNA DI VETTORI UGUALI E PARALLELI

$$\begin{cases}\bar{A}_1\\ \bar{A}_2\\ \bar{A}_3\end{cases} = \begin{cases}1\bar{A}\\ 1\bar{A}\\ 1\bar{A}\end{cases}$$





COMPONENTI SIMMETRICI

(FORTESCUE, 1917)

1) DATI:

UNA TERNA DIRETTA $\bar{A}_d \quad \alpha^2\bar{A}_d \quad \alpha\bar{A}_d$

UNA TERNA INVERSA $\bar{A}_i \quad \alpha\bar{A}_i \quad \alpha^2\bar{A}_i$

E UNA TERNA OMOPOLARE $\bar{A}_0 \quad \bar{A}_0 \quad \bar{A}_0$

SI PUÒ COMPORRE UNA QUALSIASI TERNA (SISTEMA DISS/SQ.)

$$\bar{A}_1 = \bar{A}_0 + \bar{A}_d + \bar{A}_i \qquad (1)$$

$$\bar{A}_2 = \bar{A}_0 + \alpha^2\bar{A}_d + \alpha\bar{A}_i \qquad (2)$$

$$\bar{A}_3 = \bar{A}_0 + \alpha\bar{A}_d + \alpha^2\bar{A}_i \qquad (3)$$

VICEVERSA

2) DATA

UNA QUALSIASI TERNA $\bar{A}_1$ $\bar{A}_2$ $\bar{A}_3$

ESSA SI PUÒ SCOMPORRE IN:

UNA TERNA OMOPOLARE

$$\bar{A}_0 = \frac{1}{3}\left(\bar{A}_1 + \bar{A}_2 + \bar{A}_3\right)$$

(SOMMANDO ①, ②, ③)

UNA TERNA DIRETTA

$$\bar{A}_d = \frac{1}{3}\left(\bar{A}_1 + \alpha\bar{A}_2 + \alpha^2\bar{A}_3\right)$$

(SOMMANDO ①, α②, $\alpha^2$③)

UNA TERNA INVERSA

$$\bar{A}_i = \frac{1}{3}\left(\bar{A}_1 + \alpha^2\bar{A}_2 + \alpha\bar{A}_3\right)$$

(SOMMANDO ①, $\alpha^2$②, α③)