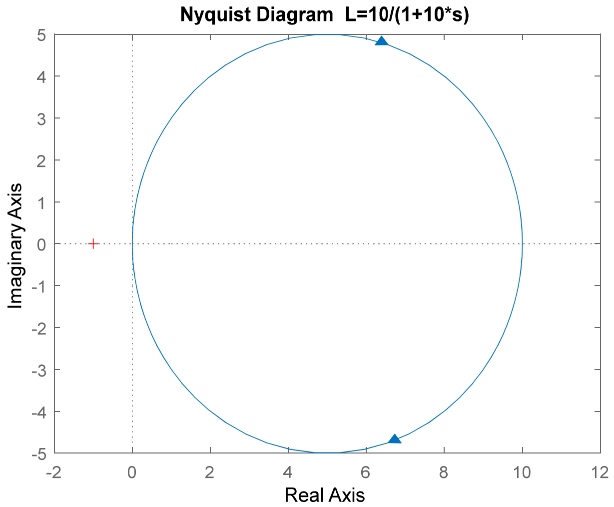
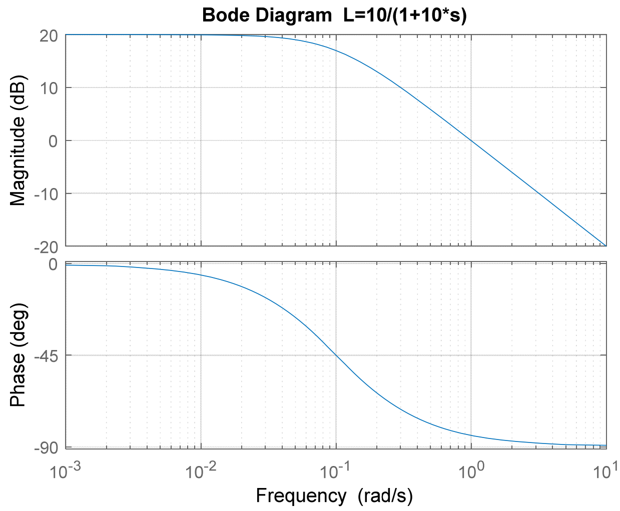
L=10/(1+10*s)

nyquist(L)

bode(L)

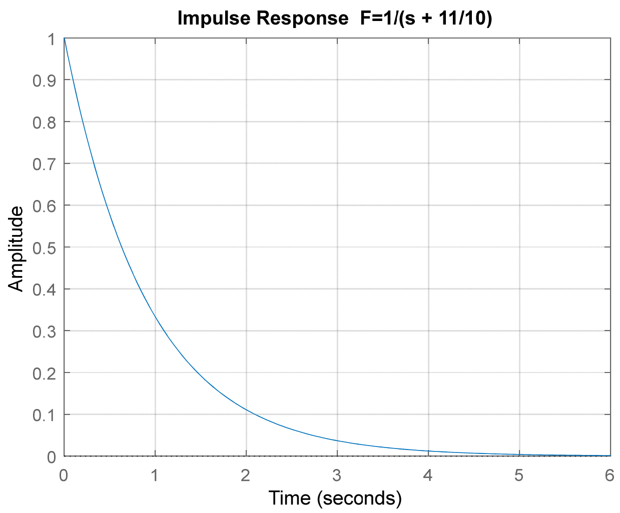


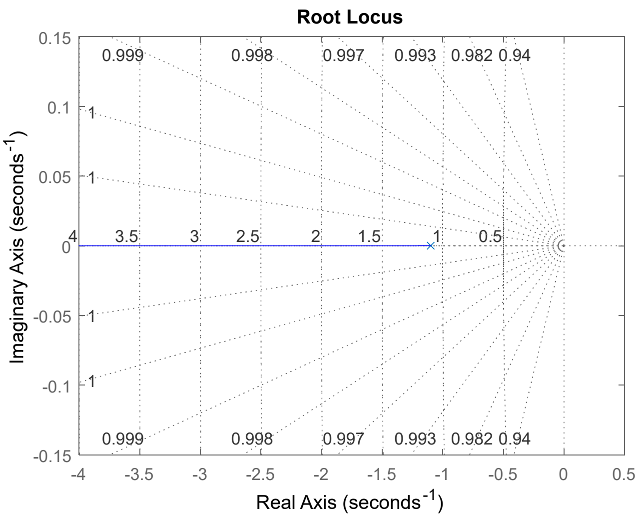


1
F = -------
(s+1.1)

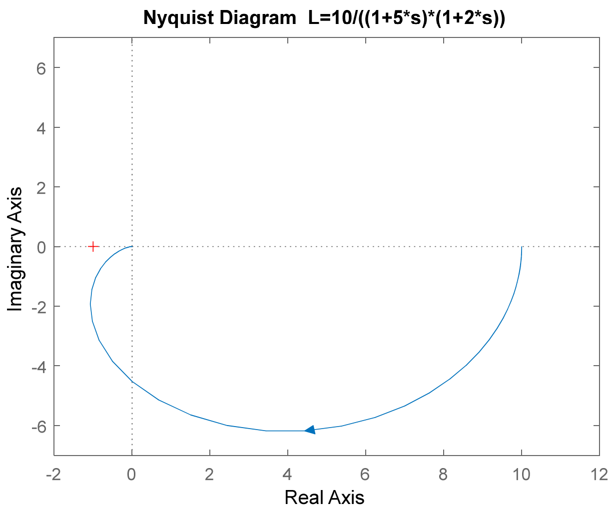
F = feedback(L,1)

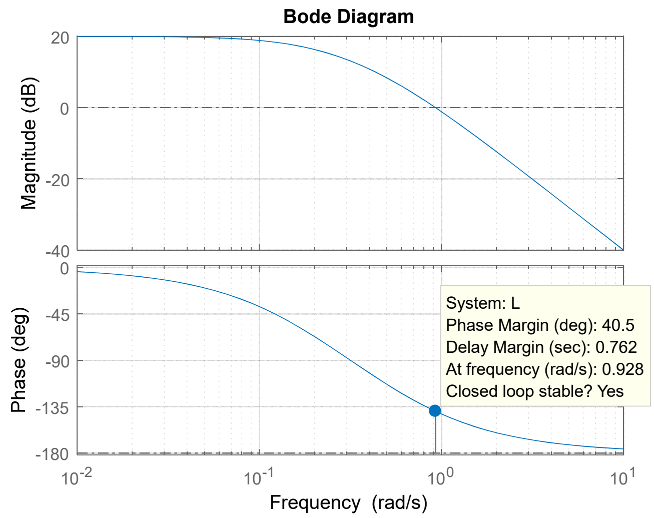
f = exp(-(11*t)/10)





L=10/((1+5*s)*(1+2*s))   nyquist(L)   bode(L)

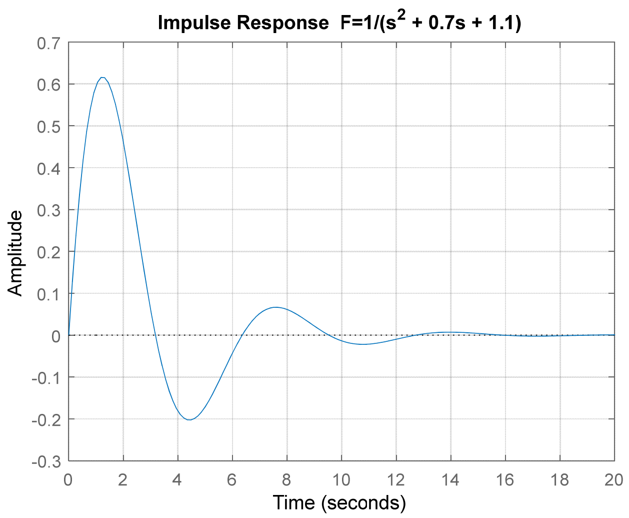


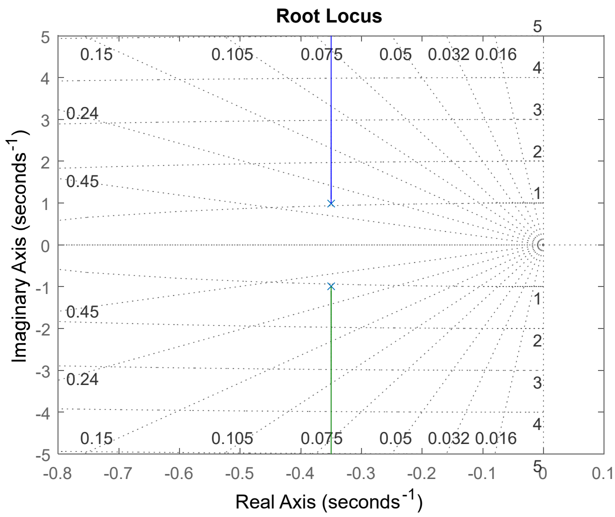


$$F = \frac{1}{(s^2 + 0.7s + 1.1)}$$

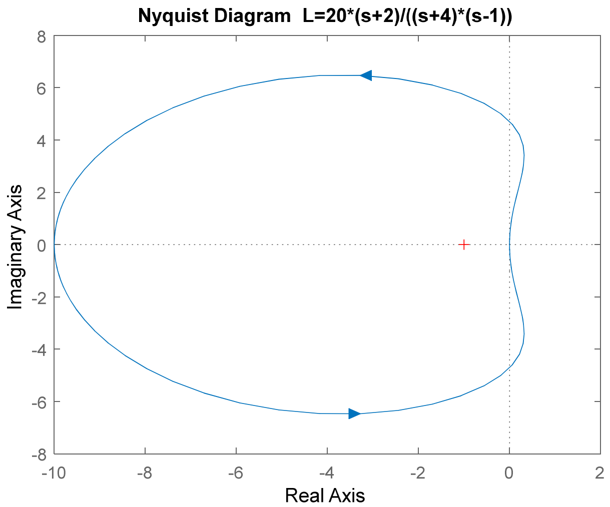
F = feedback(L,1)

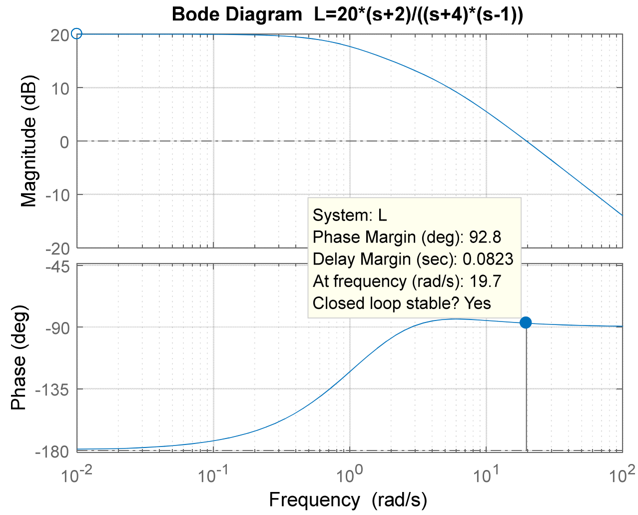
f = (20*391^(1/2)*exp(-(7*t)/20)*sin((391^(1/2)*t)/20))/391





L=20*(s+2)/((s+4)*(s-1))   nyquist(L)   bode(L)

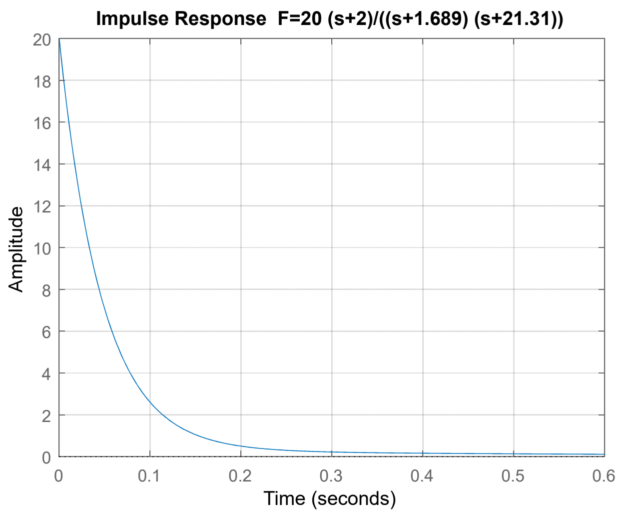


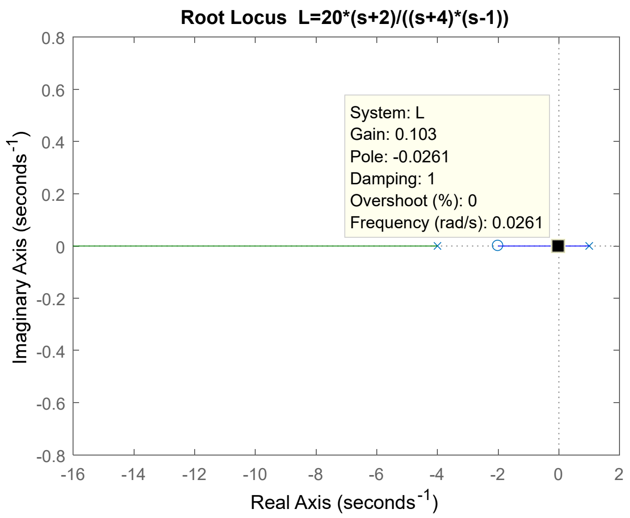


$$F = \frac{20\ (s+2)}{(s+1.689)\ (s+21.31)}$$

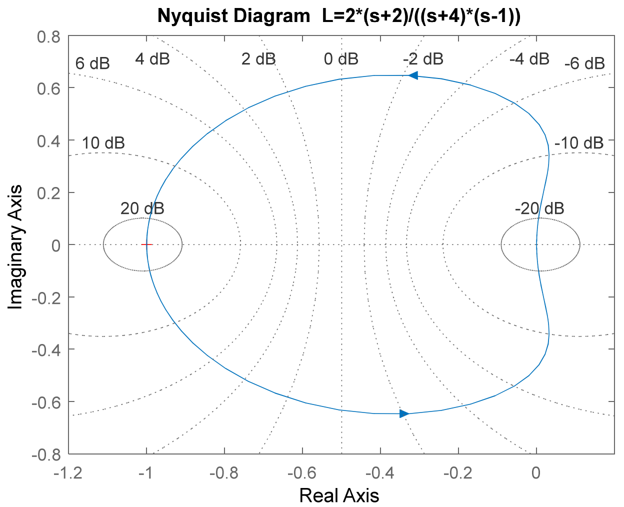
F = feedback(L,1)

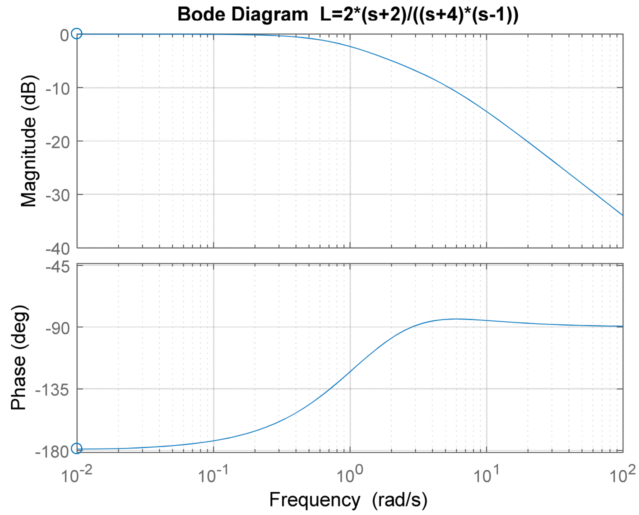
f=(386200*exp(-(2131*t)/100))/19621 + (6220*exp(-(1689*t)/1000))/19621
(per tracciare la funz f con geogebra basta cambiare la variabile t con x)





L=2*(s+2)/((s+4)*(s-1))   nyquist(L)   bode(L)



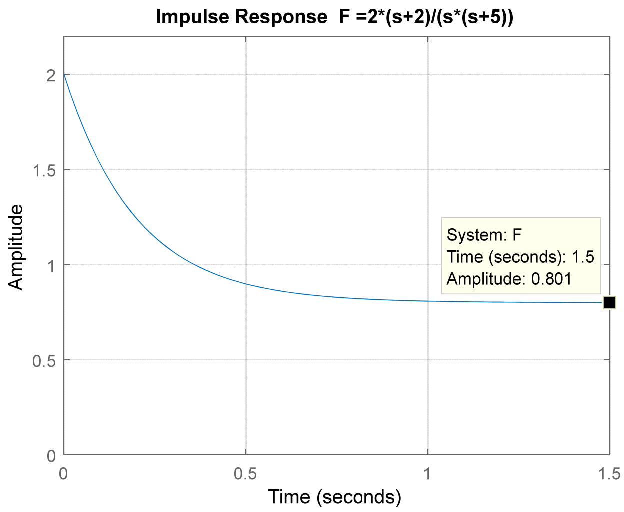


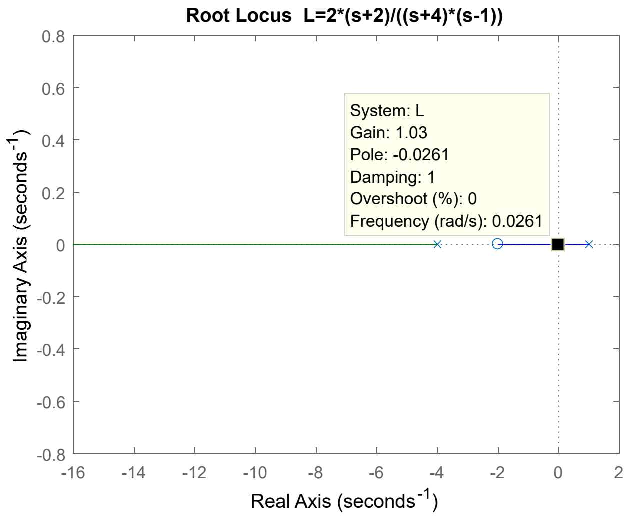
$$F = \frac{2\,(s+2)}{s\,(s+5)}$$

F = feedback(L,1)   f=(6*exp(-5*t))/5 + 4/5   con Matlab   impulseplot(F)

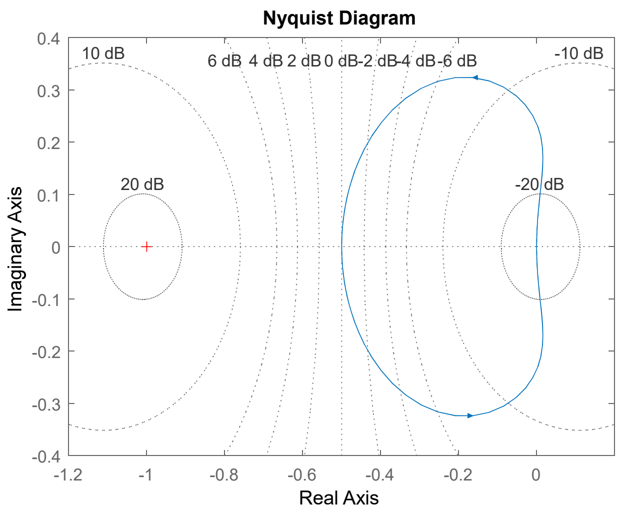
(per tracciare la funz f con geogebra basta cambiare la variabile t con x)

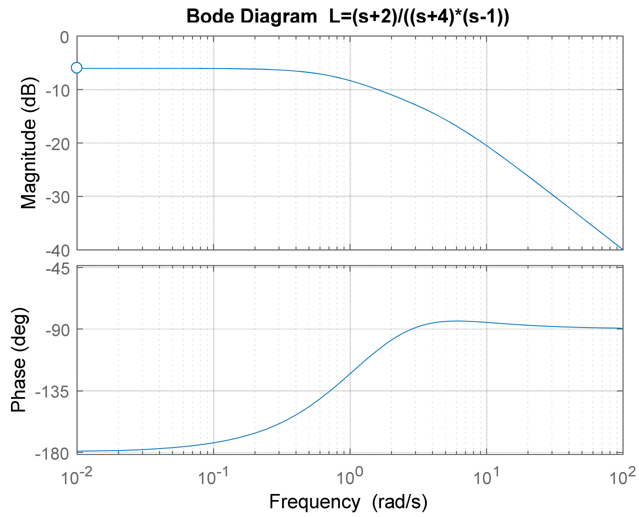
impulseplot(F)   rlocus(L)





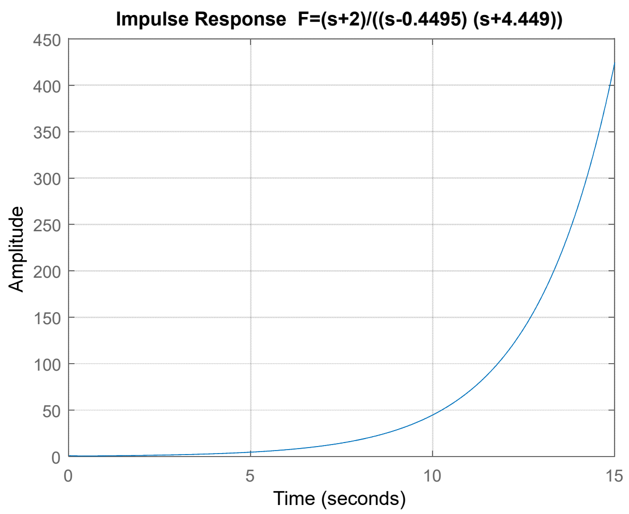
L=(s+2)/((s+4)*(s-1))   nyquist(L)   bode(L)

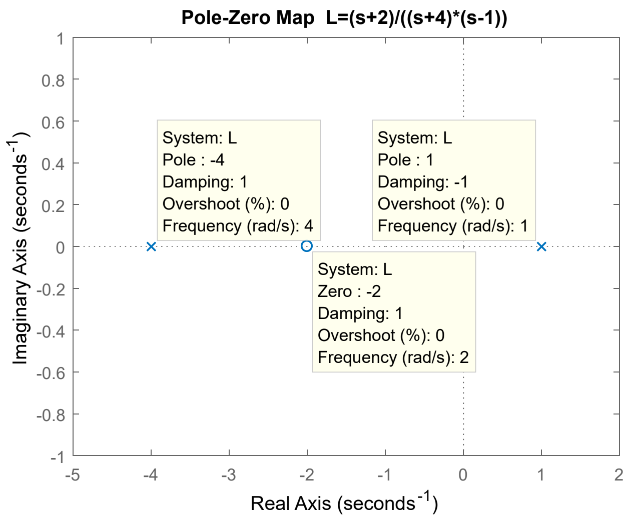





```
      (s+2)
F = -------------------        F = feedback(L,1)      f= (4899*exp((899*t)/2000))/9797 + (4898*exp(-(4449*t)/1000))/9797
   (s-0.4495) (s+4.449)
```

impulseplot(F)   pzmap(L)





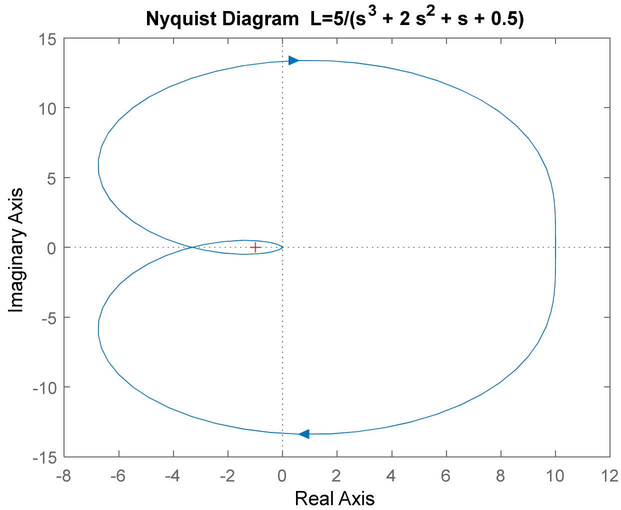
Per l’antitrasformata   [da   s=zpk('s');   oppure   s=tf('s');]

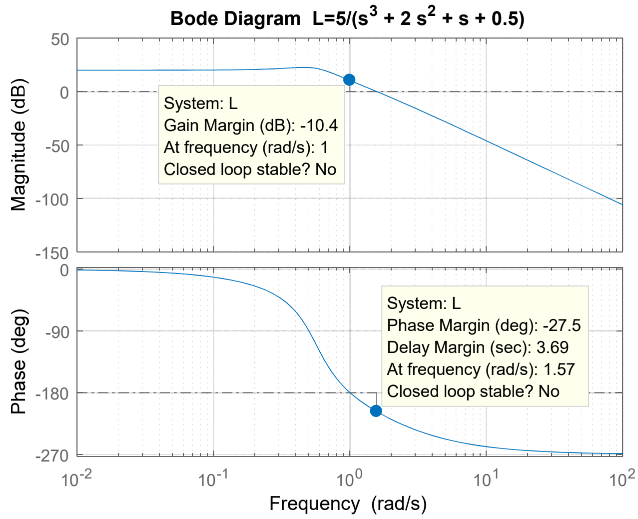
```
syms s t;
F=(s+2)/((s-0.4495)*(s+4.449));
f = ilaplace (F)
```

Nl=[0,5]; Dl=[1 2 1 0.5];

L=tf(Nl,Dl) L = 5/(s^3 + 2 s^2 + s + 0.5)

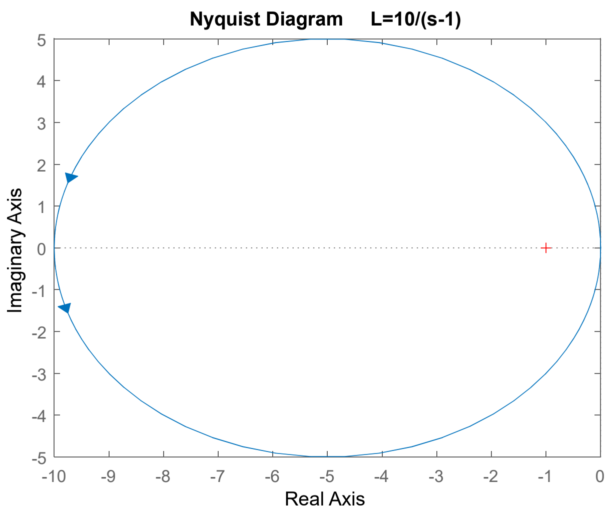
L=zpk(L) L = 5/((s+1.565) (s^2 + 0.4348s + 0.3194))

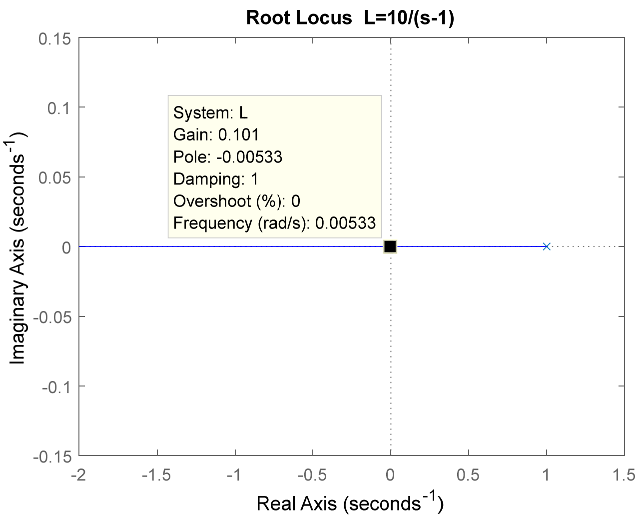




s=zpk('s');

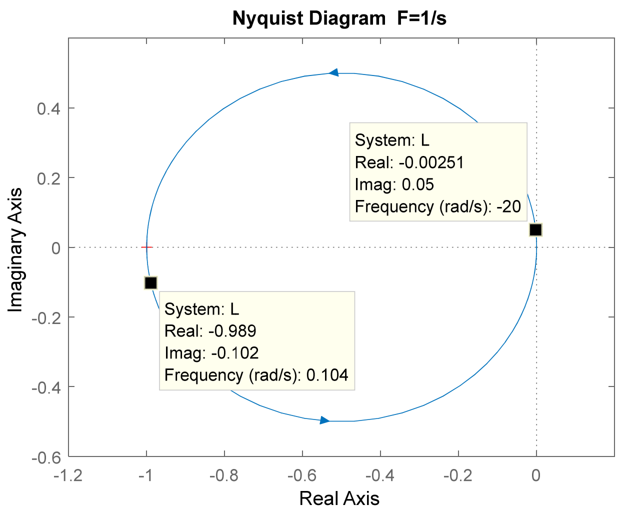
L=10/(s-1) F = 10/(s+9)

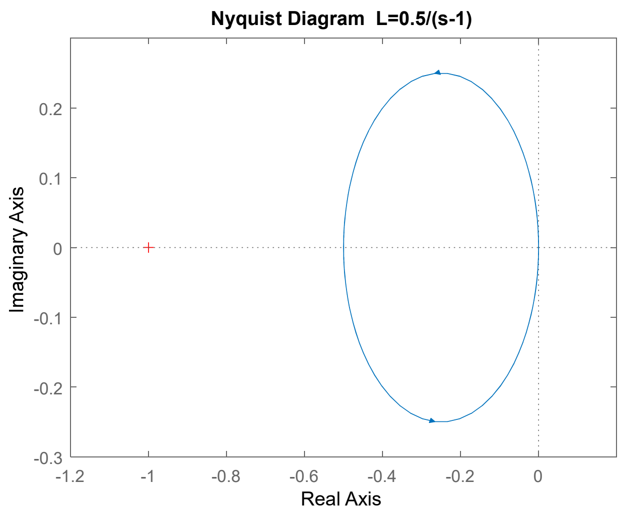




L=1/(s-1) F = 1/ s

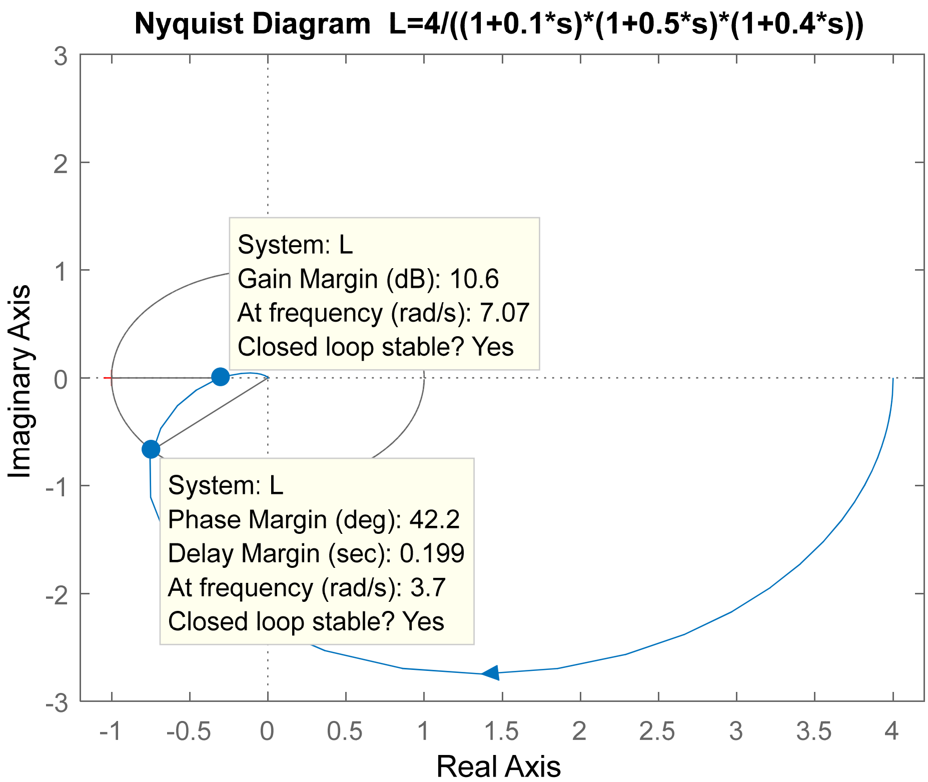
L=0.5/(s-1) F = 0.5/ (s-0.5)

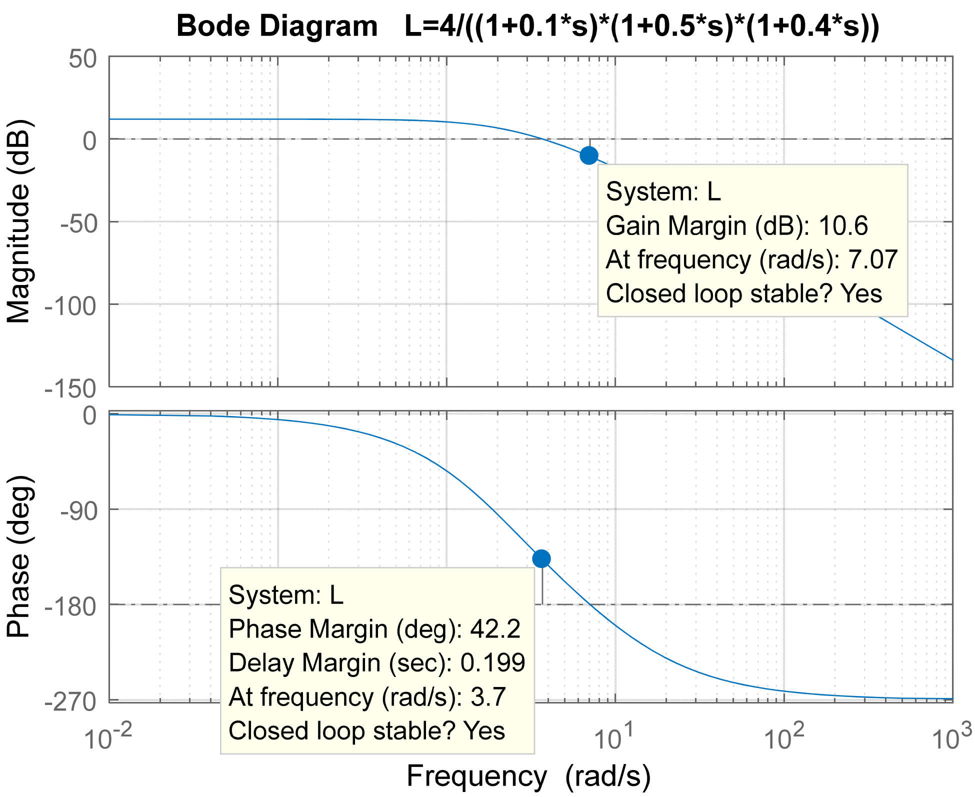




L=4/((1+0.1*s)*(1+0.5*s)*(1+0.4*s))

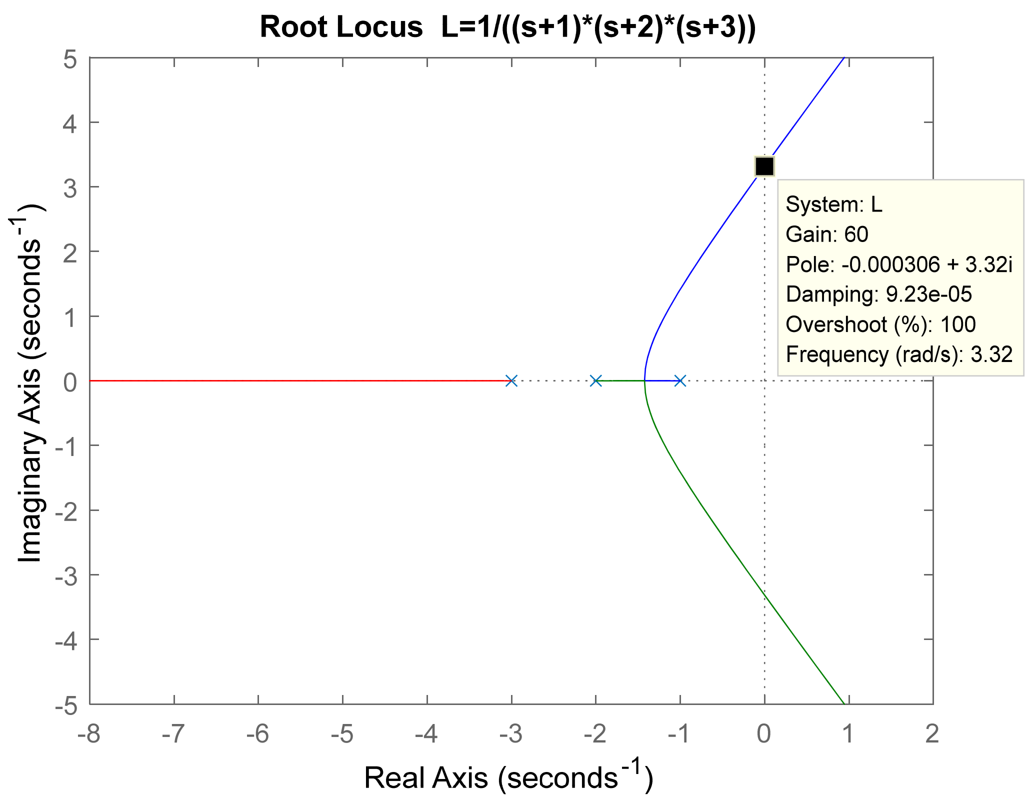
F= 200/((s+12.07) (s^2 + 2.426s + 20.71))

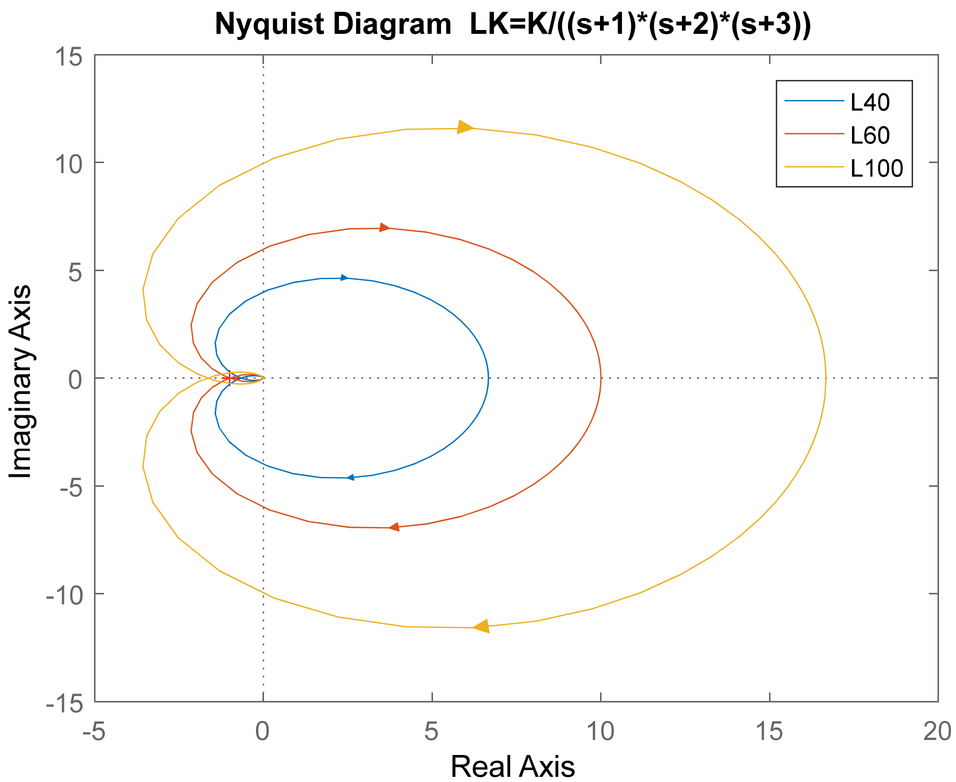


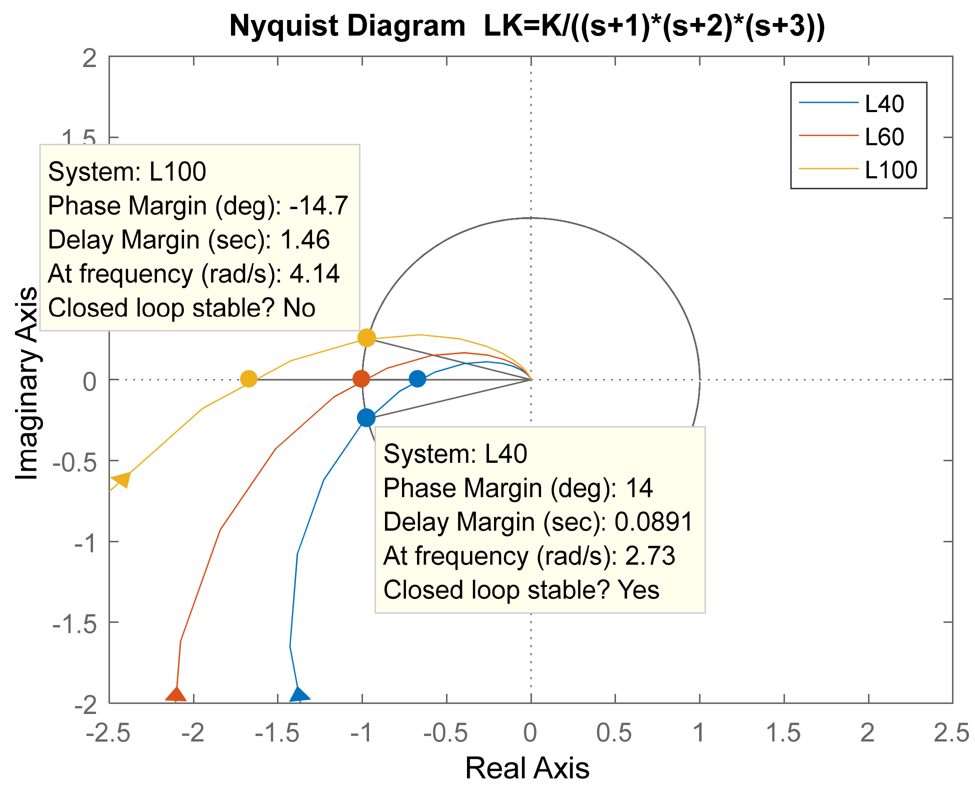


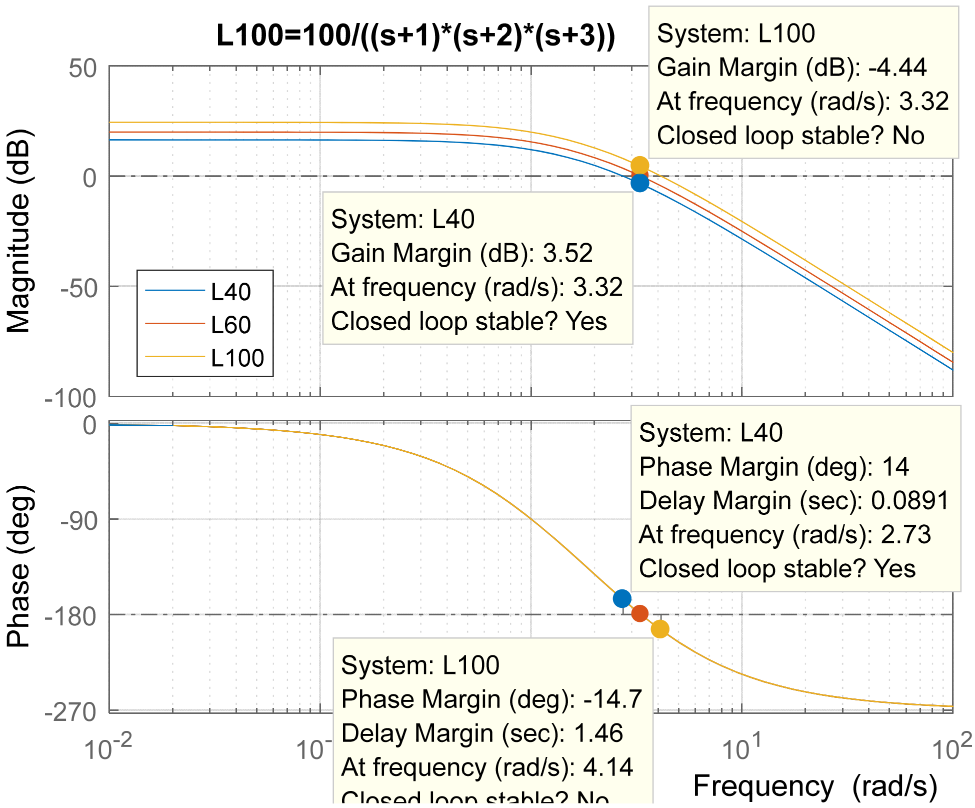
L=K/((s+1)*(s+2)*(s+3))

instabile per K>60





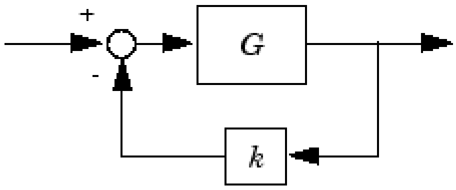

Nyquist Diagram LK=K/((s+1)*(s+2)*(s+3))
L40
L60
L100
System: L100
Phase Margin (deg): -14.7
Delay Margin (sec): 1.46
At frequency (rad/s): 4.14
Closed loop stable? No
System: L40
Phase Margin (deg): 14
Delay Margin (sec): 0.0891
At frequency (rad/s): 2.73
Closed loop stable? Yes
Imaginary Axis
Real Axis
2
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5


L100=100/((s+1)*(s+2)*(s+3))
System: L100
Gain Margin (dB): -4.44
At frequency (rad/s): 3.32
Closed loop stable? No
System: L40
Gain Margin (dB): 3.52
At frequency (rad/s): 3.32
Closed loop stable? Yes
L40
L60
L100
Magnitude (dB)
50
0
-50
-100
System: L40
Phase Margin (deg): 14
Delay Margin (sec): 0.0891
At frequency (rad/s): 2.73
Closed loop stable? Yes
System: L100
Phase Margin (deg): -14.7
Delay Margin (sec): 1.46
At frequency (rad/s): 4.14
Phase (deg)
0
-90
-180
-270
Frequency (rad/s)

Esercizio 6

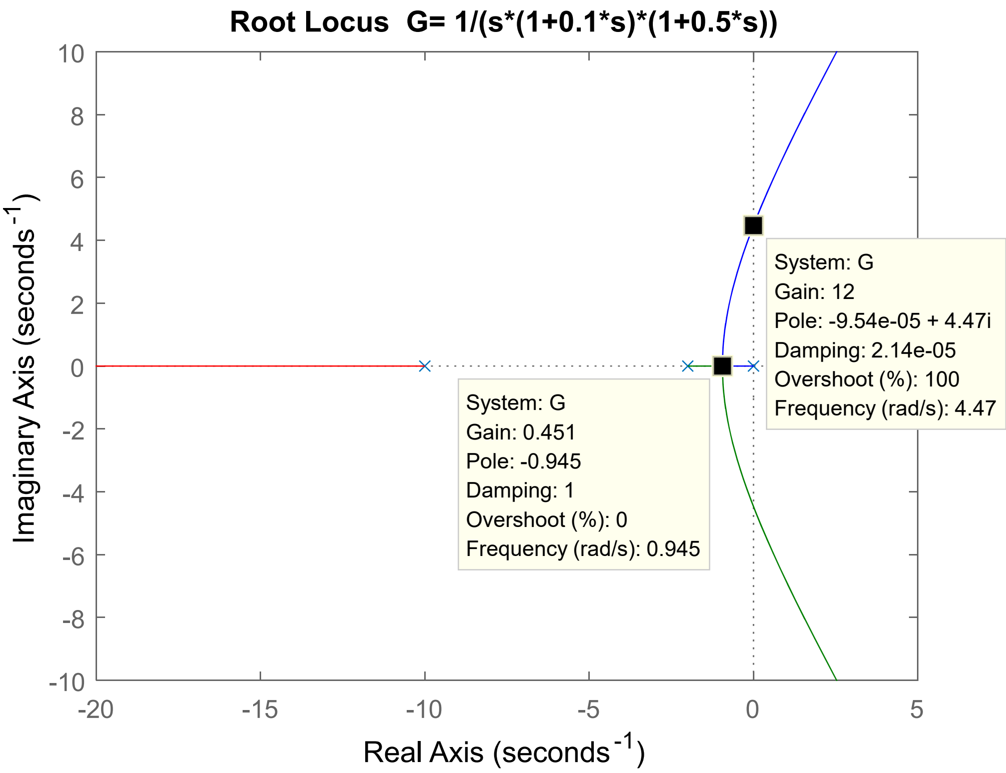
s=zpk('s');

G=1/((s*(1+0.1*s)*(1+(0.5*s)))   H=k



$$GH = \frac{20\ k}{s\ (s+10)\ (s+2)}$$

sis stabile ad anello aperto (no poli a parte reale >0)



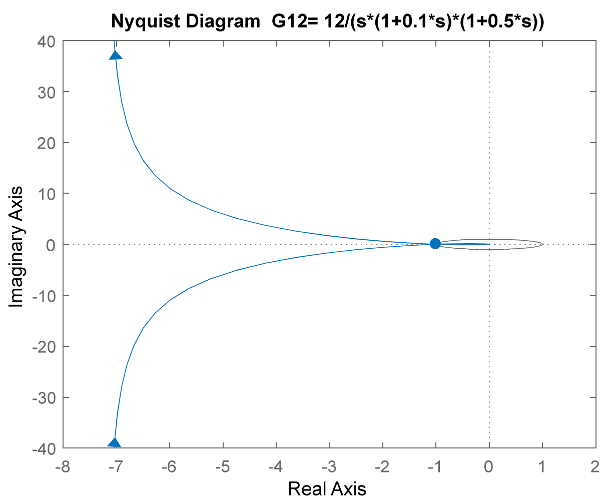
Con k=0.451   radici coincidenti

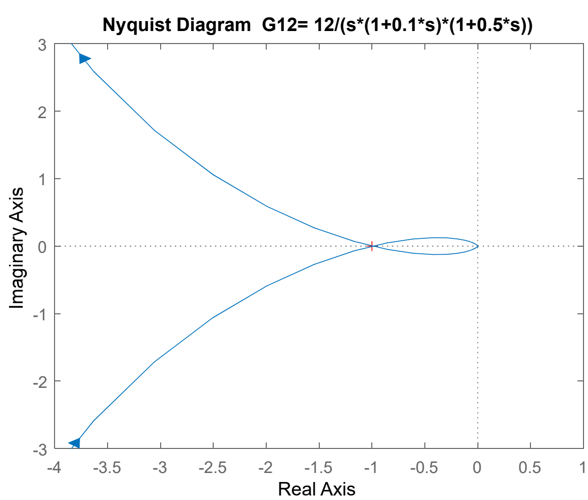
$$W = \frac{9.02}{(s+10.11)\ (s+0.945)^2}$$

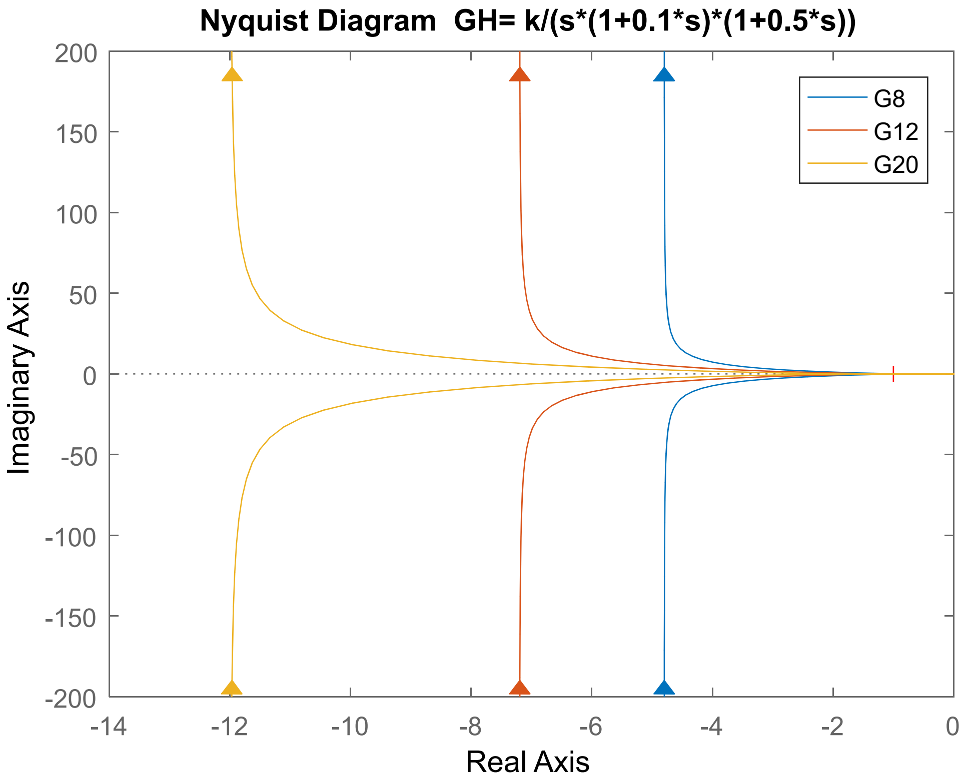
Con k=12   radici sull'asse immaginario

$$W = \frac{240}{(s+12)\ (s^2 + 20)}$$

Stabile ad anello chiuso per   0 < k < 12

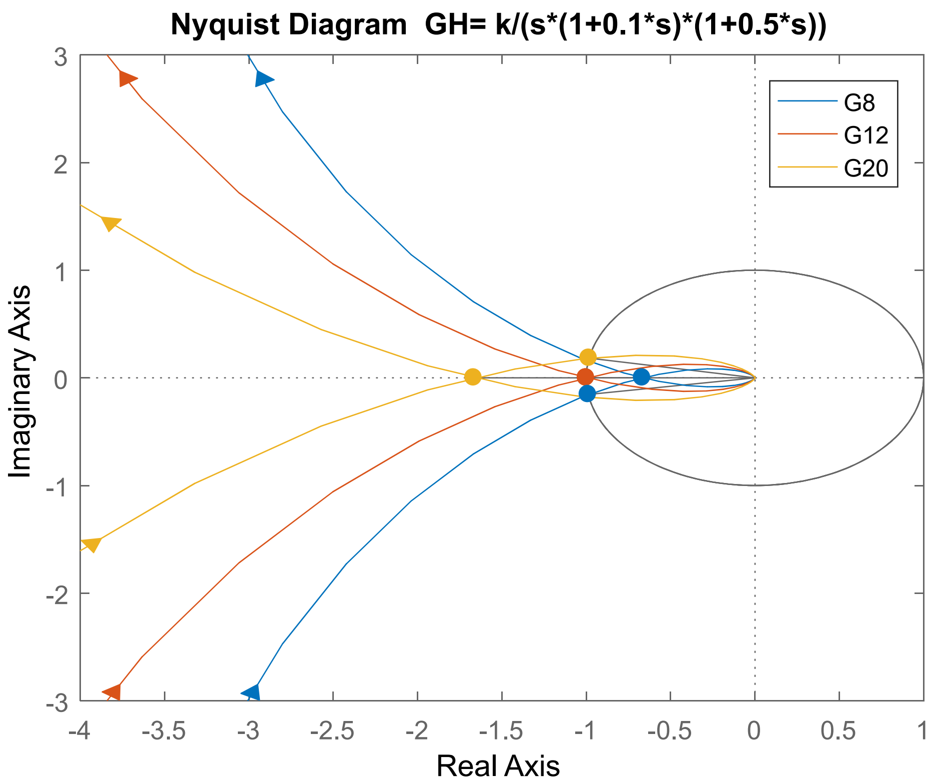


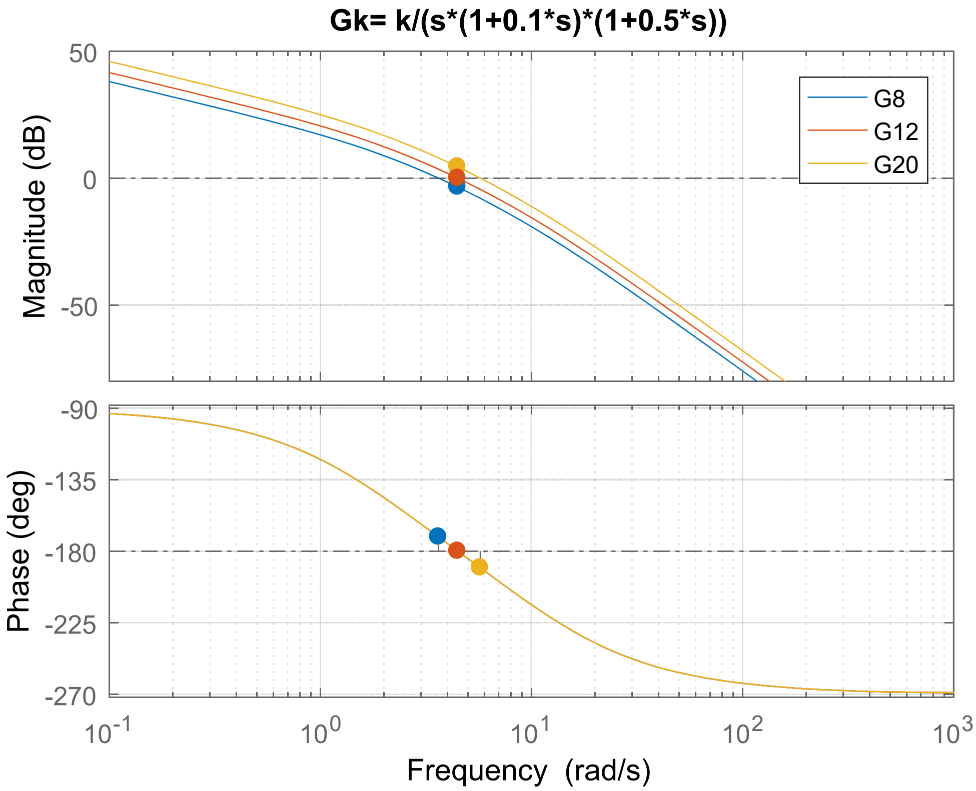


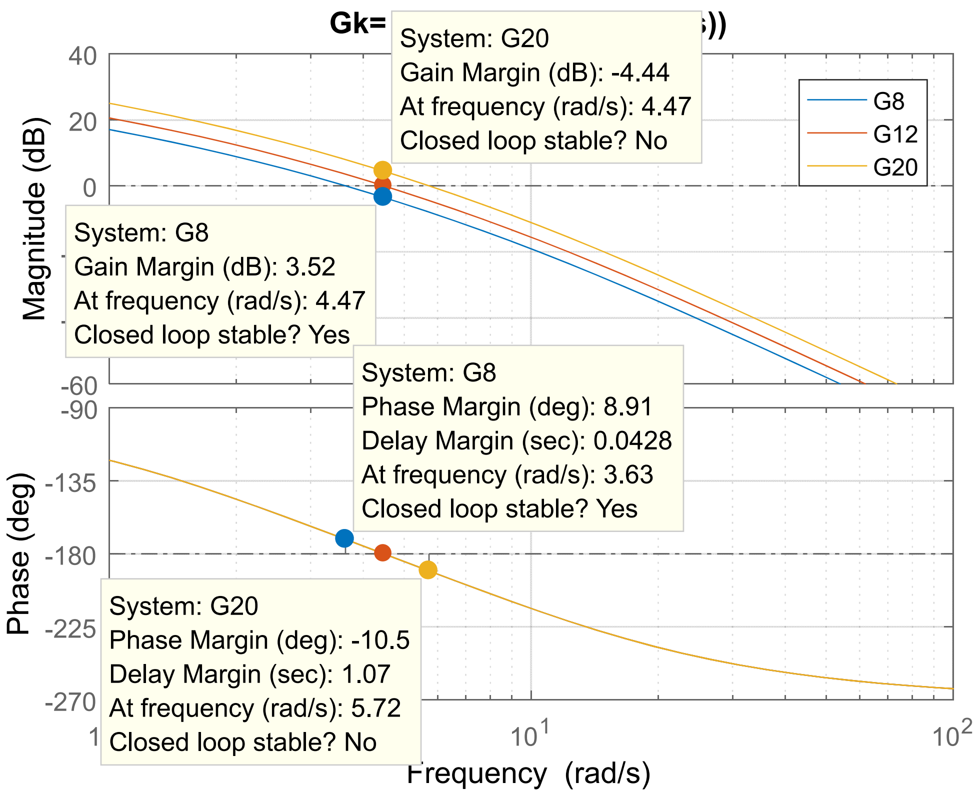


Asintoto per ω → 0 -0.6·k

Sguardo intorno al punto -1




Gk= k/(s*(1+0.1*s)*(1+0.5*s))
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
G8
G12
G20


System: G20
Gain Margin (dB): -4.44
At frequency (rad/s): 4.47
Closed loop stable? No
System: G8
Gain Margin (dB): 3.52
At frequency (rad/s): 4.47
Closed loop stable? Yes
System: G8
Phase Margin (deg): 8.91
Delay Margin (sec): 0.0428
At frequency (rad/s): 3.63
Closed loop stable? Yes
System: G20
Phase Margin (deg): -10.5
Delay Margin (sec): 1.07
At frequency (rad/s): 5.72
Closed loop stable? No
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
G8
G12
G20

Esercizio 6

La funzione di trasferimento ad anello aperto del sistema è:

$$G(s)\cdot H(s)=\frac{k}{s\cdot(1+0{,}1\cdot s)\cdot(1+0{,}5\cdot s)}$$

Il sistema è stabile ad anello aperto perché la sua funzione di trasferimento non ha poli a parte reale positiva $\left(p_1=0,\ p_2=-\frac{1}{0{,}1},\ p_3=-\frac{1}{0{,}5}\right)$.

Posto $s=j\cdot\omega$, si ha:

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{k}{j\cdot\omega\cdot(1+j\cdot 0{,}1\cdot\omega)\cdot(1+j\cdot 0{,}5\cdot\omega)}$$

$$\Phi=-\mathrm{arctg}\frac{\omega}{0}-\mathrm{arctg}\frac{0{,}1\cdot\omega}{1}-\mathrm{arctg}\frac{0{,}5\cdot\omega}{1}$$

- per $\omega=0$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|\to\infty\\ \Phi=-90°\end{cases}$

- per $\omega\to\infty$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|\to 0\\ \Phi\to-270°\end{cases}$

La pulsazione in corrispondenza della quale il diagramma interseca l'asse reale è

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{-j\cdot k\cdot(1-j\cdot 0{,}1\cdot\omega)\cdot(1-j\cdot 0{,}5\cdot\omega)}{\omega\cdot\left[1+(0{,}1\cdot\omega)^2\right]\cdot\left[1+(0{,}5\cdot\omega)^2\right]}$$

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{-j\cdot k+j\cdot 0{,}05\cdot k\cdot\omega^2-0{,}5\cdot k\cdot\omega-0{,}1\cdot k\cdot\omega}{\omega\cdot\left[1+(0{,}1\cdot\omega)^2\right]\cdot\left[1+(0{,}5\cdot\omega)^2\right]}$$

$$\mathrm{Re}\left[G(j\cdot\omega)\cdot H(j\cdot\omega)\right]=\frac{-0{,}6\cdot k\cdot\omega}{\omega\cdot\left[1+(0{,}1\cdot\omega)^2\right]\cdot\left[1+(0{,}5\cdot\omega)^2\right]}$$

$$\mathrm{Im}\left(G(j\cdot\omega)\cdot H(j\cdot\omega)\right)=\frac{j\cdot k\cdot\left(0{,}05\cdot k\cdot\omega^2-1\right)}{\omega\cdot\left[1+(0{,}1\cdot\omega)^2\right]\cdot\left[1+(0{,}5\cdot\omega)^2\right]}$$

$$\frac{k\cdot\left(0{,}05\cdot k\cdot\omega^2-1\right)}{\omega\cdot\left[1+(0{,}1\cdot\omega)^2\right]\cdot\left[1+(0{,}5\cdot\omega)^2\right]}=0$$

$$\omega_c^2=\frac{1}{0{,}1\cdot 0{,}5}$$

Il modulo della funzione di trasferimento ad anello aperto calcolato in corrispondenza di ω_c è uguale a

$$\left|G(j\cdot\omega)\cdot H(j\cdot\omega)\right|=\frac{k}{\sqrt{\frac{1}{0{,}05}}\cdot\sqrt{1+\frac{0{,}01}{0{,}05}}\cdot\sqrt{1+\frac{0{,}25}{0{,}05}}}=\frac{k}{12}$$

Il sistema è:

- stabile quando è $0<k<12$ perché il diagramma di Nyquist non circonda il punto $-1+j\cdot 0$;
- instabile quando è $k>12$ perché in quest'ultimo caso si ha $N=2$ e $P=0$ (fig. 5.5.53).

L'asintoto è il limite per $\omega\to 0$ della parte reale della funzione $G(j\cdot\omega)\cdot H(j\cdot\omega)$

$$\lim_{\omega\to 0}\mathrm{Re}\left[G(j\omega)\cdot H(j\omega)\right]=\lim_{\omega\to 0}\frac{-0{,}6\cdot k\cdot\omega}{\omega\cdot\left[1+(0{,}1\omega)^2\right]\cdot\left[1+(0{,}5\omega)^2\right]}=-0{,}6\cdot k$$

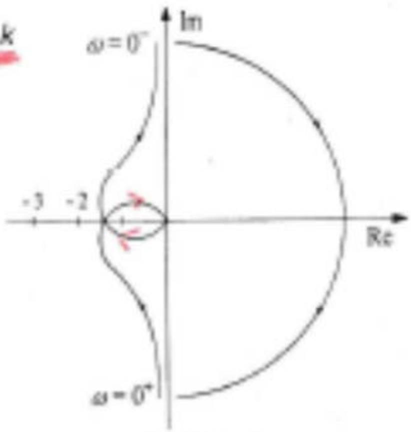


Fig. 5.5.53

MATLAB

```
% Modulo 5 Unità didattica 5 Esercizio 6
G1=tf(30, [1 0]); G2=tf(1,[0.1 1]); G3=tf(1,[0.5 1])
GH=G1*G2*G3
ltiview
```

Per osservare la zona significativa del diagramma utilizzare i pulsanti di zoom.

Esercizio 7

La funzione di trasferimento ad anello aperto del sistema è:

$$G(s)\cdot H(s)=\frac{k}{s^2\cdot(1+0{,}5\cdot s)}$$

Il sistema è stabile ad anello aperto perché la sua funzione di trasferimento non ha poli a parte reale positiva $\left(p_1=p_2=0,\ p_3=-\frac{1}{0{,}5}\right)$.

Posto $s=j\cdot\omega$, si ha:

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{k}{(j\cdot\omega)^2\cdot(1+j\cdot 0{,}5\cdot\omega)}$$

$$\Phi=-180^\circ-\mathrm{arctg}\frac{0{,}5\cdot\omega}{1}$$

- per $\omega=0$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|\to\infty\\ \Phi=-180^\circ\end{cases}$
- per $\omega\to\infty$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|\to 0\\ \Phi\to-270^\circ\end{cases}$

Il sistema è instabile ad anello chiuso perché è $N=2$ e $P=0$: il diagramma di Nyquist circonda in verso orario il punto $-1+j\cdot 0$ (fig. 5.5.54).

MATLAB

```
% Modulo 5 Unità didattica 5 Esercizio 7
G1=tf(10, [1 0 0]); G2=tf(1,[0.5 1])
GH=G1*G2
ltiview
```

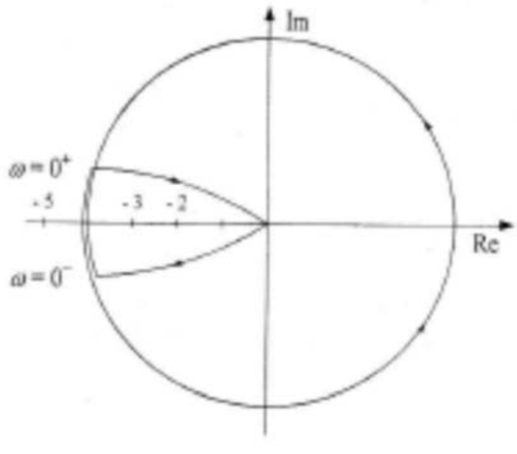
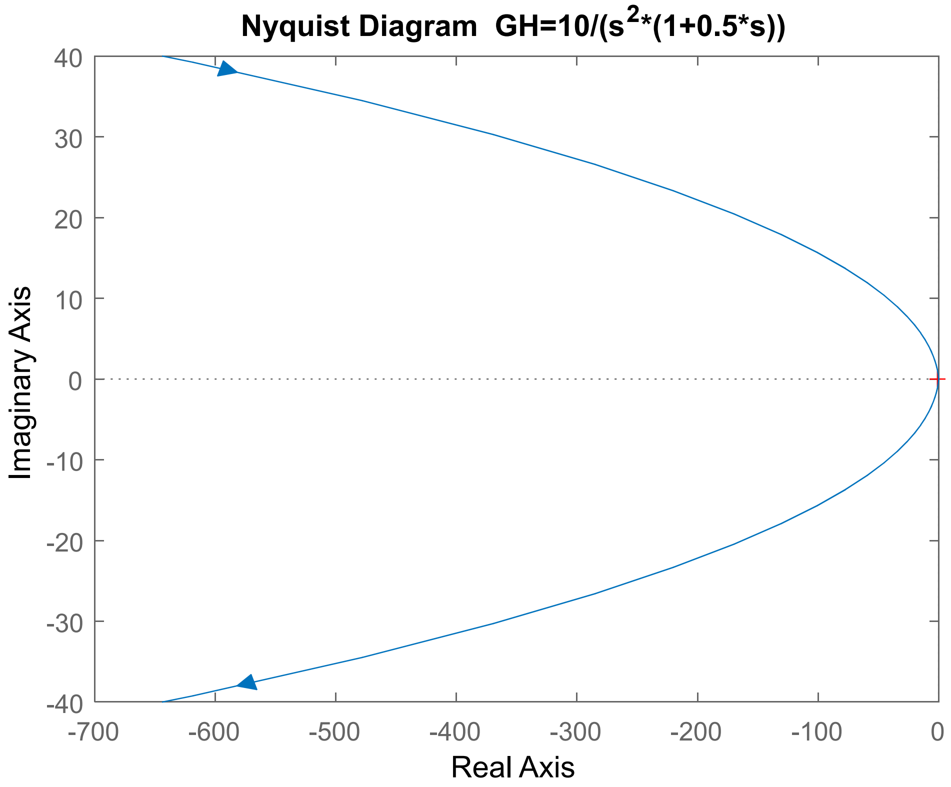


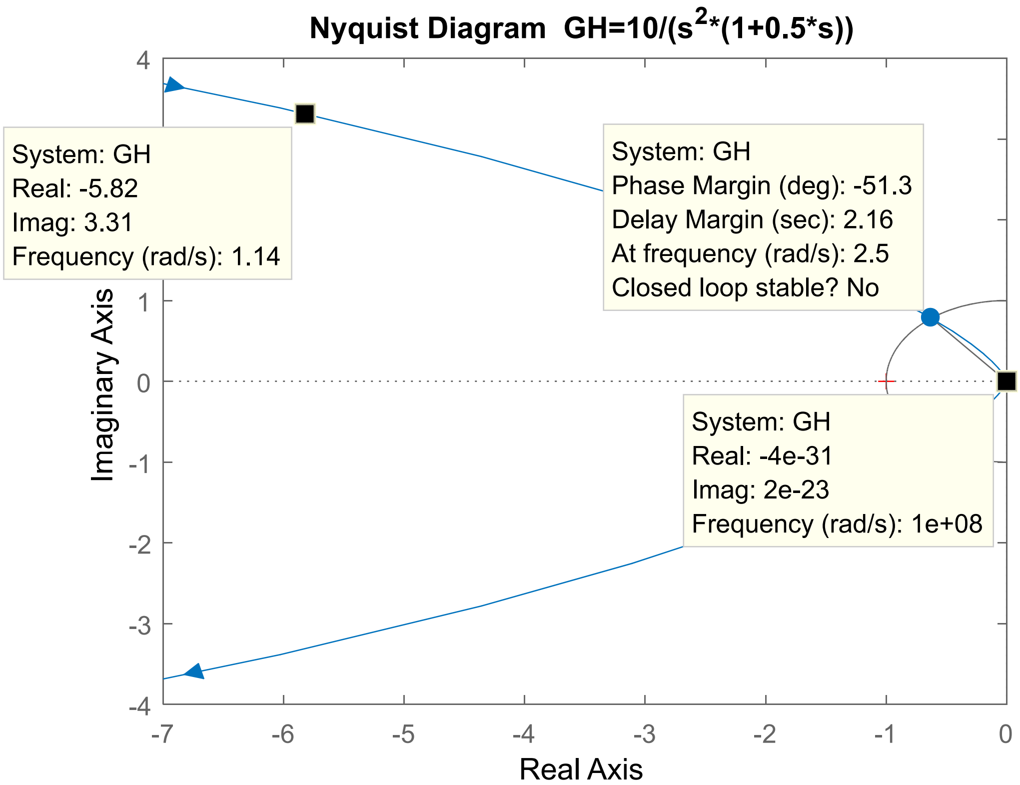
Fig. 5.5.54

Esercizio 7

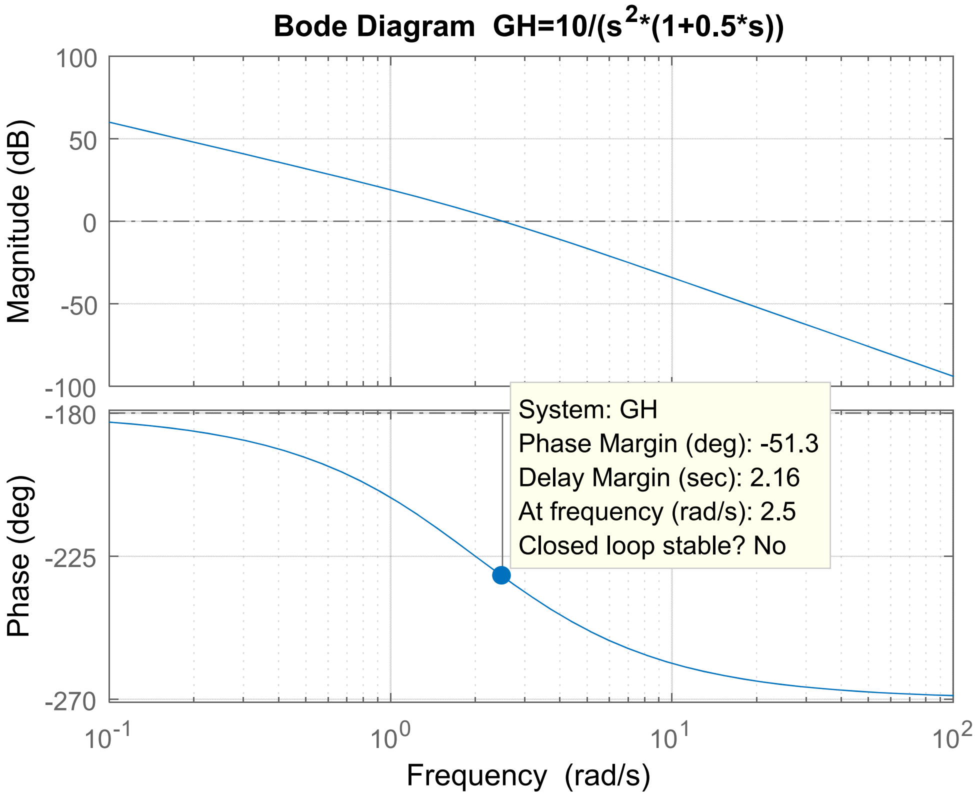
GH=10/(s^2*(1+0.5*s))

```
         20
GH = -----------
      s^2 (s+2)
```





P = 0 N = 2 stabile ad anello aperto, instabile ad anello chiuso


Bode Diagram GH=10/(s^2*(1+0.5*s))
Magnitude (dB)
100
50
0
-50
-100
Phase (deg)
-180
-225
-270
System: GH
Phase Margin (deg): -51.3
Delay Margin (sec): 2.16
At frequency (rad/s): 2.5
Closed loop stable? No
10^-1
10^0
10^1
10^2
Frequency (rad/s)

Esercizio 8

La funzione di trasferimento ad anello aperto del sistema è:

$$G(s)\cdot H(s)=\frac{10\cdot(1+5\cdot s)}{(1-3\cdot s)}$$

Il sistema è instabile ad anello aperto perché la sua funzione di trasferimento ha un polo positivo $\left(p_1=\frac{1}{3}\right)$.

Posto $s=j\cdot\omega$, si ha:

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{10\cdot(1+j\cdot 5\cdot\omega)}{(1-j\cdot 3\cdot\omega)}$$

$$\Phi=\text{arctg}\frac{5\cdot\omega}{1}-\text{arctg}\frac{-3\cdot\omega}{1}$$

- per $\omega=0$ $\begin{cases}\left|G(j\cdot\omega)\cdot H(j\cdot\omega)\right|=10\\ \Phi=0°\end{cases}$
- per $\omega\to\infty$ $\begin{cases}\left|G(j\cdot\omega)\cdot H(j\cdot\omega)\right|\to\frac{50}{3}\\ \Phi\to 180°\end{cases}$

Il sistema è stabile ad anello chiuso, $N=1$ e $P=1$, perché il diagramma di Nyquist circonda una sola volta il punto $-1+j\cdot 0$ in verso antiorario (fig. 5.5.55).

MATLAB

```
% Modulo 5 Unità didattica 5 Esercizio 8
G1=tf(10, [-3 1]); G2=tf([5 1],[0 1])
GH=G1*G2
ltiview
```

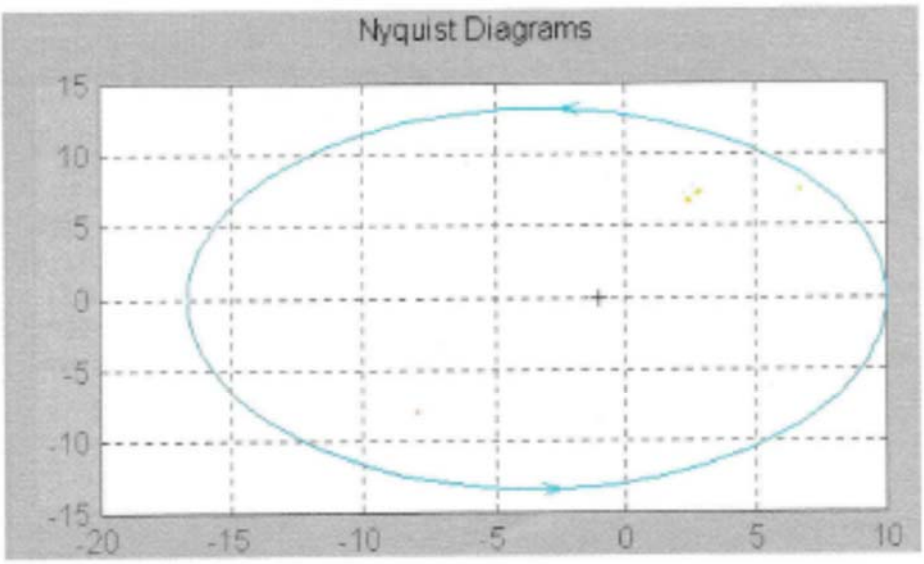
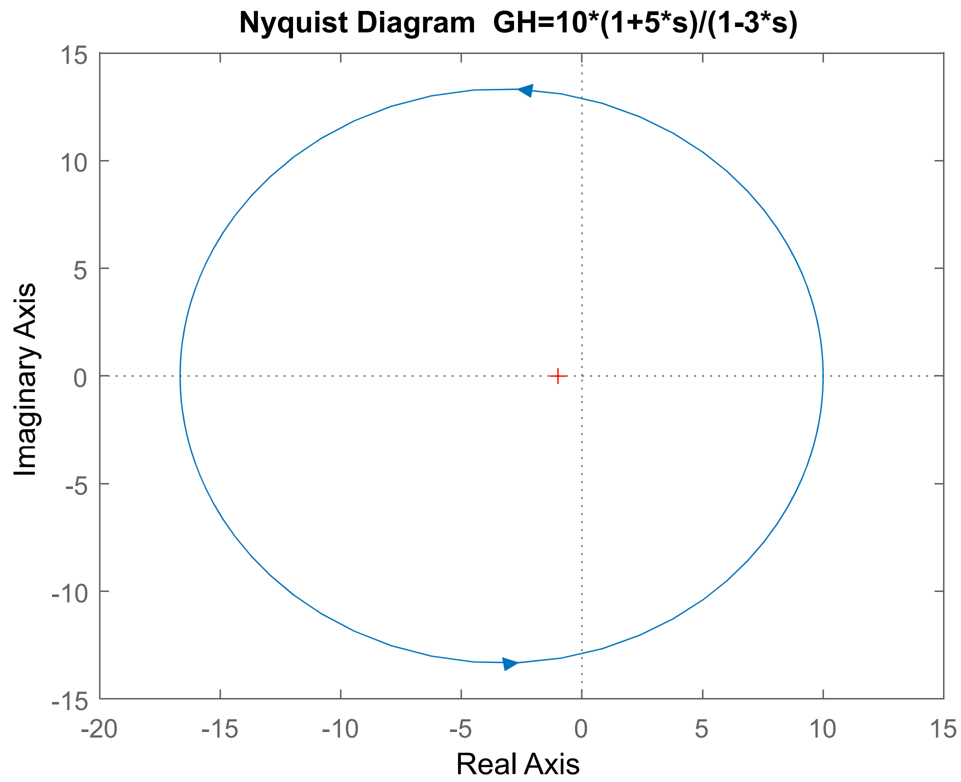


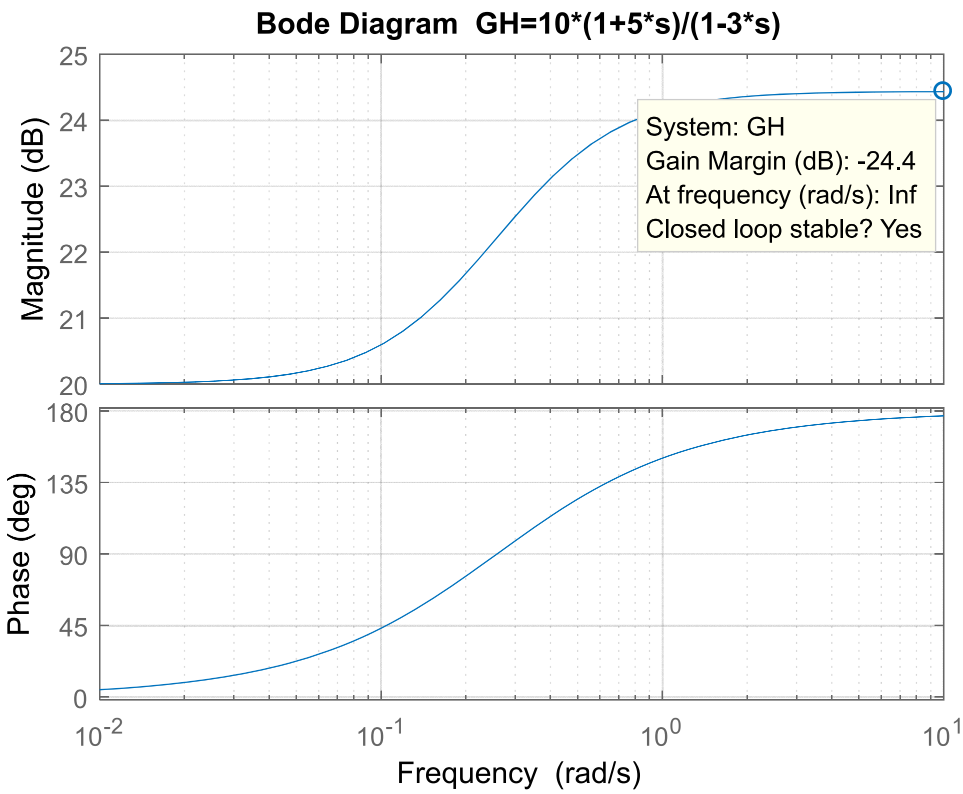
Fig. 5.5.55

Esercizio 8

GH=10*(1+5*s)/(1-3*s)

$$GH = \frac{-16.667\ (s+0.2)}{(s-0.3333)}$$





N=1 P=1 → STABILE ad anello chiuso

Esercizio 9

La funzione di trasferimento ad anello aperto del sistema è:

$$G(s)\cdot H(s)=\frac{10\cdot(1+0{,}1\cdot s)}{(1-5\cdot s)}$$

Il sistema è instabile ad anello aperto perché la sua funzione di trasferimento ha un polo reale positivo $\left(p_1=\frac{1}{5}\right)$.

Posto $s=j\cdot\omega$, si ha:

$$G(j\cdot\omega)\cdot H(j\cdot\omega)=\frac{10\cdot(1+j\cdot 0{,}1\cdot\omega)}{(1-j\cdot 5\cdot\omega)}$$

$$\Phi=\text{arctg}\frac{0{,}1\cdot\omega}{1}-\text{arctg}\frac{-5\cdot\omega}{1}$$

- per $\omega=0$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|=10\\ \Phi=0°\end{cases}$

- per $\omega\to\infty$ $\begin{cases}|G(j\cdot\omega)\cdot H(j\cdot\omega)|\to 0{,}2\\ \Phi\to 180°\end{cases}$

Il sistema è instabile ad anello chiuso, $N=0$ e $P=1$, perché il diagramma di Nyquist non circonda il punto $-1+j\cdot 0$ (fig. 5.5.56).

MATLAB

```
% Modulo 5 Unità didattica 5 Esercizio 9
G1=tf( [0.1 1], [-5 1])
GH=10*G1
ltiview
```

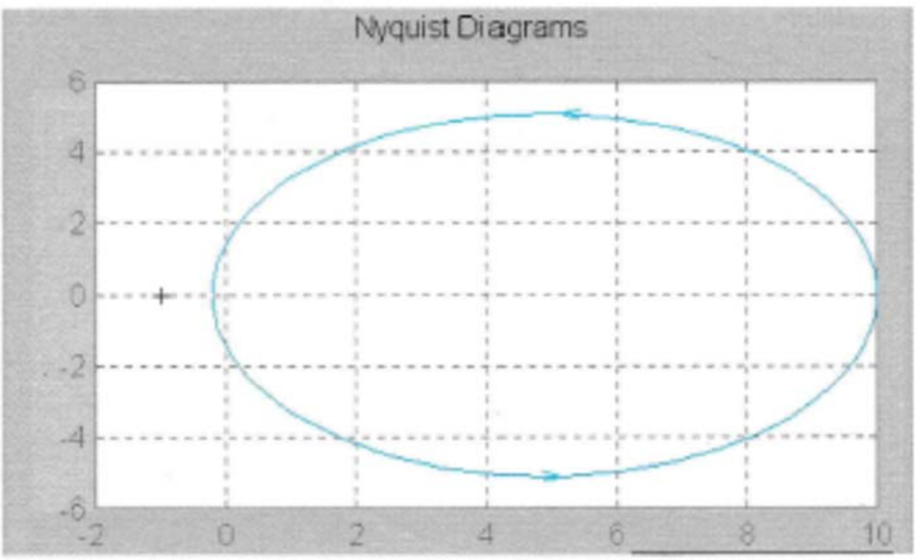


Fig. 5.5.56

Esercizio 9

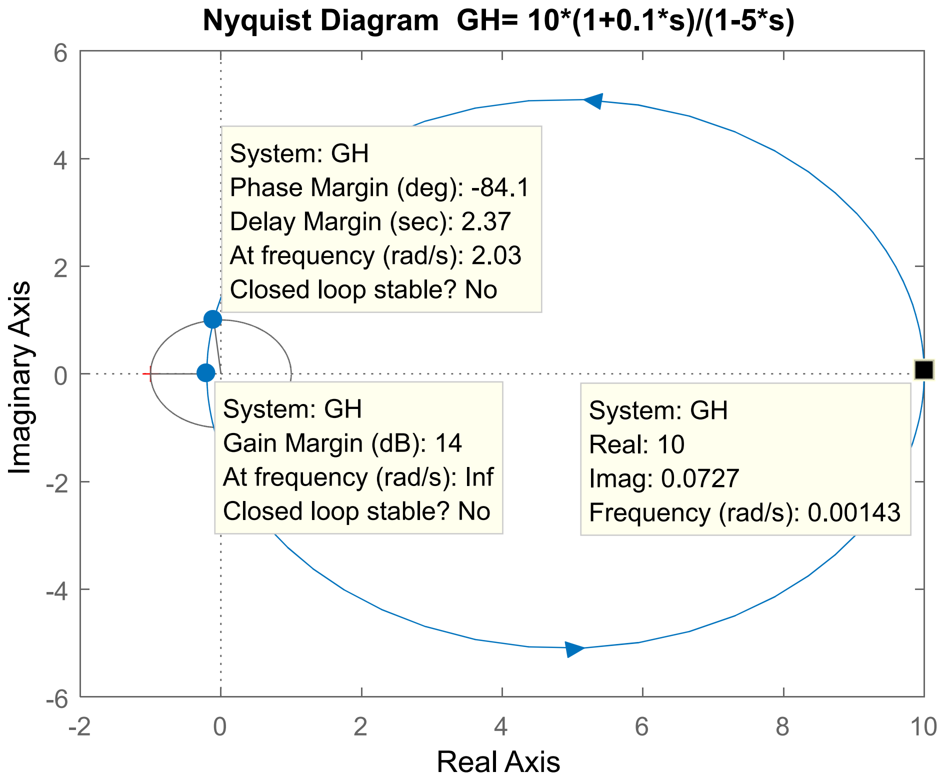
GH= 10*(1+0.1*s)/(1-5*s)

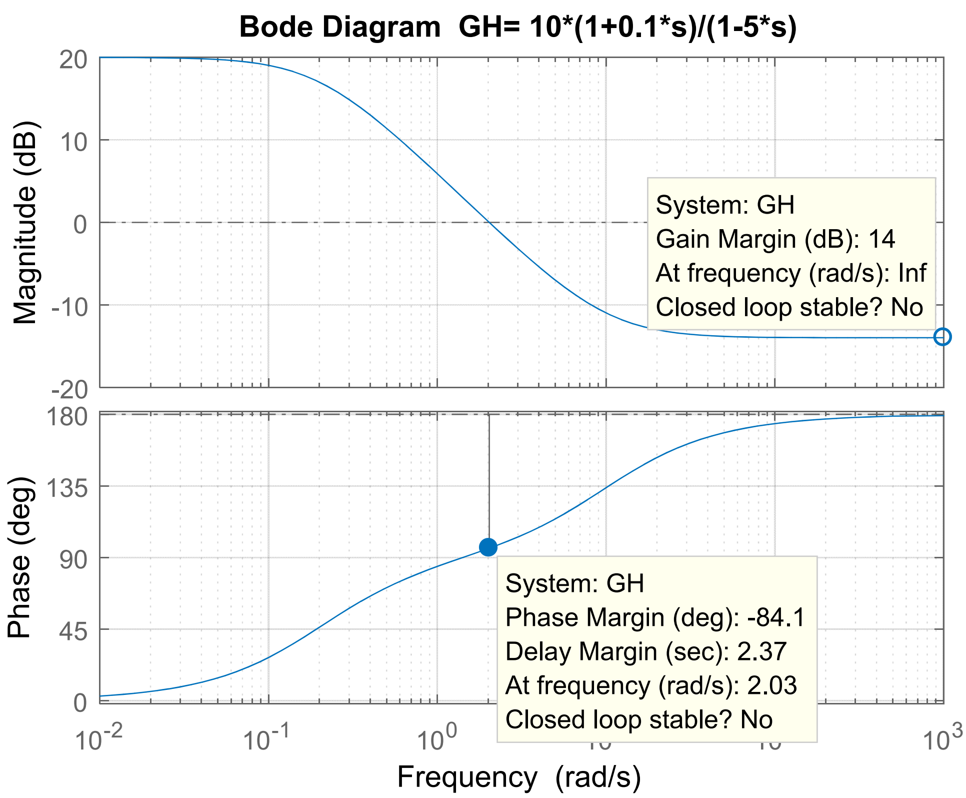
$$GH = \frac{-0.2\,(s+10)}{(s-0.2)}$$

P = 1 N = 0

instabile ad anello aperto

instabile ad anello chiuso





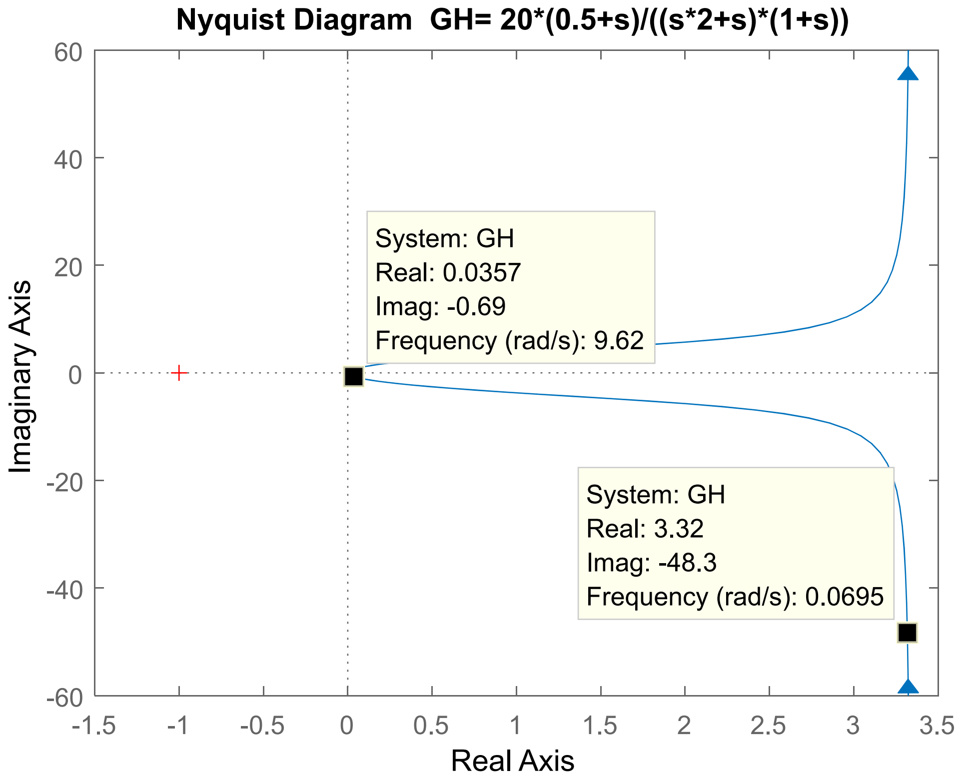
Margine di guadagno > 0 margine di fase < 0 instabile ad anello chiuso

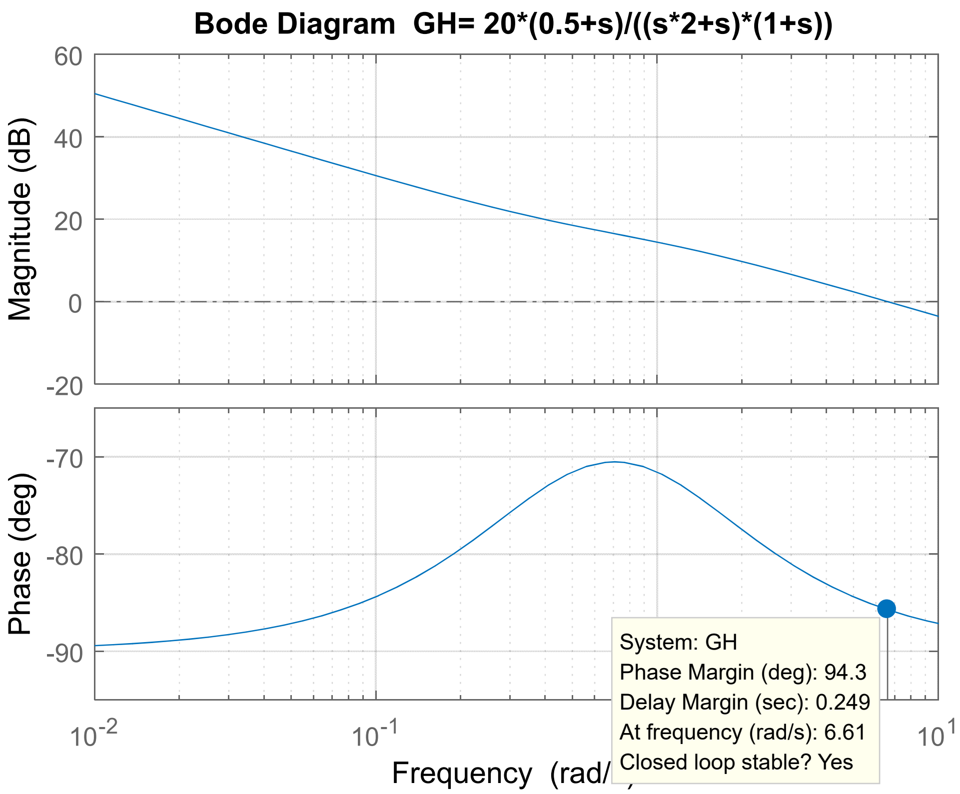
Esercizio XX

GH= 20*(0.5+s)/((s*2+s)*(1+s))

$$GH = \frac{6.6667\ (s+0.5)}{s\ (s+1)}$$

stabile ad anello aperto

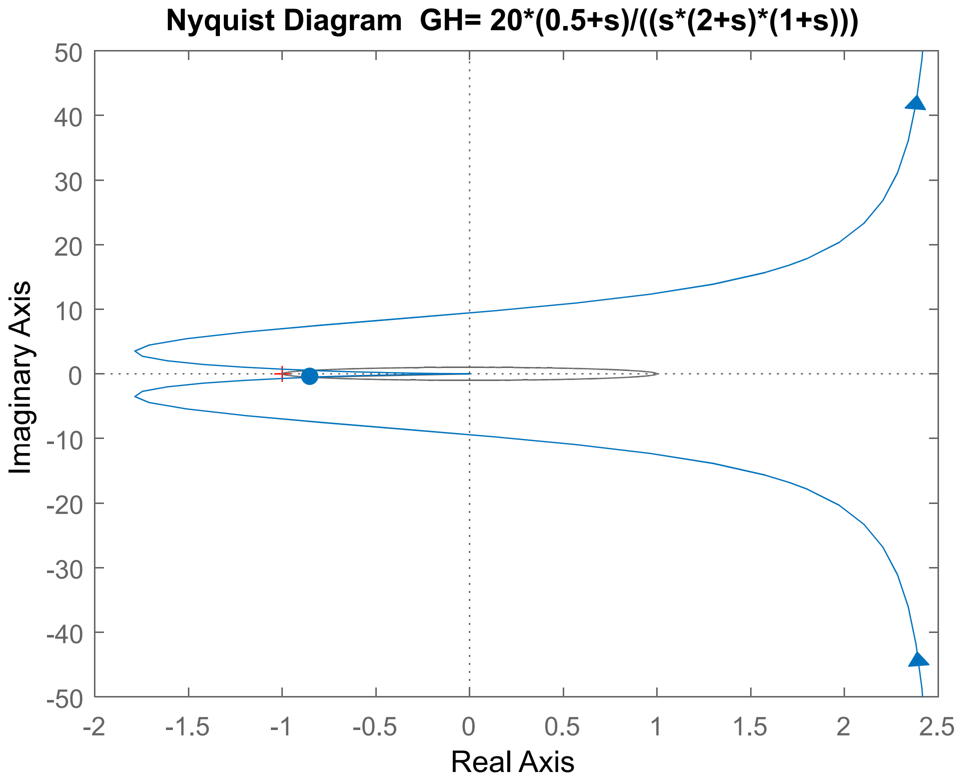




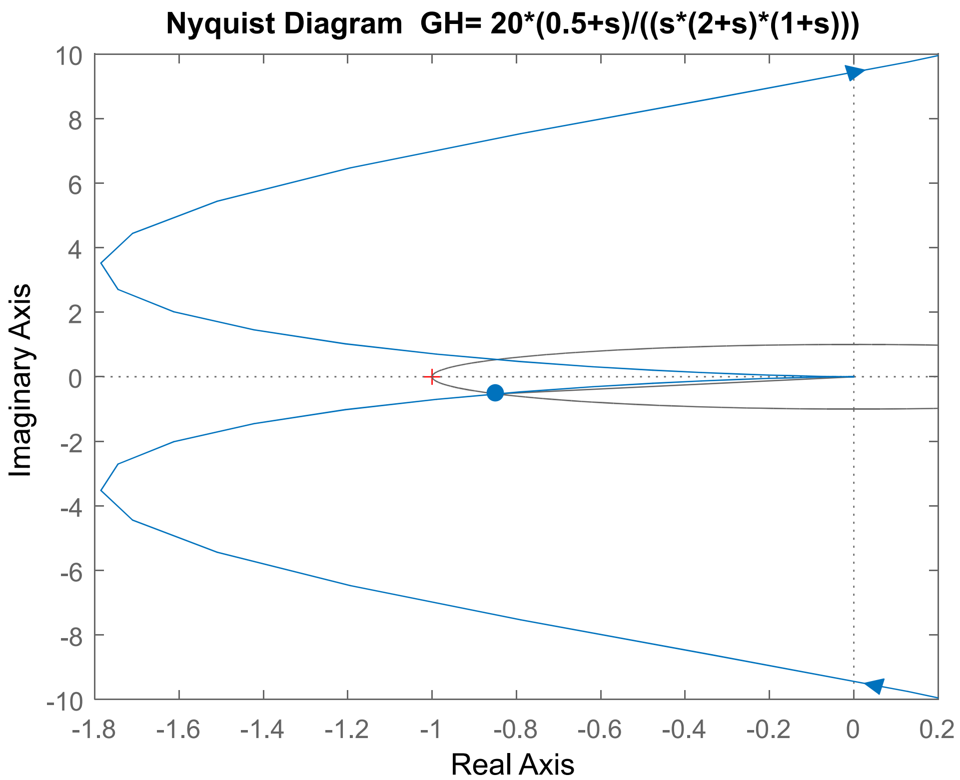
Esercizio 10

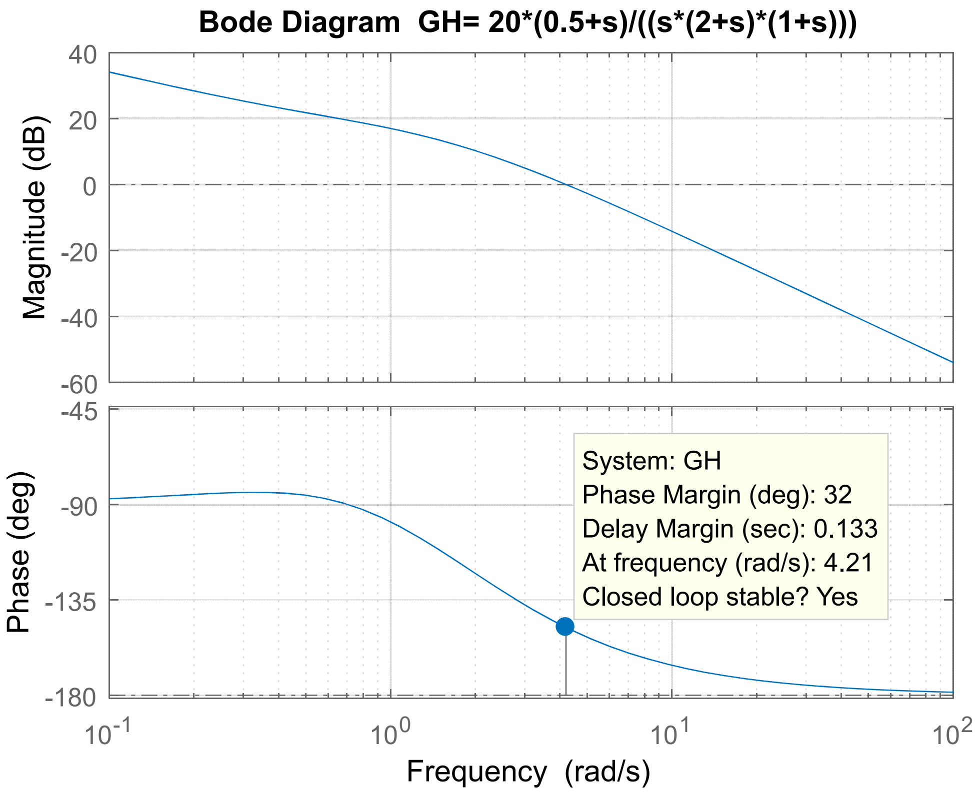
GH= 20*(0.5+s)/((s*(2+s)*(1+s)))

$$GH = \frac{20\,(s+0.5)}{s\,(s+2)\,(s+1)}$$



Zoom



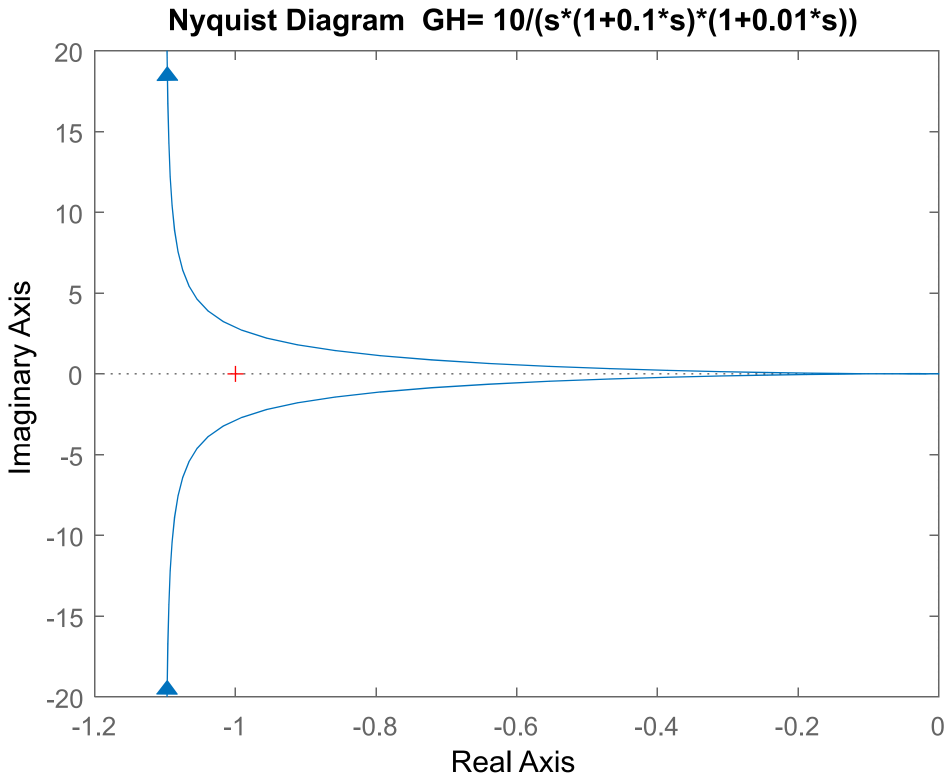


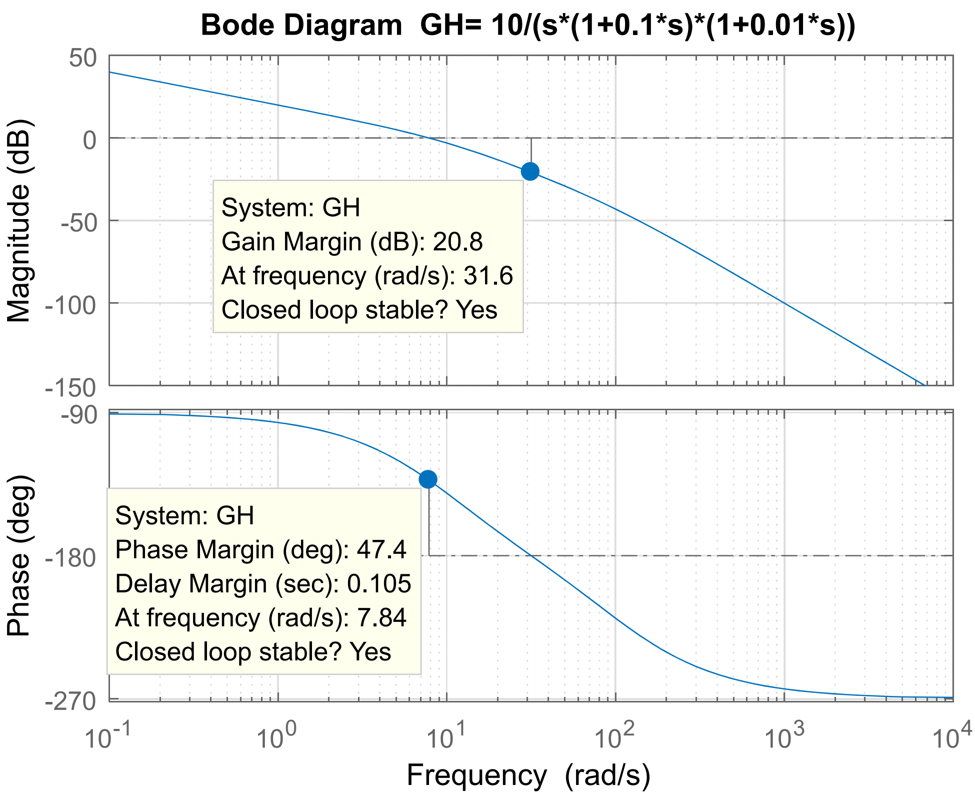
Stabile ad anello aperto e ad anello chiuso

Esercizio 11

GH= 10/(s*(1+0.1*s)*(1+0.01*s))

```
           10000
GH = ---------------------                          stabile ad anello aperto
        s (s+10) (s+100)
```



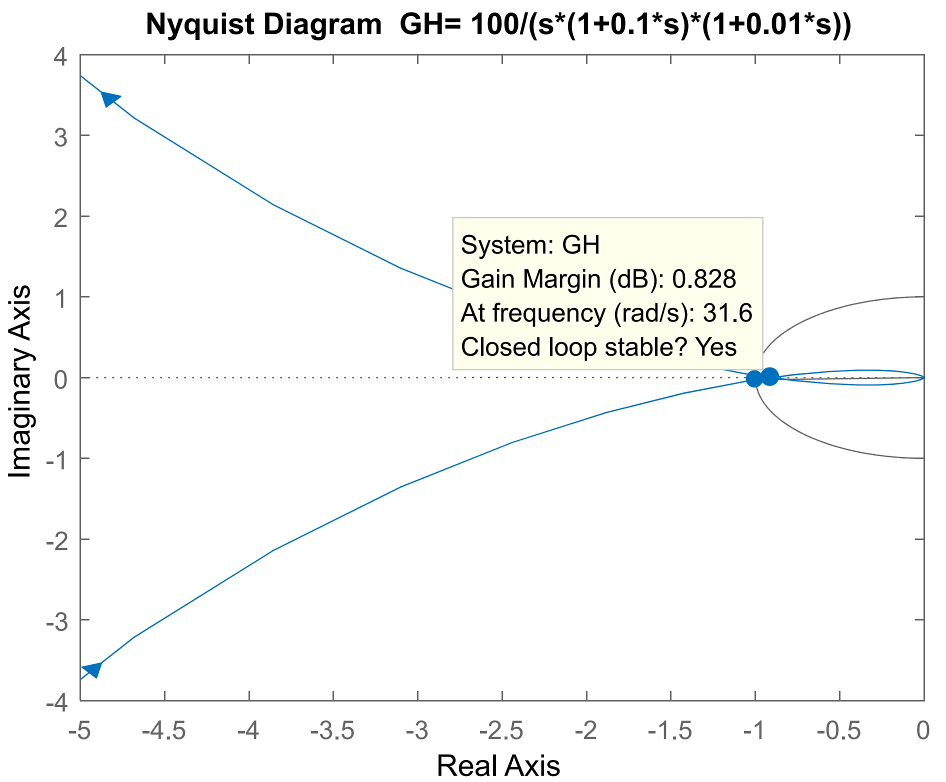


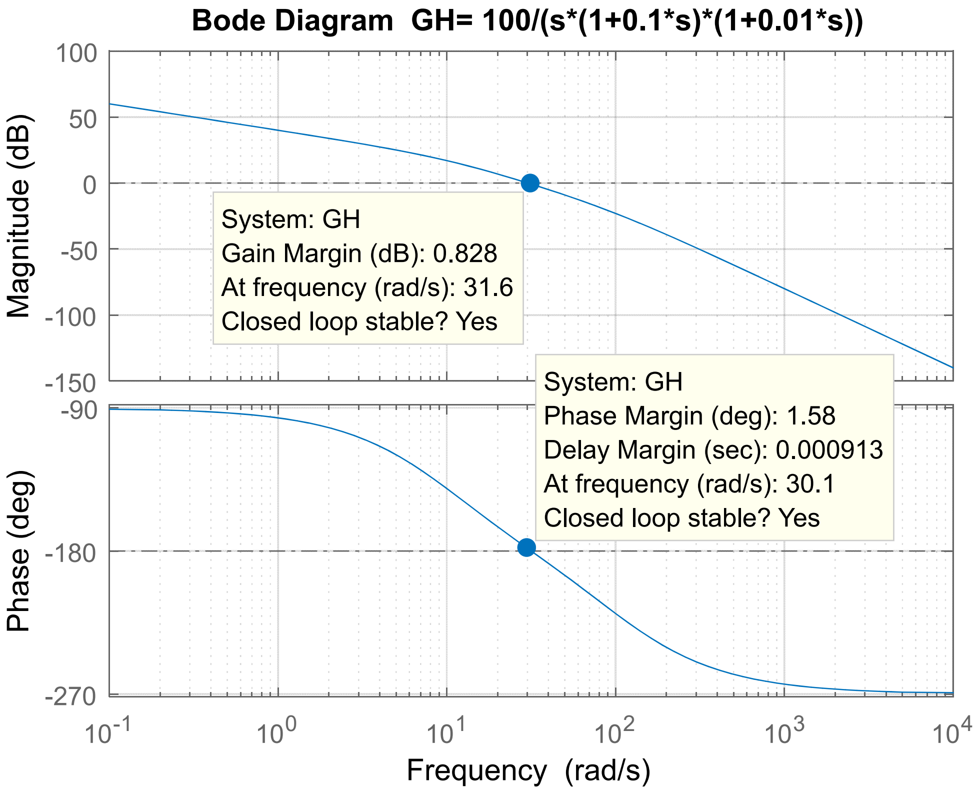
ES 11 bis

GH= 100/(s*(1+0.1*s)*(1+0.01*s))

```
          1e+05
GH = --------------------
       s (s+10) (s+100)
```

stabile di poco ad anello chiuso
(non sufficientemente stabile)





ES 11 ter

GH= 400/(s*(1+0.1*s)*(1+0.01*s))

```
           4e+05
GH = --------------------
       s (s+10) (s+100)
```

stabile ad anello aperto – instabile ad anello chiuso