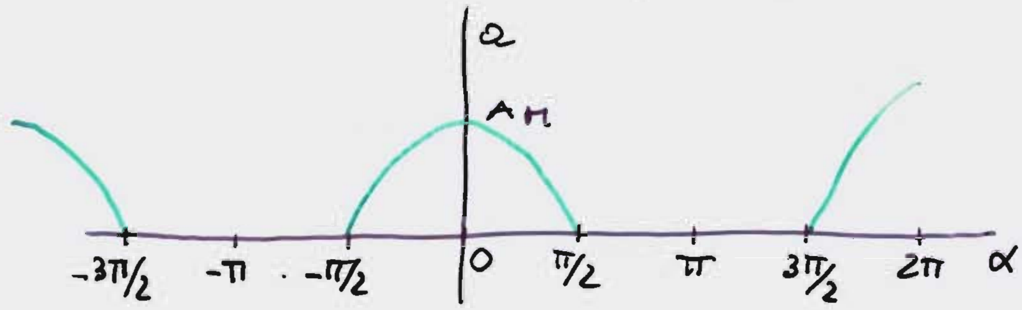
SVILUPPARE IN SERIE TRIGONOMETRICA DI FOURIER




α = ωt

$$a(\alpha) = A_M \cos\alpha \qquad -\pi/2 \leq \alpha \leq \pi/2$$

$$a(\alpha) = 0 \qquad \pi/2 \leq \alpha \leq \pi \ ; \ -\pi \leq \alpha \leq -\pi/2$$

V. MEDIO

$$A_m = \frac{1}{2\pi}\left(A_M \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha\right) = \frac{1}{2\pi} A_M \left(\sin\alpha\right)_{-\pi/2}^{\pi/2}$$

$$= \frac{A_M}{2\pi}(1+1) = \frac{2A_M}{2\pi} = \frac{A_M}{\pi}$$

V. EFFICACE

$$A = \sqrt{\frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} A_M^2 \cos^2\alpha \, d\alpha} = A_M \sqrt{\frac{1}{2} \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos^2\alpha \, d\alpha}$$

$$= A_M \sqrt{\frac{1}{2} \cdot \frac{1}{2}} = \frac{A_M}{2}$$

$$a(\alpha) = A_m + \sum_{n=1}^{\infty} a_n \cos n\alpha + \sum_{n=1}^{\infty} b_n \sin n\alpha$$

$a(\alpha)$ È PARI $\rightarrow$ SOLO $a_n$

$$a_n = \frac{1}{\pi} A_M \int_{-\pi/2}^{\pi/2} \cos\alpha \cos n\alpha \, d\alpha$$

RICORDIAMO CHE:

$$\cos\alpha \cos n\alpha = \frac{1}{2}\cos(\alpha - n\alpha) + \frac{1}{2}\cos(\alpha + n\alpha)$$
$$= \frac{1}{2}\cos(1-n)\alpha + \frac{1}{2}\cos(1+n)\alpha$$

$$a_n = \frac{A_M}{\pi}\cdot\frac{1}{2}\left[\int_{-\pi/2}^{\pi/2}\cos(1-n)\alpha\, d\alpha + \int_{-\pi/2}^{\pi/2}\cos(1+n)\alpha\, d\alpha\right]$$

$$= \frac{A_M}{\pi}\cdot\frac{1}{2}\left\{\left[\frac{\sin(1-n)\alpha}{1-n}\right]_{-\pi/2}^{\pi/2} + \left[\frac{\sin(1+n)\alpha}{1+n}\right]_{-\pi/2}^{\pi/2}\right\}$$

$$= \frac{A_M}{\pi}\cdot\frac{1}{2}\left\{\frac{\sin(1-n)\frac{\pi}{2} + \sin(1-n)\frac{\pi}{2}}{1-n} + \frac{\sin(1+n)\frac{\pi}{2} + \sin(1+n)\frac{\pi}{2}}{1+n}\right\}$$

$$= \frac{A_M}{\pi}\,\frac{1}{\cancel{2}}\left\{\frac{\cancel{2}\sin(1-n)\frac{\pi}{2}}{1-n} + \frac{\cancel{2}\sin(1+n)\frac{\pi}{2}}{1+n}\right\}$$

RICORDIAMO CHE:

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(1-n)\frac{\pi}{2} = \sin\frac{\pi}{2}\cos n\frac{\pi}{2} - \cancel{\cos\frac{\pi}{2}}\sin n\frac{\pi}{2} = \cos n\frac{\pi}{2}$$

$$\sin(1+n)\frac{\pi}{2} = \sin\frac{\pi}{2}\cos n\frac{\pi}{2} + \cancel{\cos\frac{\pi}{2}}\sin n\frac{\pi}{2} = \cos n\frac{\pi}{2}$$

$$a_n = \frac{A_M}{\pi}\left\{\frac{\cos n\pi/2}{1-n} + \frac{\cos n\pi/2}{1+n}\right\}$$

$$= \frac{A_M}{\pi}\cos n\frac{\pi}{2}\left\{\frac{1}{1-n}+\frac{1}{1+n}\right\} = \frac{A_M}{\pi}\cos n\frac{\pi}{2}\left\{\frac{1+n+1-n}{1-n^2}\right\}$$

$$= \frac{A_M}{\pi}\,\frac{2}{1-n^2}\cos n\frac{\pi}{2}$$

n DISPARI $\neq 1$ $\quad \cos n\frac{\pi}{2}=0 \qquad a_n=0$

n PARI

$$n=2 \qquad a_2 = \frac{A_M}{\pi}\cdot\frac{2}{-3}(-1) = \frac{2A_M}{3\pi}$$

$$n=4 \qquad a_4 = \frac{A_M}{\pi}\,\frac{2}{-15}\cdot 1 = -\frac{2A_M}{15\pi}$$

$n=1$

$$a_1 = \frac{A_M}{\pi}\int_{-\pi/2}^{\pi/2}\cos^2\alpha\, d\alpha$$

RICORDANDO (INT × PARTI) CHE

$$\int_0^{\alpha}\cos^2\alpha\, d\alpha = \frac{1}{2}\alpha + \frac{1}{4}\sin 2\alpha$$

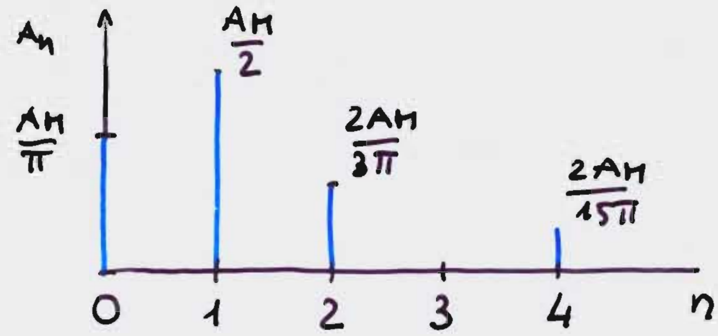
$$a_1 = \frac{A_M}{\pi}\left[\frac{1}{2}\alpha + \frac{1}{4}\sin 2\alpha\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{A_M}{\pi}\left[\frac{\pi}{4}+\frac{1}{4}\sin\pi + \frac{\pi}{4} - \frac{1}{4}\sin(-\pi)\right] = \frac{A_M}{\pi}\cdot\frac{\pi}{2}$$

$$= \frac{A_M}{2}$$

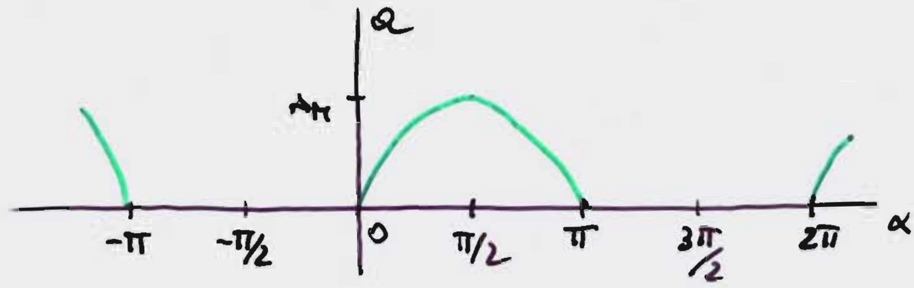
$$b_n = 0 \qquad A_n = \sqrt{a_n^2+b_n^2} = |a_n|$$

## SPETTRO ARMONICO




$$a(\alpha) = \frac{A_M}{\pi}\left\{1 + \frac{\pi}{2}\cos\alpha + \frac{2}{3}\cos 2\alpha - \frac{2}{15}\cos 4\alpha + \cdots\right\}$$

$$a(\omega t) = \frac{A_M}{\pi}\left\{1 + \frac{\pi}{2}\cos\omega t + \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \cdots\right\}$$




$$a(\alpha) = A_M \sin\alpha \qquad 0 \le \alpha \le \pi$$

$$a(\alpha) = 0 \qquad \pi \le \alpha \le 2\pi$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} A_M \sin\alpha \cos n\alpha \, d\alpha$$

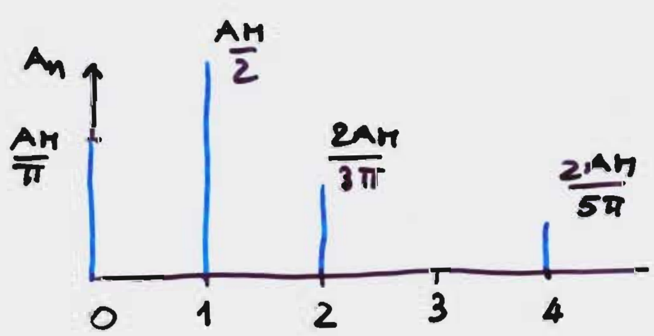
$$b_n = \frac{1}{\pi}\int_0^{\pi} A_M \sin\alpha \sin n\alpha \, d\alpha$$

RISULTATI

$n$ PARI : $a_n = \dfrac{2 A_M}{\pi(1-n^2)}$ ; $n$ DISPARI $a_n = 0$

$b_n = 0 \quad n \neq 1 \qquad n = 1 \quad b_1 = \dfrac{A_M}{2}$

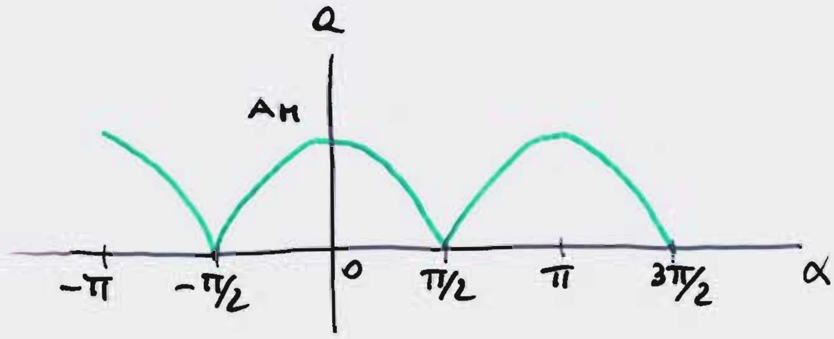
SPETTRO ARMONICO




NON CAMBIA!

$$a(t) = \frac{A_M}{\pi}\left\{ 1 + \frac{\pi}{2}\sin\omega t - \frac{2}{3}\cos 2\omega t - \frac{2}{15}\cos 4\omega t + \dots \right\}$$

SVILUPPARE IN SERIE TRIGONOMETRICA DI FOURIER




V. MEDIO

$$A_m = \frac{1}{2\pi}\, 2 \left\{ A_M \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha \right\} = \frac{2A_M}{\pi}$$

V. EFFICACE

$$A = \sqrt{\frac{1}{\cancel{2}\pi}\, \cancel{2} \int_{-\pi/2}^{\pi/2} A_M^2 \cos^2\alpha \, d\alpha} = A_M \sqrt{\frac{1}{2}} = \frac{A_M}{\sqrt{2}}$$

$a(\alpha)$ È PARI $\rightarrow$ SOLI $a_n$

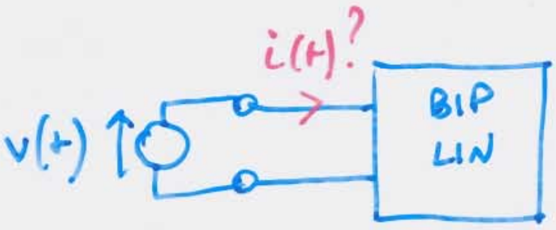
$$a_n = \frac{1}{\pi} A_M \left\{ \int_{-\pi/2}^{\pi/2} \cos\alpha \cos n\alpha \, d\alpha + \int_{\pi/2}^{\frac{3\pi}{2}} \cos(\alpha - \pi) \cos n\alpha \, d\alpha \right\}$$

. . . . . . . . . . . . . .

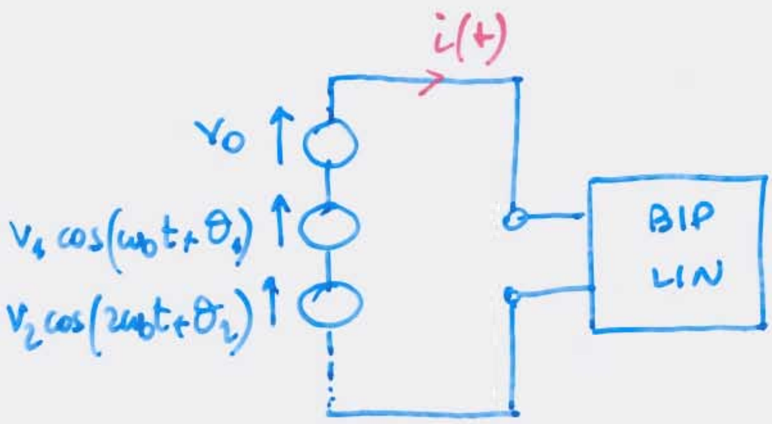
RISULTATO

$$a(\alpha) = \frac{2A_M}{\pi} \left\{ 1 + \frac{2}{3}\cos 2\alpha - \frac{2}{15}\cos 4\alpha + \cdots \right\}$$

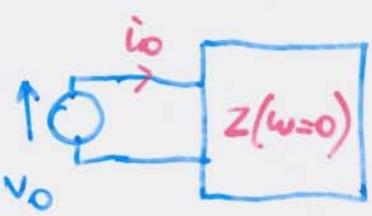
# APPLICAZIONE

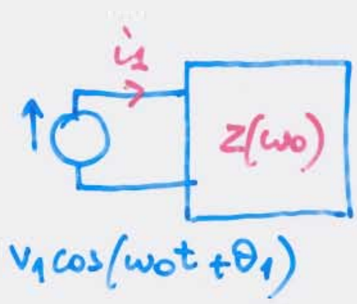


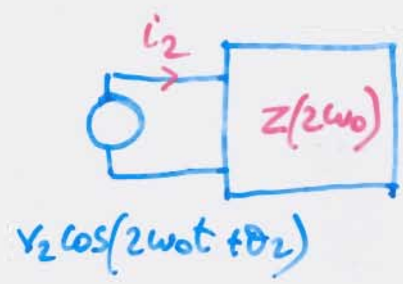

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$




SOVRAPP.
DEGLI EFFETTI




$$i(t) = i_0 + i_1(t) + i_2(t) + \dots$$

$v(t)$ e $i(t)$ hanno lo stesso $\omega_0$

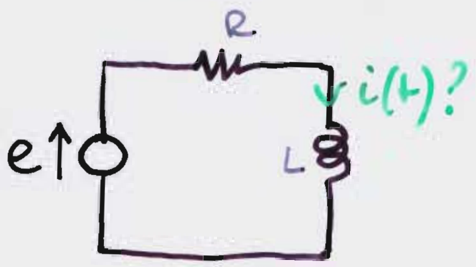
LA FORMA DI $i(t)$ È $\neq$ DALLA FORMA DI $v(t)$

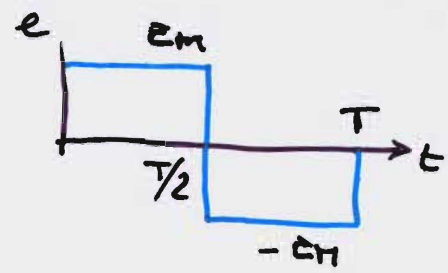
VALORE EFFICACE

$$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = \sqrt{V_0^2 + \sum \frac{V_n^2}{2}} = \sqrt{V_0^2 + \sum V_{en}^2}$$

$$I = \sqrt{I_0^2 + \sum I_{en}^2}$$

ESEMPIO




$\omega = \frac{2\pi}{T}$

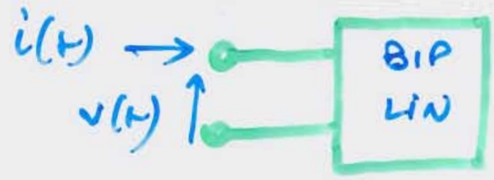
$$e(t) = e_1(t) + e_3(t) + e_5(t) +$$
$$= \frac{4E_M}{\pi}\sin(\omega t) + \frac{4E_M}{3\pi}\sin(3\omega t) + \frac{4E_M}{5\pi}\sin(5\omega t) +$$

$$e_1(t) \rightarrow \bar{E}_1 = \frac{4E_M}{\sqrt{2}\pi}\angle 0 = E_1\angle 0$$
$$\bar{I}_1 = \frac{\bar{E}_1}{R+j\omega L} = \frac{E_1}{\sqrt{R^2+\omega^2L^2}}\angle -\operatorname{atan}\frac{\omega L}{R}$$
$$i_1(t) = \sqrt{2}\,\frac{E_1}{\sqrt{R^2+\omega^2L^2}}\cdot\sin\left(\omega t - \operatorname{atan}\frac{\omega L}{R}\right)$$
$$= \frac{4E_M}{\pi\sqrt{R^2+\omega^2L^2}}\cdot\sin\left(\omega t - \operatorname{atan}\frac{\omega L}{R}\right)$$

$$e_3(t) \rightarrow \bar{E}_3 = \frac{4E_M}{\sqrt{2}\,3\pi}\angle 0 = E_3\angle 0$$
$$\bar{I}_3 = \frac{\bar{E}_3}{R+j3\omega L} = \frac{E_3}{\sqrt{R^2+9\omega^2L^2}}\angle -\operatorname{atan}\frac{3\omega L}{R}$$
$$i_3(t) = \sqrt{2}\,\frac{E_3}{\sqrt{R^2+9\omega^2L^2}}\cdot\sin\left(3\omega t - \operatorname{atan}\frac{3\omega L}{R}\right)$$
$$= \frac{4E_M}{3\pi\sqrt{R^2+9\omega^2L^2}}\cdot\sin\left(3\omega t - \operatorname{atan}\frac{3\omega L}{R}\right)$$

$$i(t) = i_1(t) + i_3(t) + \dots$$

# POTENZA MEDIA P ATTIVA




$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$

$$i(t) = I_0 + \sum_{m=1}^{\infty} I_m \cos(m\omega_0 t + \varphi_m)$$

$$P = \frac{1}{T}\int_0^T v\, i\, dt$$

$$P = \frac{1}{T}\int_0^T V_0 I_0\, dt + \cancel{\sum_{m=1}^{\infty} \frac{I_m V_0}{T}\int_0^T \cos(m\omega_0 t + \varphi_m)\, dt}$$

$$+ \cancel{\sum_{n=1}^{\infty} \frac{V_n I_0}{T}\int_0^T \cos(n\omega_0 t + \theta_n)\, dt}$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{V_n I_m}{T}\int_0^T \cos(n\omega_0 t - \theta_n)\cos(m\omega_0 t - \varphi_m)\, dt$$

$$P = V_0 I_0 + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \varphi_n) = V_0 I_0 + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos\phi_n$$

$$= V_0 I_0 + \sum_{n=1}^{\infty} V_{en} I_{en} \cos\phi_n$$

LA POTENZA MEDIA È = POTENZA IN C.C. +
POTENZA MEDIA DI
CIASCUNA ARMONICA

PER UN RESISTORE $\theta_n = \varphi_n$

$$P = V_0 I_0 + \frac{1}{2}\sum_{n=1}^{\infty} V_n I_n$$

TEOREMA DI PARSEVAL

POTENZA REATTIVA Q

$$Q = \sum_{r=1}^{\infty} V_{er} I_{er} \operatorname{sen} \phi_r$$

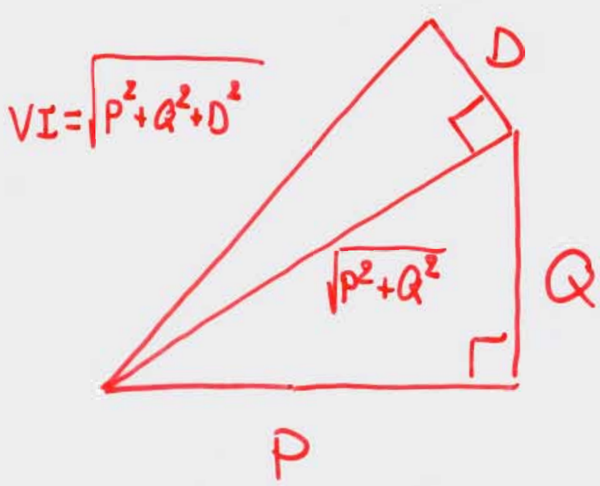
POTENZA APPARENTE S

$S = VI$   $V$ = VAL EFF DI $v(t)$
$I$ = " " " $i(t)$

SI VERIFICA CHE   $\cos\phi = \dfrac{P}{S}$

$$S^2 = P^2 + Q^2 + D^2$$

D = POTENZA DEFORMANTE
($=0$, SE $\vartheta_n = \varphi_n$)




BIPOLO R-L

$$S^2 = (V_0^2 + V_1^2 + \cdots)(I_0^2 + I_1^2 + \cdots)$$

$$= V_0^2 I_0^2 + V_0^2 I_1^2 + V_1^2 I_0^2 + V_1^2 I_1^2 + \cdots$$

$$P^2 = (V_0 I_0 + V_1 I_1 \cos\varphi_1 + \cdots)^2$$

$$= V_0^2 I_0^2 + V_1^2 I_1^2 \cos^2\varphi_1 + 2 V_0 I_0 V_1 I_1 \cos\varphi_1 + \cdots$$

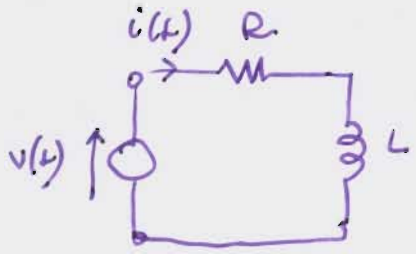
$$Q^2 = V_1^2 I_1^2 \sin^2\varphi_1$$

$$D^2 = S^2 - P^2 - Q^2$$

PER UN RESISTORE: $\cos\varphi_1 = 1 \quad \sin\varphi_1 = 0 \quad \ldots$

$$\frac{V_0}{I_0} = \frac{V_1}{I_1}$$

$$D^2 = 0$$




$$V = \sqrt{V_0^2 + V_1^2 + V_3^2 + }$$
$$I = \sqrt{I_0^2 + I_1^2 + I_3^2 + }$$

V, I = V. EFF.

LA FORMA DI i(t) È DEFORMATA RISPETTO ALLA FORMA DI v(t) E VICEVERSA

POT. ATTIVA

$P = R I^2$ OPPURE

$P = P_0 + P_1 + P_3 +$

$P_0 = V_0 I_0$

$P_1 = V_1 I_1 \cos\varphi_1$

$P_3 = V_3 I_3 \cos\varphi_3$

POT. REATTIVA

$$Q = V_1 I_1 \sin\varphi_1 + V_3 I_3 \sin\varphi_3 + \dots$$

POT. APPARENTE

$$S = V I$$

POT. DEFORMANTE

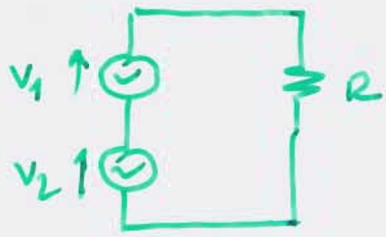
$$D^2 = S^2 - P^2 - Q^2$$

OSS.

SOLO $\omega = 0$ : $P = P_0$, $Q = 0$, $S = V_0 I_0$, $D^2 = 0$

SOLO $\omega_1$ : $P = V_1 I_1 \cos\varphi_1$, $Q = V_1 I_1 \sin\varphi_1$; $S = V_1 I_1$, $D^2 = 0$

ATTENZIONE




1. $v_1 = \sqrt{2} V_1 \cos(\omega_1 t + \theta_1)$ $\qquad$ $v_2 = \sqrt{2} V_2 \cos(\omega_2 t + \theta_2)$

$i_1 = \frac{v_1}{R} = \sqrt{2} I_1 \cos(\omega_1 t + \theta_1)$ $\qquad$ $i_2 = \frac{v_2}{R} = \sqrt{2} I_2 \cos(\omega_2 t + \theta_2)$

$$P = V_1 I_1 + V_2 I_2 \qquad (W)$$

2. $v_1 = \sqrt{2} V_1 \cos(\omega t + \theta_1)$ $\qquad$ $v_2 = \sqrt{2} V_2 \cos(\omega t + \theta_2)$

$\downarrow \bar{V}_1$ $\qquad$ $\downarrow \bar{V}_2$

$$\bar{V} = \bar{V}_1 + \bar{V}_2$$

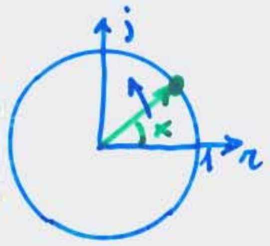
$i_1 = \frac{v_1}{R} \rightarrow \bar{I}_1$ $\qquad$ $i_2 = \frac{v_2}{R} \rightarrow \bar{I}_2$

$$\bar{I} = \bar{I}_1 + \bar{I}_2$$

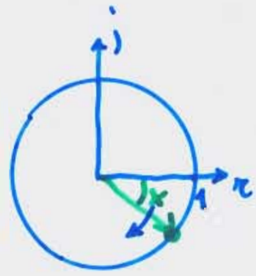
$$P = R\left(|\bar{I}|\right)^2$$

$$\neq V_1 I_1 + V_2 I_2$$

FUNZ. ESPON. COMPLESSA




$e^{jx}$

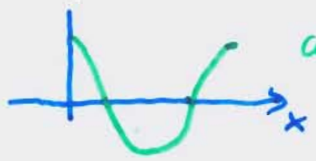



$e^{-jx}$

$x = \omega t$ MOTO CIRCOLARE

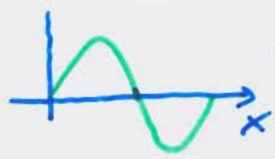
FUNZ. ARMONICHE REALI : PROIEZIONI

su r




cosx

su j




sinx

MOTO ARMONICO

FORMULA DI EULERO

$$e^{jx} = \cos x + j \sin x$$

$$e^{-jx} = \cos x - j \sin x$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \; ; \qquad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

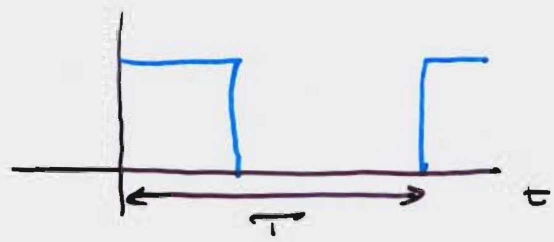
FUNZ. ESP. COMPLESSA $e^{jx}$ E

FUNZ. ARM. REALI $\cos x$ E $\sin x$

SONO PARENTI !

# DALLA SERIE DI FOURIER

# ALL' INTEGRALE DI FOURIER




$T \to \infty$

DA SEGNALE PERIODICO A

SEGNALE APERIODICO

$\omega = \frac{1}{T} \to 0$

DISTANZA TRA ARMONICHE

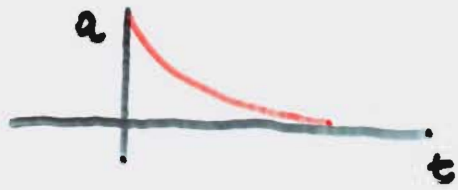
$$(n+1)\omega - n\omega = \omega = \frac{2\pi}{T} \to 0$$

SPETTRO ARMONICO DISCRETO

→ " CONTINUO

$\Sigma \to \int$

## SEGNALE NON PERIODICO




SE

ESISTE ED È FINITO $\int_{-\infty}^{+\infty} |a(t)| \, dt$

ALLORA

$$a(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} P(\omega) \operatorname{sen} \omega t \, d\omega + \frac{1}{2\pi} \int_{-\infty}^{+\infty} Q(\omega) \cos \omega t \, d\omega$$

$$a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{S}(\omega) \, e^{j\omega t} \, d\omega$$

DOVE

$$P(\omega) = \int_{-\infty}^{\infty} a(t) \operatorname{sen} \omega t \, dt$$

$$Q(\omega) = \int_{-\infty}^{\infty} a(t) \cos \omega t \, dt$$

$$S^2(\omega) = P^2(\omega) + Q^2(\omega)$$

$$\phi(\omega) = \arctan \left( P(\omega) / Q(\omega) \right)$$

SPETTRO ARMONICO


ESEMPIO 3


$a(t) = e^{-\alpha t} \qquad \alpha > 0$

$0 < t < \infty$

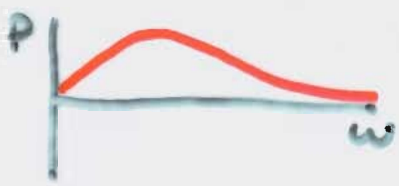
$$P(\omega) = \int_{-\infty}^{\infty} e^{-\alpha t} \operatorname{sen} \omega t \, dt = \left[ (-\alpha \operatorname{sen} \omega t - \omega \cos \omega t) \, e^{-\alpha t} / (\alpha^2 + \omega^2) \right]_0^{\infty}$$

$$= \omega / (\alpha^2 + \omega^2)$$

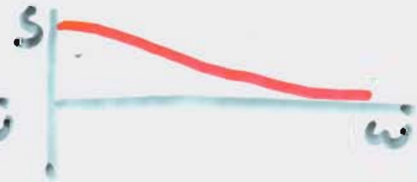
$$Q(\omega) = \int_{-\infty}^{\infty} e^{-\alpha t} \cos \omega t \, dt = \left[ (-\alpha \cos \omega t + \omega \operatorname{sen} \omega t) \, e^{-\alpha t} / (\alpha^2 + \omega^2) \right]_0^{\infty}$$

$$= \alpha / (\alpha^2 + \omega^2)$$

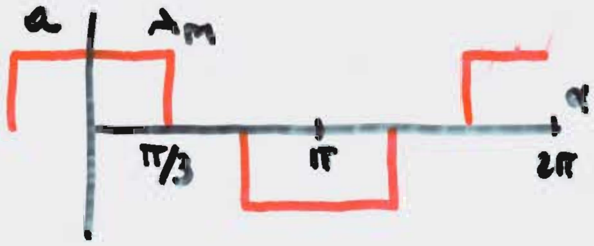
$$S(\omega) = \sqrt{P^2(\omega) + Q^2(\omega)} = \frac{1}{\sqrt{\alpha^2 + \omega^2}}$$







## ESEMPIO 2




$a(t) = A_M \qquad -\pi/3 < \alpha < \pi/3$

$a(t) = 0 \qquad \pi/3 < \alpha < 2\pi/3$

$a(t) = -A_M \qquad 2\pi/3 < \alpha < 4\pi/3$

$A_m = 0$

$$A_{ma} = \frac{1}{2\pi}\left[\int_{-\pi/3}^{\pi/3} A_M\, d\alpha + \int_{2\pi/3}^{4\pi/3} A_M\, d\alpha\right] = \frac{1}{2\pi}\cdot 2\cdot\frac{2\pi}{3}A_M = \frac{2A_M}{3}$$

$$A = \sqrt{\frac{1}{2\pi}\left[\int_{-\pi/3}^{\pi/3} A_M^2\, d\alpha + \int_{2\pi/3}^{4\pi/3} A_M^2\, d\alpha\right]} = \sqrt{\frac{1}{2\pi}\cdot 2\cdot\frac{2\pi}{3}A_M^2} = A_M\frac{\sqrt{2}}{\sqrt{3}}$$

FUNZIONE PARI : $b_n = 0$ ; $n = 1, 2, 3, \dots$

FUNZIONE EMISIM : $a_n = 0$ ; $n = 2, 4, 6, \dots$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi/3}^{\pi/3} A_M \cos n\omega t\, d\alpha - \int_{2\pi/3}^{4\pi/3} A_M \cos n\omega t\, d\alpha\right]$$

$$= \frac{1}{\pi}\,\frac{A_M}{n}\left[\left(\operatorname{sen} n\omega t\right)_{-\pi/3}^{\pi/3} - \left(\operatorname{sen} n\omega t\right)_{2\pi/3}^{4\pi/3}\right]$$

$$= \frac{2A_M}{n\pi}\cdot 2\operatorname{sen} n\frac{\pi}{3}$$


# SERIE ESPONENZIALE DI FOURIER

USANDO L'IDENTITA' DI EULERO

$$\cos n\omega t = \frac{1}{2}\left(e^{jn\omega t} + e^{-jn\omega t}\right)$$

$$\operatorname{sen} n\omega t = \frac{1}{2j}\left(e^{jn\omega t} - e^{-jn\omega t}\right)$$

SOSTITUENDO NELLA SERIE TRIGONOMETRICA SENO-COSENO

$$a(t) = A_m + \frac{1}{2}\sum_{n=1}^{\infty}\left\{(a_n - jb_n)e^{jn\omega t} + (a_n + jb_n)e^{-jn\omega t}\right\}$$

DEFINITI

$$\bar{C}_n = \frac{a_n - jb_n}{2} \quad ; \quad \bar{C}_{-n} = \frac{a_n + jb_n}{2}$$

$$a(t) = A_m + \sum_{n=1}^{\infty}\left\{\bar{C}_n e^{jn\omega t} + \bar{C}_{-n} e^{-jn\omega t}\right\}$$

$$a(t) = A_m + \sum_{n=-\infty}^{\infty} \bar{C}_n e^{jn\omega t}$$

RISULTA $\bar{C}_{-m} = \bar{C}_m^*$ , $\bar{C}_m \bar{C}_{-m} = |\bar{C}_m|^2$

$\bar{C}_n$ PUÒ ESSERE RICAVATO DA $a_n$ e $b_n$

OPPURE DIRETTAMENTE

$$\bar{C}_n = \frac{1}{T}\int_0^T a(t)\, e^{-jn\omega t}\, dt$$

$$|\bar{C}_n| = \frac{|A_n|}{2} \quad \text{MA} \quad -\infty < n < \infty$$

INFATTI

SOMMANDO

$$\bar{c}_n + \bar{c}_{-n} = \frac{1}{2}\left(2a_n - \cancel{jb_n} + \cancel{jb_n}\right)$$

$$a_n = \bar{c}_n + \bar{c}_{-n}$$

SOTTRAENDO

$$\bar{c}_n - \bar{c}_{-n} = \frac{1}{2}\left(-2j\, b_n + \cancel{a_n} - \cancel{a_n}\right)$$

$$b_n = j\left(\bar{c}_n - \bar{c}_{-n}\right)$$

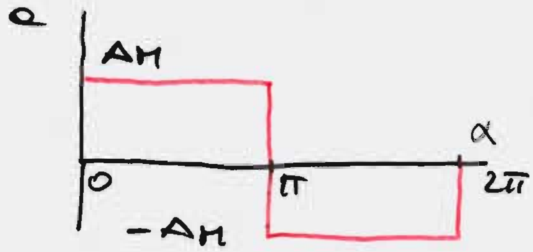
LEGAME SERIE TRIGONOMETRICA - SERIE ESPONENZIALE

$$|A_n| = \sqrt{a_n^2 + b_n^2} = 2\left|\bar{c}_n\right| = 2\left|\bar{c}_{-n}\right|$$

$a(t)$ PARI $\rightarrow$ $\bar{c}_n$ REALE

$a(t)$ DISPARI $\rightarrow$ $\bar{c}_n$ IMMAGINARIO

ESEMPIO 1




SERIE ESPONENZIALE

$$\bar{C}_n = \frac{1}{2\pi}\left(\int_0^{\pi} A_M e^{-jn\alpha}\, d\alpha - \int_{\pi}^{2\pi} A_M e^{-jn\alpha}\, d\alpha\right)$$

$$= \frac{1}{2\pi}\left[\frac{A_M}{jn}\left(-e^{-jn\alpha}\right)_0^{\pi} - \frac{A_M}{jn}\left(-e^{-jn\alpha}\right)_{\pi}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\cdot\frac{A_M}{jn}\left[-e^{-jn\pi} + 1 - \left(-e^{-jn2\pi} + e^{-jn\pi}\right)\right]$$

n pari $e^{-jn\pi} = 1$

$\rightarrow \bar{C}_n = 0$

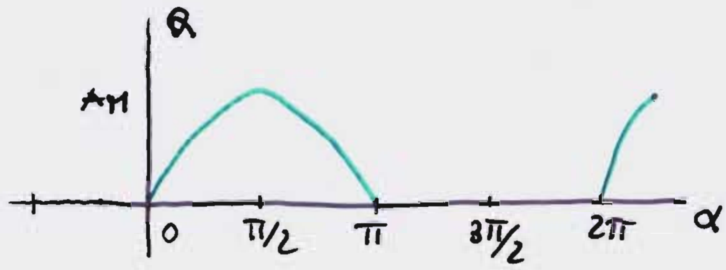
n dispari $e^{-jn\pi} = -1$

$\bar{C}_n = \frac{4A_M}{j2\pi n}$ $n = 1, 3, 5, \ldots$ $\bar{C}_n = 0$ $n = 2, 4, 6$

$|\bar{C}_n| = \frac{2A_M}{\pi n}$

$|A_n| = b_n = 2|\bar{C}_n| = \frac{4A_M}{\pi n}$

SVILUPPARE IN SERIE ESPONENZIALE DI FOURIER




$$\bar{c}_n = \frac{1}{2\pi}\int_0^{2\pi} a(\alpha)\, e^{-jn\alpha}\, d\alpha = \frac{1}{2\pi}\int_0^{\pi} A_M \sin\alpha\, e^{-jn\alpha}\, d\alpha$$

RICORDIAMO CHE:

$$\int_0^x e^{ax} \sin bx\, dx = \frac{a \sin bx - b\cos bx}{a^2+b^2} \cdot e^{ax}$$

$$\int_0^\alpha e^{-jn\alpha} \sin\alpha\, d\alpha = \frac{-jn\sin\alpha - \cos\alpha}{-n^2+1}\, e^{-jn\alpha}$$

$$\bar{c}_n = \frac{A_M}{2\pi}\left[\frac{-jn\sin\pi - \cos\pi}{-n^2+1}\, e^{-jn\pi} - \frac{-jn\sin 0 - \cos 0}{-n^2+1}\, e^{-jn0}\right]$$

$$= \frac{A_M}{2\pi(1-n^2)} \cdot \left[e^{-jn\pi} + 1\right]$$

$n$ PARI $\quad e^{-jn\pi} = 1 \rightarrow \bar{c}_n = \frac{A_M}{\cancel{2}\pi(1-n^2)} \cdot \cancel{2} = \frac{A_M}{\pi(1-n^2)}$

$n$ DISPARI $\neq 1 \quad e^{-jn\pi} = -1 \rightarrow \bar{c}_n = 0$

$$n=1 \qquad \bar{c}_1 = -j\,\frac{A_M}{4}$$

$$a(t) = \frac{A_M}{\pi} + j\,\frac{A_M}{4}\, e^{j\omega t} - j\,\frac{A_M}{4}\, e^{-j\omega t} - \frac{A_M}{3\pi}\, e^{j2\omega t} - \frac{A_M}{3\pi}\, e^{-j2\omega t} + \dots$$