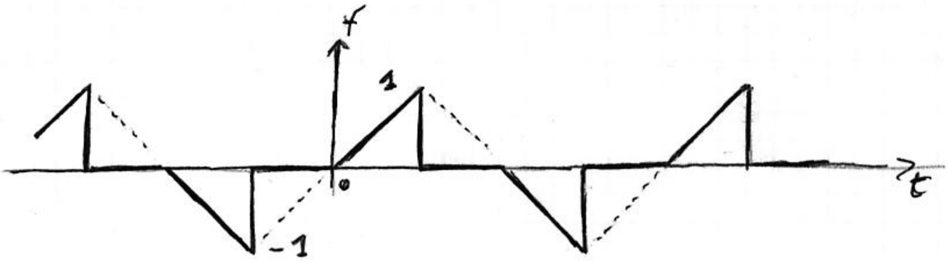
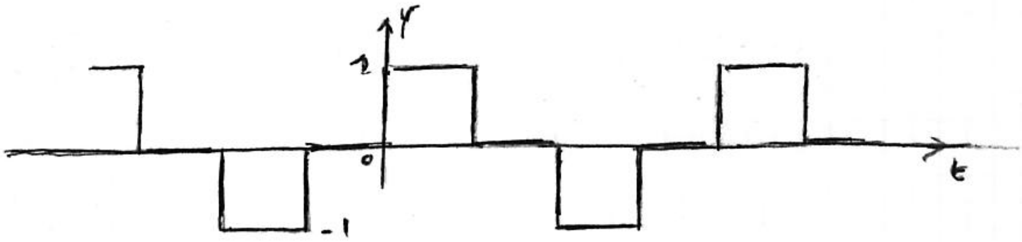
①



1A SCRIVERE LE RELAZIONI CHE CONSENTONO IL CALCOLO DIRETTO DEI COEFFICIENTI DELLO SVILUPPO IN SERIE DI FOURIER DELLA f.d.O. (IMPOSTARE, MA NON SVILUPPARE I CALCOLI)

1B OSSERVANDO LA fdO, COSA SI PUÒ DIRE, A PRIORI, RELATIVAMENTE AI COEFFICIENTI DELLO SVILUPPO IN SERIE ?

②

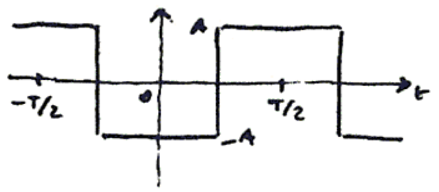


2A SCOMPORRE LA fdO NELLA SOMMA DI UNA FUNZIONE PARI E UNA FUNZIONE DISPARI E DETERMINARNE LO SVILUPPO IN SERIE DI FOURIER (SONO NOTI GLI SVILUPPI IN SERIE ALLEGATI)

2B DETERMINARE IL VALORE EFFICACE DELLA fdO —

ALLEGATO ALL'ES. 2

7)



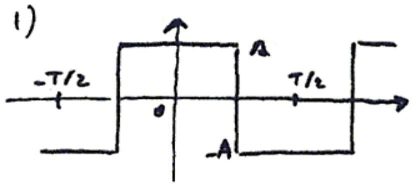
ONDA QUADRA PARI

LA FUNZIONE È OPPOSTA AD f_1

$f_7(t) = -f_1(t)$

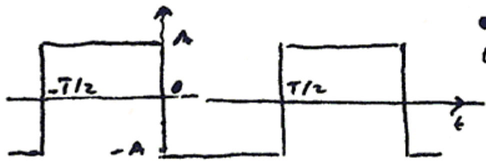
$$f_7(t) = -\frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

1)



$$f_1(t) = \frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

8)



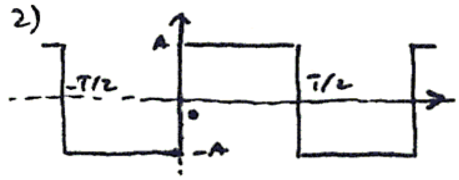
ONDA QUADRA DISPARI

LA FUNZIONE È OPPOSTA AD f_2

$f_8(t) = -f_2(t)$

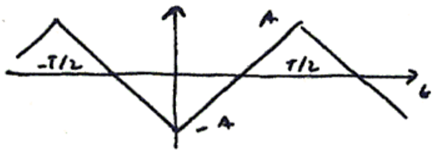
$$f_8(t) = -\frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

2)



$$f_2(t) = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

9)



ONDA TRIANGOLARE PARI

LA FUNZIONE È OPPOSTA AD f_3

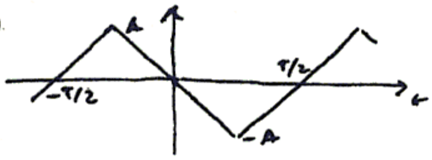
$f_9(t) = -f_3(t)$

$$f_9(t) = -\frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

3)

$$f_3(t) = \frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

10)



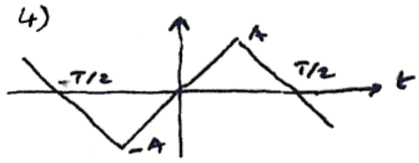
ONDA TRIANGOLARE DISPARI

LA FUNZIONE È OPPOSTA AD f_4

$f_{10}(t) = -f_4(t)$

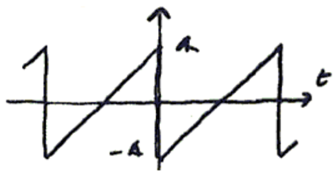
$$f_{10}(t) = -\frac{8A}{\pi^2}\left(\sin\omega t - \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t - \dots\right)$$

4)



$$f_4(t) = \frac{8A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \dots\right]$$

11)



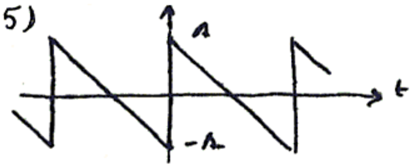
ONDA A DENTE DI SEGA

LA FUNZIONE È OPPOSTA AD f_5

$f_{11}(t) = -f_5(t)$

$$f_{11}(t) = -\frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

5)



$$f_5(t) = \frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

12)

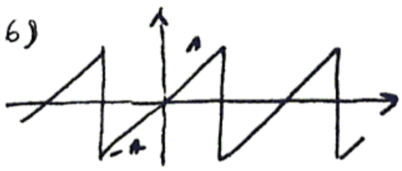
ONDA A DENTE DI SEGA

LA FUNZIONE È OPPOSTA AD f_6

$f_{12}(t) = -f_6(t)$

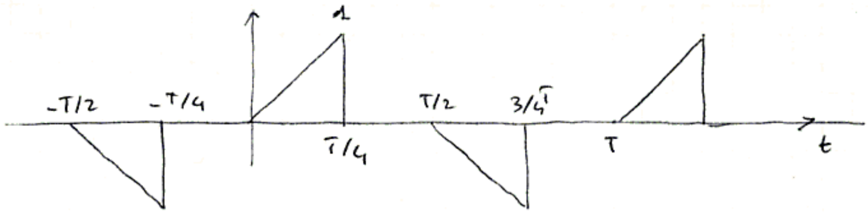
$$f_{12}(t) = -\frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \dots\right)$$

6)



$$f_6(t) = \frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \right)$$

1



1A $f(t) = A_0 + \sum_1^{\infty} a_n \cos n\omega t + \sum_1^{\infty} b_n \sin n\omega t$ con $\omega = \frac{2\pi}{T}$ $n = 1, 2, 3 \dots$

$$f(t) = \begin{cases} f_1 = \frac{1}{T/4} t = \frac{4}{T} t & \text{per } 0 < t < T/4 \\ f_2 = -\frac{4}{T}(t - T/2) & \text{per } T/2 < t < \frac{3}{4}T \\ 0 & \text{per } T/4 < t < T/2 \\ 0 & \text{per } \frac{3}{4}T < t < T \end{cases} \quad \text{oppure} \quad \begin{cases} f_1 = \frac{4}{T} t & \text{per } 0 < t < T/4 \\ f_3 = -\frac{4}{T}\left(t + \frac{T}{2}\right) & \text{per } -\frac{T}{2} < t < -T/4 \\ 0 & \text{per } -T/4 < t < 0 \\ 0 & \text{per } \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\left[\int_0^{T/4} f_1(t)\,dt + \int_{T/2}^{3/4T} f_2(t)\,dt\right] = 0 \quad \text{oppure} \quad A_0 = \frac{1}{T}\left[\int_0^{T/4} f_1(t)\,dt + \int_{-T/2}^{-T/4} f_3(t)\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\cos n\omega t\,dt + \int_{T/2}^{\frac{3}{4}T} f_2(t)\cos n\omega t\,dt\right]$$

oppure

$$a_n = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\cos n\omega t\,dt + \int_{-T/2}^{-T/4} f_3(t)\cos n\omega t\,dt\right]$$

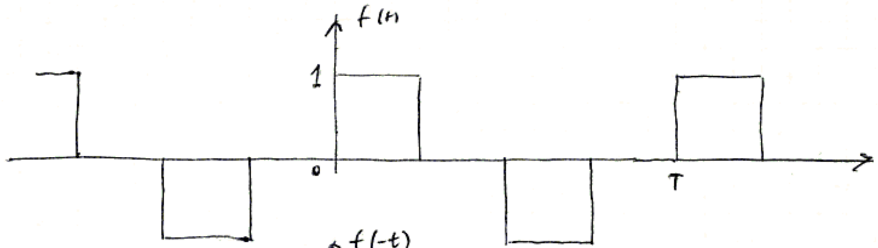
$$b_n = \frac{2}{T}\int_T f(t)\sin n\omega t\,dt = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\sin n\omega t\,dt + \int_{T/2}^{\frac{3}{4}T} f_2(t)\cos n\omega t\,dt\right]$$

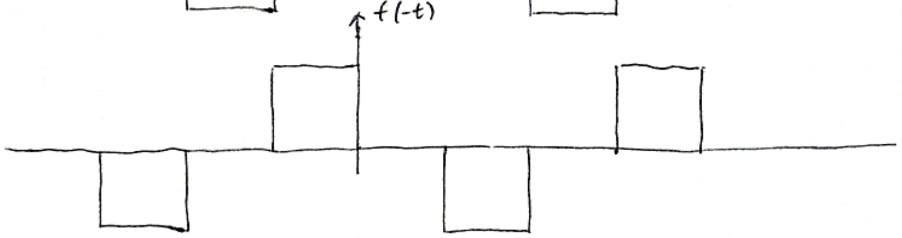
oppure

$$b_n = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\sin n\omega t\,dt + \int_{-T/2}^{-T/4} f_3(t)\sin n\omega t\,dt\right]$$

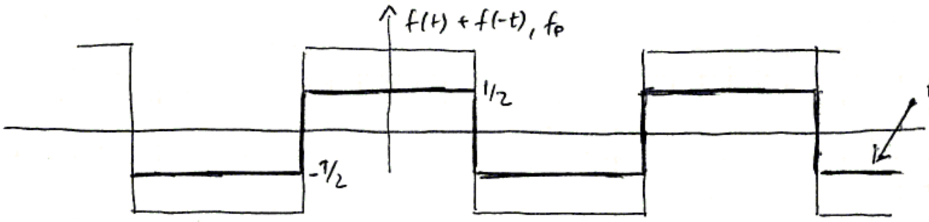
1B
- $f(t)$ NON È NÉ PARI NÉ DISPARI $\rightarrow$ $f(t) \neq f(-t)$ E $f(t) \neq -f(-t)$
  $\exists$ SIA COEFF. a_n SIA COEFF. b_n $[a_n \neq 0,\ b_n \neq 0]$
- $f(t)$ È EMISIMMETRICA (SIMMETRIA DI SEMIONDA) $\rightarrow f(t) = -f(t + T/2)$
  $\exists$ SOLO ARMONICHE DISPARI $[a_{2n} = 0$ E $b_{2n} = 0]$

2A)





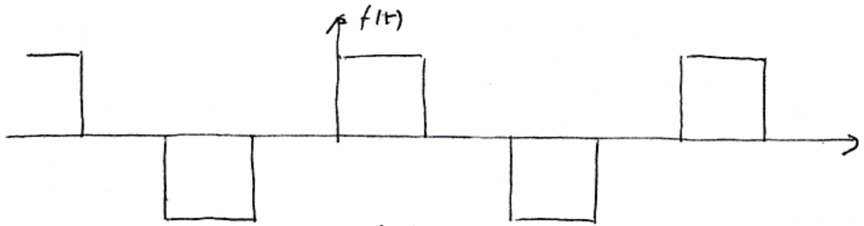
f. SPECCHIO
RISPETTO ALL'ASSE DELLE ORDINATE

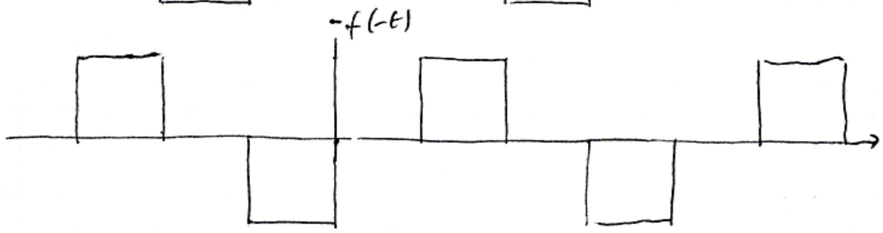


$$f_P = \frac{f(t) + f(-t)}{2}$$

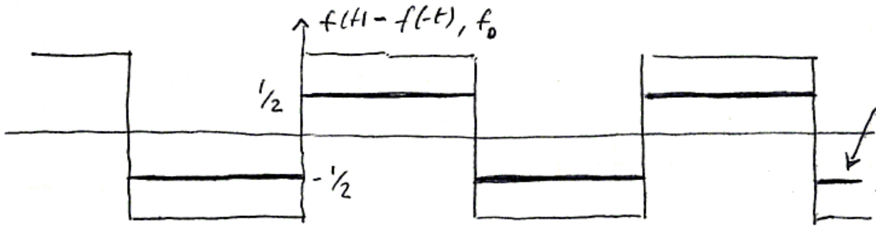
$$f_P = \frac{2}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t \dots\right]$$

(ALLEGATO CON A=1/2)





f. OPPOSTA
DELLA f. SPECCHIO



$$f_D = \frac{f(t) - f(-t)}{2}$$

$$f_D = \frac{2}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t \dots\right]$$

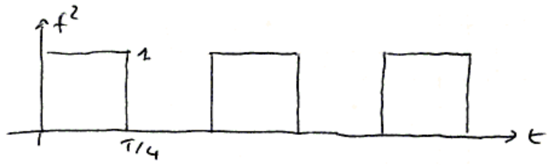
$$f(t) = f_P + f_D = \frac{2}{\pi}\left[\cos\omega t + \sin\omega t - \frac{1}{3}(\cos 3\omega t - \sin 3\omega t) + \frac{1}{5}(\cos 5\omega t + \sin 5\omega t) - \dots\right]$$

2B

$$f_{rms} = \sqrt{\frac{1}{T}\int_T f^2 dt} =$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T/4} dt + \int_{T/2}^{3/4 T} dt\right]} =$$

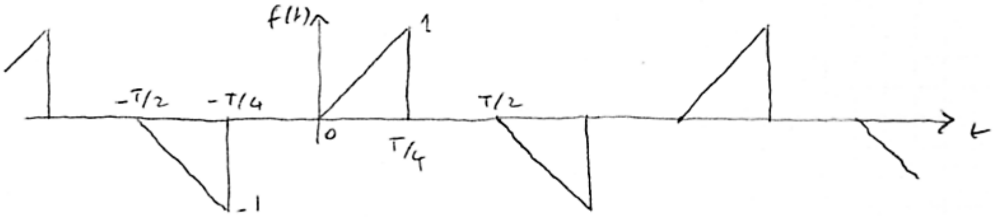
$$= \sqrt{\frac{1}{T}\, 2\,\frac{T}{4}} = \sqrt{\frac{1}{2}}$$



$f^2(t) = 1$ per $0 < t < T/4$

$f^2(t) = 1$ per $\frac{T}{2} < t < \frac{3}{4}T$

SVILUPPO IN SERIE DI FOURIER DELLA SEG. FORMA D'ONDA



a) SOLUZIONE MEDIANTE CALCOLO DIRETTO DEI COEFFICIENTI

$$f(t) = \begin{cases} -\frac{4}{T}\left(t+\frac{T}{2}\right) = -\frac{4}{T}t - 2 & \text{per } -\frac{T}{2}<t<-\frac{T}{4} \\ \frac{4}{T}t & \text{per } 0<t<T/4 \\ 0 & \text{per } -\frac{T}{4}<t<0 \ \text{ e per } \frac{T}{4}<t<\frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_1^\infty a_n \cos n\omega t + \sum_1^\infty b_n \sin n\omega t \qquad \text{con } \omega = \frac{2\pi}{T} \quad n=1,2,\dots$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)dt + \int_0^{T/4}\frac{4}{T}t\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)\cos n\omega t\,dt + \int_0^{T/4}\frac{4}{T}t\cos n\omega t\,dt\right] =$$

$$= -\frac{8}{T^2}\int_{-T/2}^{-T/4} t\cos n\omega t\,dt - \frac{4}{T}\int_{-T/2}^{-T/4}\cos n\omega t\,dt + \frac{8}{T^2}\int_0^{T/4} t\cos n\omega t\,dt =$$

PER PARTI $\int t\cos n\omega t\,dt = \int t\, d\left(\frac{\sin n\omega t}{n\omega}\right) = t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}$

$$= -\frac{8}{T^2}\left[t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}\right]_{-T/2}^{-T/4} - \frac{4}{T}\frac{\sin n\omega t}{n\omega}\Bigg|_{-T/2}^{-T/4} + \frac{8}{T^2}\left[t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right]_0^{T/4} =$$

$$= -\frac{8}{T^2}\left[-\frac{T}{4}\frac{\sin n\omega(-T/4)}{n\omega} + \frac{\cos n\omega(-T/4)}{n^2\omega^2} + \frac{T}{2}\frac{\sin n\omega(-T/2)}{n\omega} - \frac{\cos n\omega(-T/2)}{n^2\omega^2}\right] - \frac{4}{T}\frac{\sin n\omega(-T/4)}{n\omega} +$$

$$+ \frac{4}{T}\frac{\sin n\omega(-T/2)}{n\omega} + \frac{8}{T^2}\left[\frac{T}{4}\frac{\sin n\omega T/4}{n\omega} + \frac{\cos n\omega T/4}{n^2\omega^2} - \frac{1}{n^2\omega^2}\right] =$$

$$= \cancel{-\frac{2}{T}\frac{\sin n\omega T/4}{n\omega}} \cancel{- \frac{8\cos n\omega T/4}{n^2\omega^2T^2}} \cancel{+ \frac{4}{T}\frac{\sin n\omega T/2}{n\omega}} + \frac{8}{T^2}\frac{\cos n\omega T/2}{n^2\omega^2} + \frac{4}{T}\frac{\sin n\omega T/4}{n\omega} +$$

$$\cancel{-\frac{4}{T}\frac{\sin n\omega T/2}{n\omega}} \cancel{+ \frac{2}{T}\frac{\sin n\omega T/4}{n\omega}} \cancel{+ \frac{8\cos n\omega T/4}{n^2\omega^2T^2}} - \frac{8}{n^2\omega^2T^2} =$$

$$\begin{cases} \omega\frac{T}{4} = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2} \\ \omega\frac{T}{2} = \pi \\ \omega T = 2\pi \end{cases}$$

$$= \frac{8}{n^2 4\pi^2}\cos n\pi + \frac{4}{2n\pi}\sin n\pi/2 - \frac{8}{4\pi^2 n^2} =$$

$$= \frac{2}{n\pi}\sin n\frac{\pi}{2} - \frac{2}{n^2\pi^2}\left(1-\cos n\pi\right)$$

$n=1 \qquad a_1 = \frac{2}{\pi} - \frac{4}{\pi^2}$

$n=2 \qquad a_2 = 0$

$n=3 \qquad a_3 = -\frac{2}{3\pi} - \frac{4}{3^2\pi^2}$

$n=5 \qquad a_5 = \frac{2}{5\pi} - \frac{4}{5^2\pi^2}$

$$b_n = \frac{2}{T}\int_T f(t)\sin n\omega t\,dt = \frac{2}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)\sin n\omega t + \int_0^{T/4}\frac{4}{T}t\sin n\omega t\,dt\right] =$$

$$= -\frac{8}{T^2}\int_{-T/2}^{-T/4} t\sin n\omega t\,dt - \frac{4}{T}\int_{-T/2}^{-T/4}\sin n\omega t\,dt + \frac{8}{T^2}\int_0^{T/4} t\sin n\omega t\,dt =$$

PER PARTI $\int t\sin n\omega t\,dt = \int t\,d\left(-\frac{\cos n\omega t}{n\omega}\right) = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}$

$$= -\frac{8}{T^2}\left[-t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_{-T/2}^{-T/4} + \frac{4}{T}\frac{\cos n\omega t}{n\omega}\Bigg|_{-T/2}^{-T/4} + \frac{8}{T^2}\left[-t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_0^{T/4} =$$

$$= -\frac{8}{T^2}\left[\frac{T}{4}\frac{\cos n\omega(-T/4)}{n\omega} + \frac{\sin n\omega(-T/4)}{n^2\omega^2} - \frac{T}{2}\frac{\cos n\omega(-T/2)}{n\omega} - \frac{\sin n\omega(-T/2)}{n^2\omega^2}\right] + \frac{4}{T}\frac{\cos n\omega(-T/4)}{n\omega} +$$

$$- \frac{4}{T}\frac{\cos n\omega(-T/2)}{n\omega} + \frac{8}{T^2}\left[-\frac{T}{4}\frac{\cos n\omega T/4}{n\omega} + \frac{\sin n\omega T/4}{n^2\omega^2}\right] =$$

$$= -\frac{2}{T}\cancel{\frac{\cos n\omega T/4}{n\omega}} + 8\frac{\sin n\omega T/4}{n^2\omega^2T^2} + \frac{4}{T}\cancel{\frac{\cos n\omega T/2}{n\omega}} - \frac{8}{T^2}\frac{\sin n\omega T/2}{n^2\omega^2} + \frac{4}{T}\cancel{\frac{\cos n\omega T/4}{n\omega}} +$$

$$- \frac{4}{T}\cancel{\frac{\cos n\omega T/2}{n\omega}} - \frac{2}{T}\cancel{\frac{\cos n\omega T/4}{n\omega}} + \frac{8\sin n\omega T/4}{n^2\omega^2T^2} =$$

$$= 16\frac{\sin n\omega T/4}{n^2\omega^2T^2} - \frac{8\sin n\omega T/2}{n^2\omega^2T^2}$$

$\omega\frac{T}{4} = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$

$\omega\frac{T}{2} = \frac{2\pi}{T}\cdot\frac{T}{2} = \pi$

$n\omega T = 2\pi$

$$= \frac{16\sin n\pi/2}{4n^2\pi^2} - \frac{8}{4n^2\pi^2}\cdot\sin n\pi$$

$$= \frac{4}{n^2\pi^2}\sin n\pi/2$$

$n=1$ $\quad a_1 = \frac{4}{\pi^2}$

$n=2$ $\quad a_2 = 0$

$n=3$ $\quad a_3 = -\frac{4}{3^2\pi^2}$

$n=5$ $\quad a_5 = \frac{4}{5^2\pi^2}$

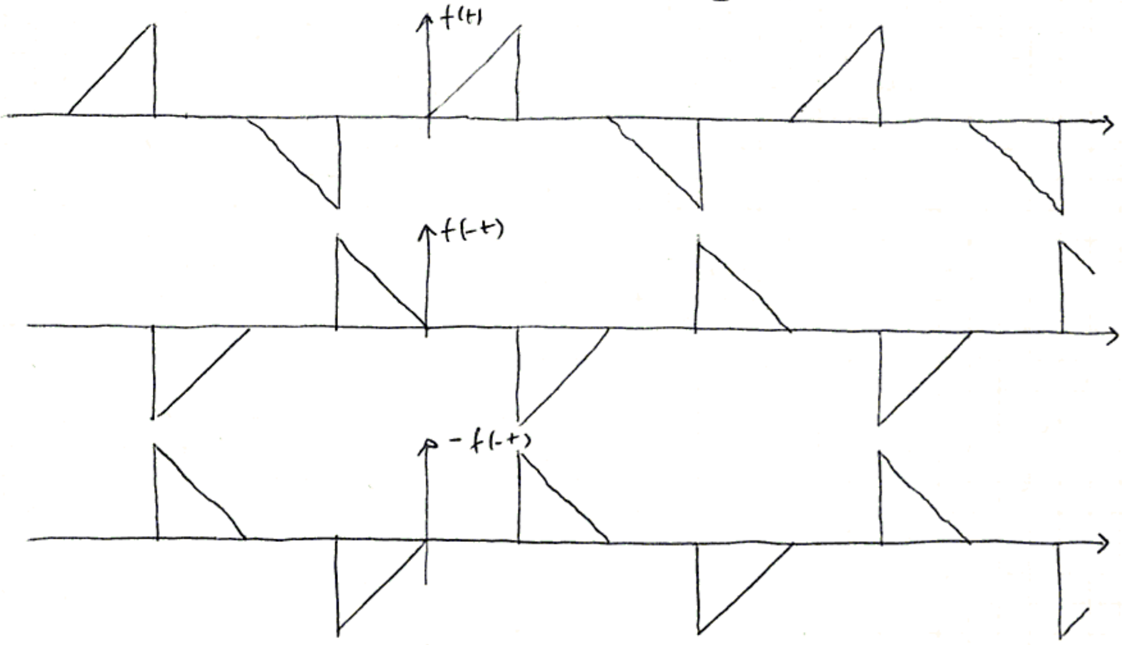
$n=7$ $\quad a_7 = -\frac{4}{7^2\pi^2}$

$$f(t) = \sum_1^\infty{}_n\left[\frac{2}{n\pi}\sin n\frac{\pi}{2} - \frac{2}{n^2\pi^2}(1-\cos n\pi)\right]\cos n\omega t + \sum_1 {}_n\frac{4}{n^2\pi^2}\sin n\frac{\pi}{2}\sin n\omega t =$$

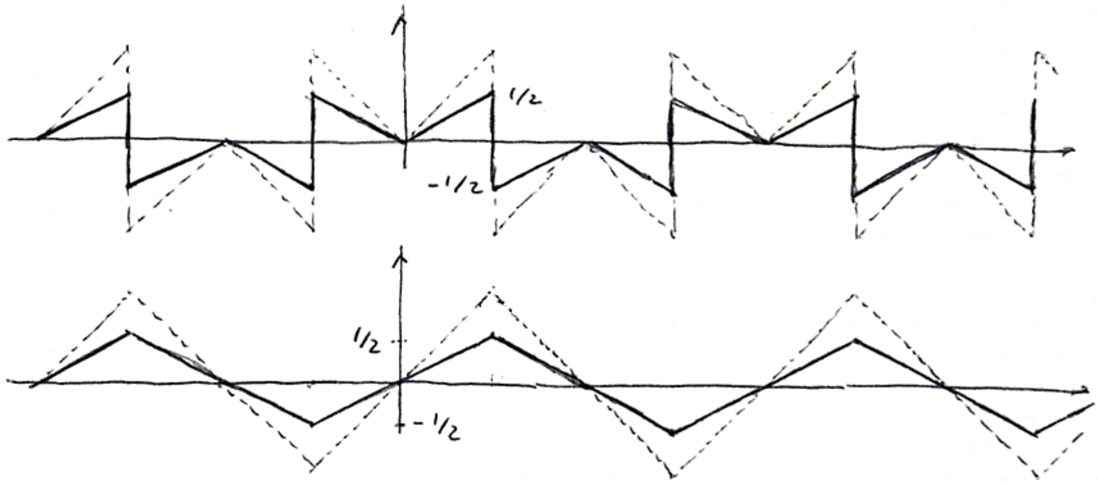
$$= \left(\frac{2}{\pi}-\frac{4}{\pi^2}\right)\cos\omega t - \left(\frac{2}{3\pi}-\frac{4}{3^2\pi^2}\right)\cos 3\omega t + \left(\frac{2}{5\pi}-\frac{4}{5^2\pi^2}\right)\cos 5\omega t - \dots + \frac{4}{\pi^2}\sin\omega t - \frac{4}{3^2\pi^2}\sin 3\omega t + \frac{4}{5^2\pi^2}\sin 5\omega t - \dots$$

b) SOLUZIONE MEDIANTE SCOMPOSIZIONE DELLA f d'O in SOMMA DI f d'O PARI E DISPARI

$$f_P = \frac{f(t) + f(-t)}{2} \qquad f_D = \frac{f(t) - f(-t)}{2}$$



P



D

IN TRATTEGGIO $f(t) + f(-t)$

IN TRATTO CONTINUO f_P

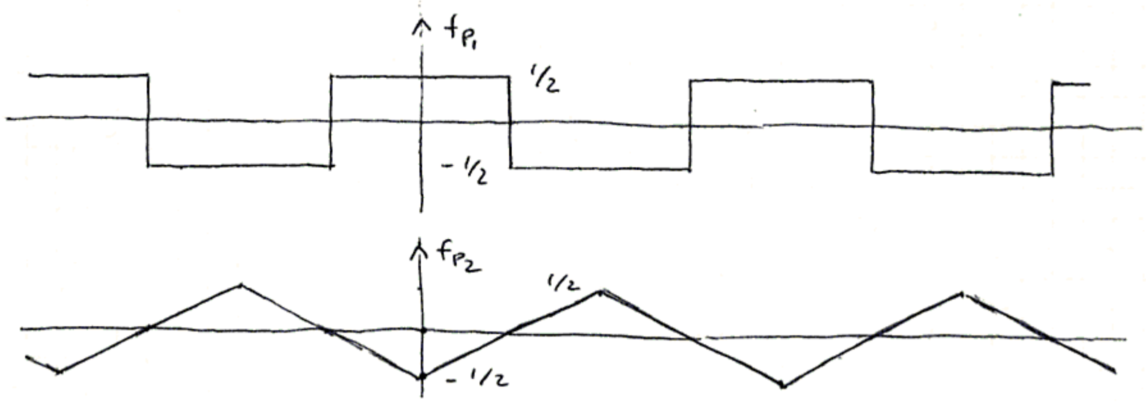
IN TRATTEGGIO $f(t) - f(-t)$

IN TRATTO CONTINUO f_D

LA FORMA D'ONDA PARI PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI ALTRE 2 f.d'O PARI DI CUI È NOTO LO SVILUPPO IN SERIE.

P1

P2

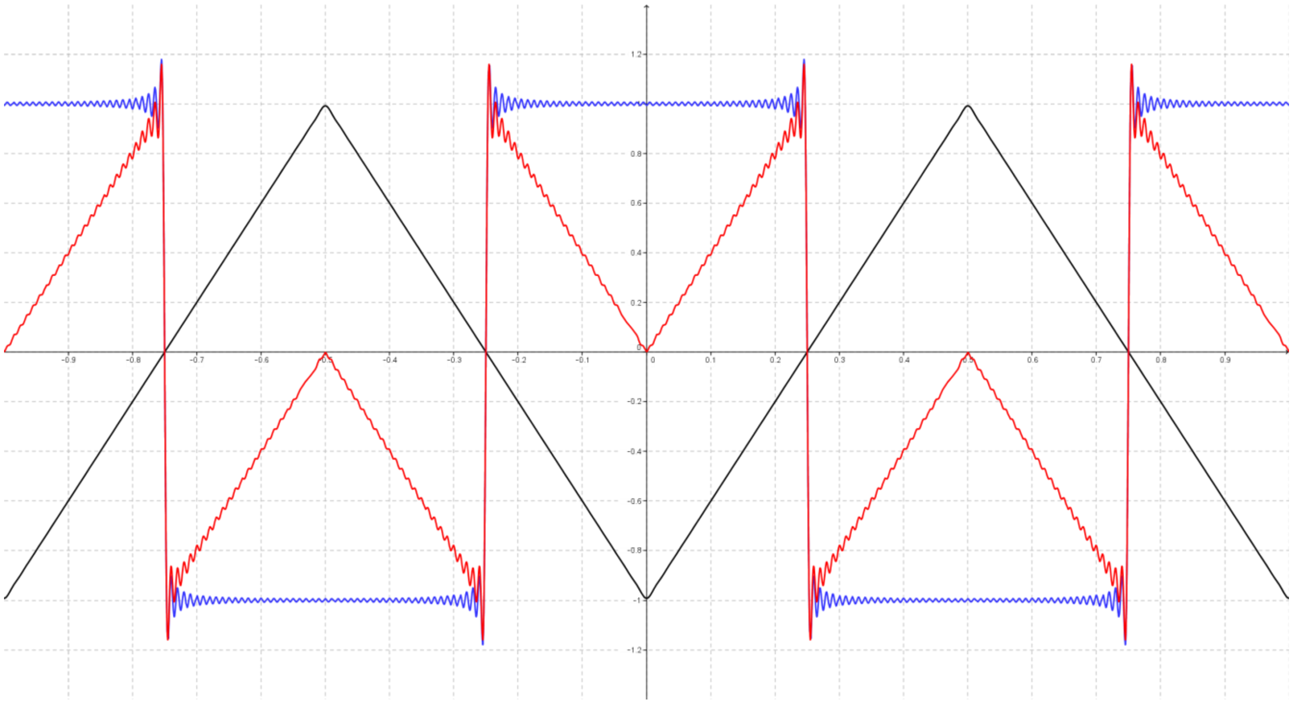


$$f(t) = f_P + f_D = f_{P_1} + f_{P_2} + f_D$$

(FORME D'ONDA 1), 9) E 4) IN ALLEGATO 2 CON $A = \frac{1}{2}$)

CON

$$\begin{cases} f_{P_1} = \frac{2}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right] \\ f_{P_2} = -\frac{4}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right] \\ f_D = \frac{4A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t + \dots\right] \end{cases}$$

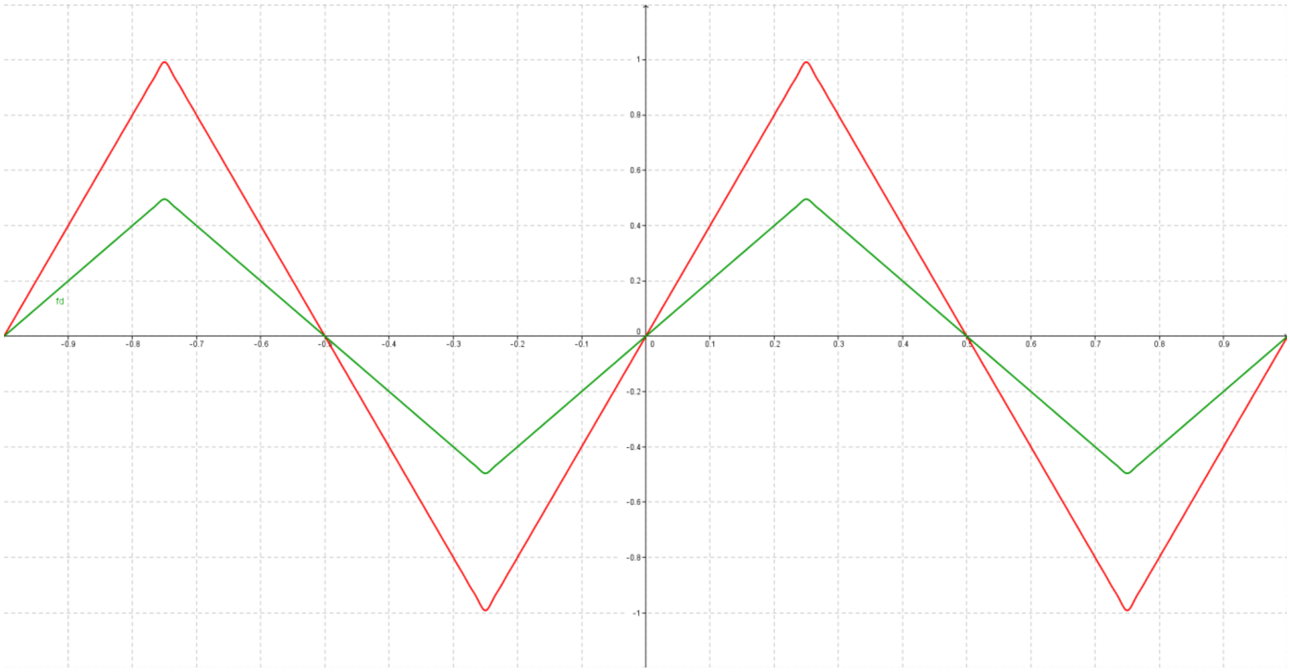


$$\mathrm{QuadraPari}(x) = 4 / \pi \, (\cos(2\pi x) - 1/3 \cos(2 (3) \pi x) + 1/5 \cos(2 (5) \pi x) - 1/7 \cos(2 (7) \pi x) + 1/9 \cos(2 (9) \pi x) - 1/11 \cos(2 (11) \pi x) + 1/13 \cos(2 (13) \pi x) - 1/15 \cos(2 (15) \pi x) + 1/17 \cos(2 (17) \pi x) - 1/19 \cos(2 (19) \pi x) + 1/21 \cos(2 (21) \pi x) - 1/23 \cos(2 (23) \pi x) + 1/25 \cos(2 (25) \pi x) - 1/27 \cos(2 (27) \pi x) + 1/29 \cos(2 (29) \pi x) - 1/31 \cos(2 (31) \pi x) + 1/33 \cos(2 (33) \pi x) - 1/35 \cos(2 (35) \pi x) + 1/37 \cos(2 (37) \pi x) - 1/39 \cos(2 (39) \pi x) + 1/41 \cos(2 (41) \pi x) - 1/43 \cos(2 (43) \pi x) + 1/45 \cos(2 (45) \pi x) - 1/47 \cos(2 (47) \pi x) + 1/49 \cos(2 (49) \pi x) - 1/51 \cos(2 (51) \pi x) + 1/53 \cos(2 (53) \pi x) - 1/55 \cos(2 (55) \pi x) + 1/57 \cos(2 (57) \pi x) - 1/59 \cos(2 (59) \pi x) + 1/61 \cos(2 (61) \pi x) - 1/63 \cos(2 (63) \pi x) + 1/65 \cos(2 (65) \pi x) - 1/67 \cos(2 (67) \pi x) + 1/69 \cos(2 (69) \pi x) - 1/71 \cos(2 (71) \pi x) + 1/73 \cos(2 (73) \pi x) - 1/75 \cos(2 (75) \pi x) + 1/77 \cos(2 (77) \pi x) - 1/79 \cos(2 (79) \pi x) + 1/81 \cos(2 (81) \pi x) - 1/83 \cos(2 (83) \pi x) + 1/85 \cos(2 (85) \pi x) - 1/87 \cos(2 (87) \pi x) + 1/89 \cos(2 (89) \pi x) - 1/91 \cos(2 (91) \pi x) + 1/93 \cos(2 (93) \pi x) - 1/95 \cos(2 (95) \pi x) + 1/97 \cos(2 (97) \pi x) - 1/99 \cos(2 (99) \pi x))$$

$$\mathrm{OTriangolarePari}(x) = (-8) / \pi^2 \, (\cos(2\pi x) + 1/3^2 \cos(2 (3) \pi x) + 1/5^2 \cos(2 (5) \pi x) + 1/7^2 \cos(2 (7) \pi x) + 1/9^2 \cos(2 (9) \pi x) + 1/11^2 \cos(2 (11) \pi x) + 1/13^2 \cos(2 (13) \pi x) + 1/15^2 \cos(2 (15) \pi x) + 1/17^2 \cos(2 (17) \pi x) + 1/19^2 \cos(2 (19) \pi x) + 1/21^2 \cos(2 (21) \pi x) + 1/23^2 \cos(2 (23) \pi x) + 1/25^2 \cos(2 (25) \pi x) + 1/27^2 \cos(2 (27) \pi x) + 1/29^2 \cos(2 (29) \pi x) + 1/31^2 \cos(2 (31) \pi x) + 1/33^2 \cos(2 (33) \pi x) + 1/35^2 \cos(2 (35) \pi x) + 1/37^2 \cos(2 (37) \pi x) + 1/39^2 \cos(2 (39) \pi x) + 1/41^2 \cos(2 (41) \pi x) + 1/43^2 \cos(2 (43) \pi x) + 1/45^2 \cos(2 (45) \pi x) + 1/47^2 \cos(2 (47) \pi x) + 1/49^2 \cos(2 (49) \pi x))$$

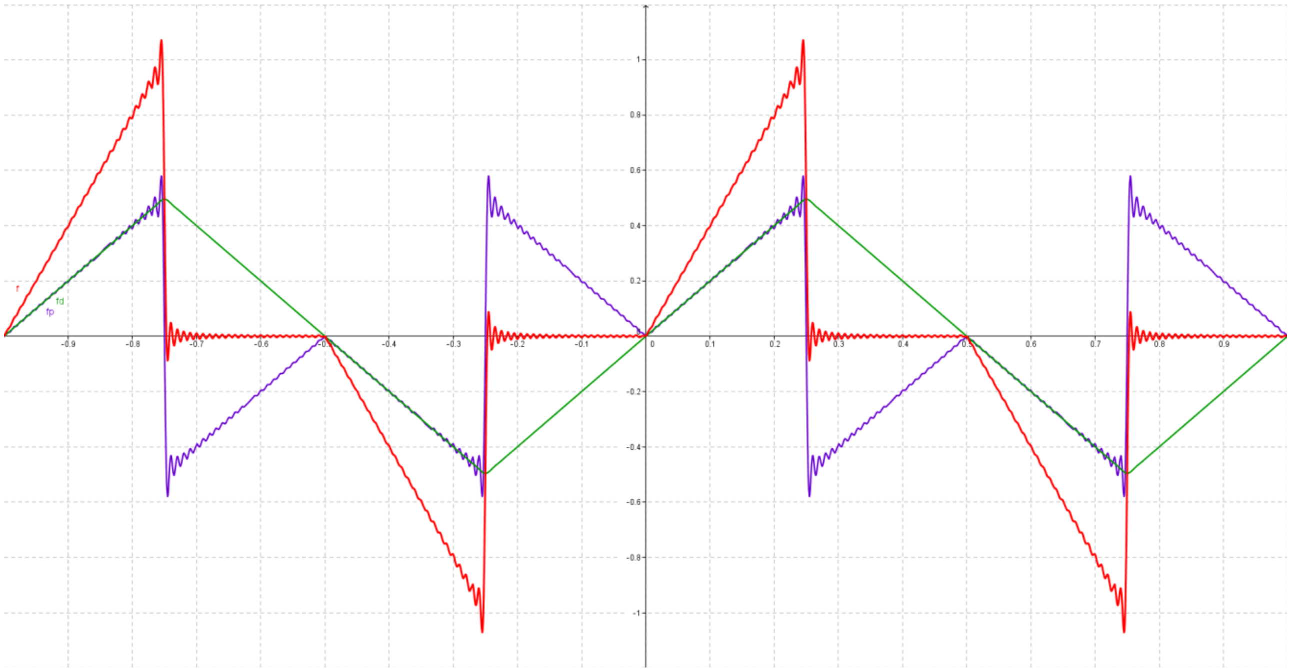
$$\mathrm{fp2}(x) = \mathrm{QuadraPari}(x) + \mathrm{OTriangolarePari}(x)$$

$$\mathrm{fp}(x) = 1/2 \, \mathrm{fp2}(x)$$



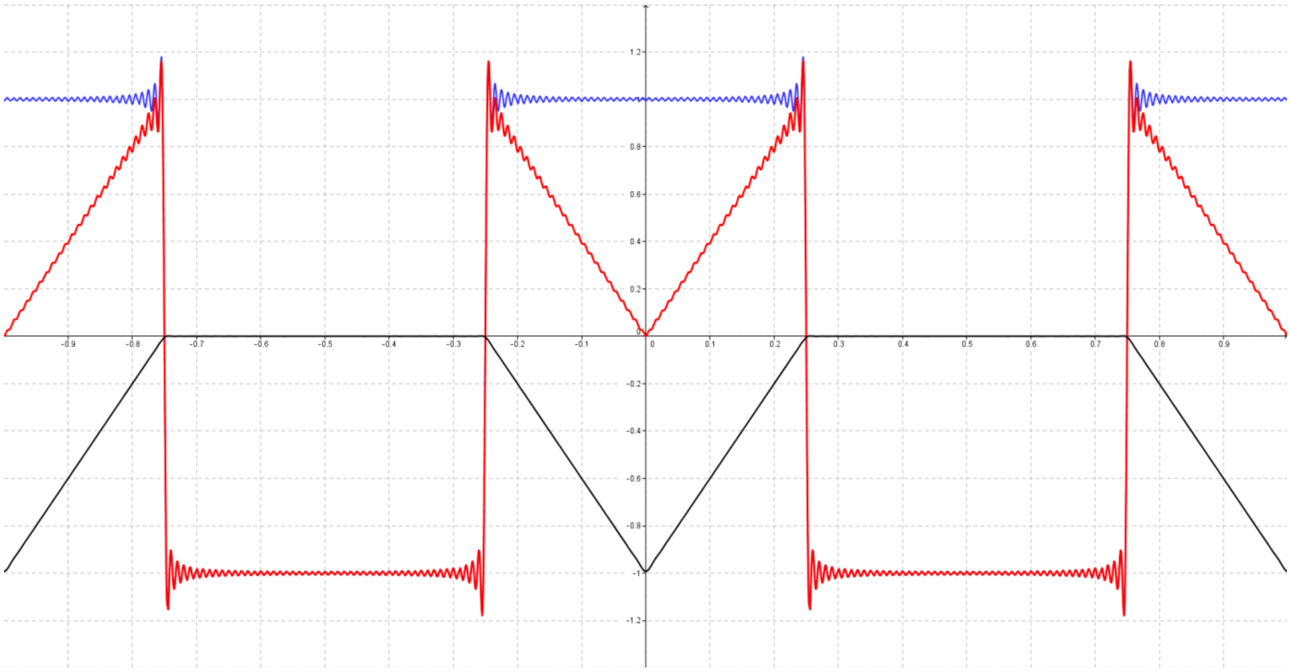
TriangolareDispari(x) = 8 / π² (sen(2π x) - 1 / 3² sen(2 (3) π x) + 1 / 5² sen(2 (5) π x) - 1 / 7² sen(2 (7) π x) + 1 / 9² sen(2 (9) π x) - 1 / 11² sen(2 (11) π x) + 1 / 13² sen(2 (13) π x) - 1 / 15² sen(2 (15) π x) + 1 / 17² sen(2 (17) π x) - 1 / 19² sen(2 (19) π x) + 1 / 21² sen(2 (21) π x) - 1 / 23² sen(2 (23) π x) + 1 / 25² sen(2 (25) π x) - 1 / 27² sen(2 (27) π x) + 1 / 29² sen(2 (29) π x) - 1 / 31² sen(2 (31) π x) + 1 / 33² sen(2 (33) π x) - 1 / 35² sen(2 (35) π x) + 1 / 37² sen(2 (37) π x) - 1 / 39² sen(2 (39) π x) + 1 / 41² sen(2 (41) π x) - 1 / 43² sen(2 (43) π x) + 1 / 45² sen(2 (45) π x) - 1 / 47² sen(2 (47) π x) + 1 / 49² sen(2 (49) π x))

fd(x) =1/2 TriangolareDispari(x)



f
fd
fp
-0.9
-0.8
-0.7
-0.6
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4
0.6
0.7
0.8
0.9
1
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-1

$f(x) = fp(x) + fd(x)$



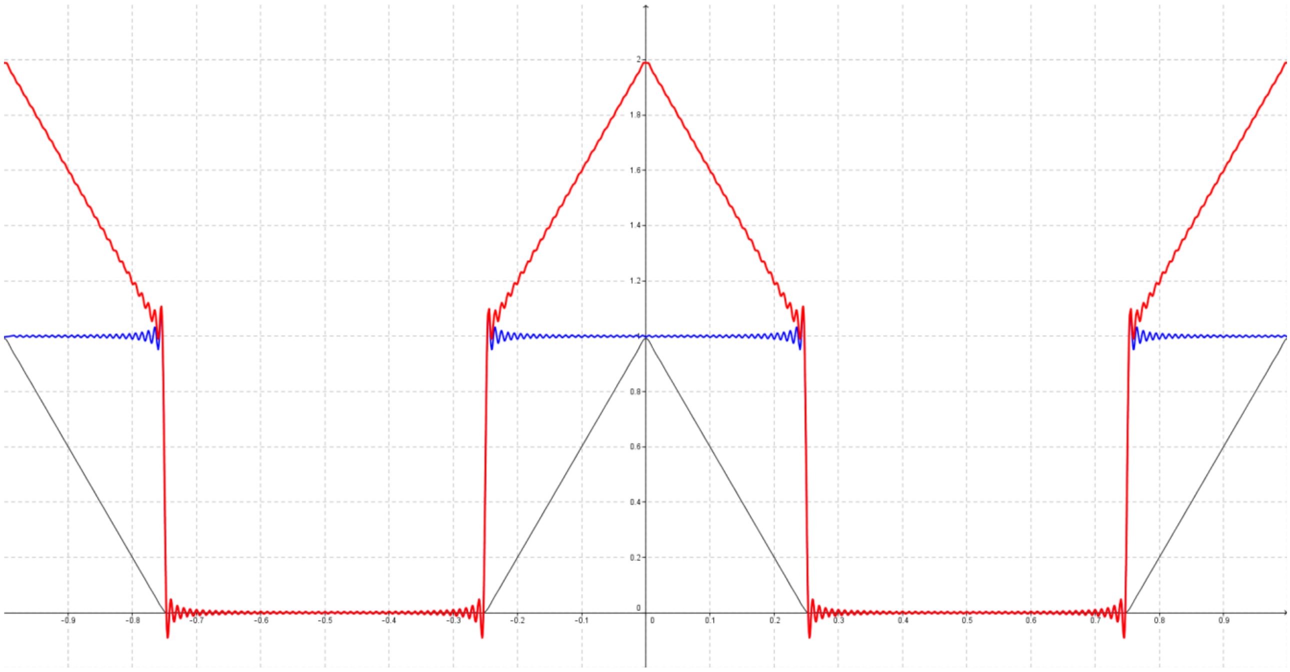
QuadraPari(x) = $4 / \pi (\cos(2\pi x) - 1 / 3 \cos(2 (3) \pi x) + 1 / 5 \cos(2 (5) \pi x) - 1 / 7 \cos(2 (7) \pi x) + 1 / 9 \cos(2 (9) \pi x) - 1 / 11 \cos(2 (11) \pi x) + 1 / 13 \cos(2 (13) \pi x) - 1 / 15 \cos(2 (15) \pi x) + 1 / 17 \cos(2 (17) \pi x) - 1 / 19 \cos(2 (19) \pi x) + 1 / 21 \cos(2 (21) \pi x) - 1 / 23 \cos(2 (23) \pi x) + 1 / 25 \cos(2 (25) \pi x) - 1 / 27 \cos(2 (27) \pi x) + 1 / 29 \cos(2 (29) \pi x) - 1 / 31 \cos(2 (31) \pi x) + 1 / 33 \cos(2 (33) \pi x) - 1 / 35 \cos(2 (35) \pi x) + 1 / 37 \cos(2 (37) \pi x) - 1 / 39 \cos(2 (39) \pi x) + 1 / 41 \cos(2 (41) \pi x) - 1 / 43 \cos(2 (43) \pi x) + 1 / 45 \cos(2 (45) \pi x) - 1 / 47 \cos(2 (47) \pi x) + 1 / 49 \cos(2 (49) \pi x) - 1 / 51 \cos(2 (51) \pi x) + 1 / 53 \cos(2 (53) \pi x) - 1 / 55 \cos(2 (55) \pi x) + 1 / 57 \cos(2 (57) \pi x) - 1 / 59 \cos(2 (59) \pi x) + 1 / 61 \cos(2 (61) \pi x) - 1 / 63 \cos(2 (63) \pi x) + 1 / 65 \cos(2 (65) \pi x) - 1 / 67 \cos(2 (67) \pi x) + 1 / 69 \cos(2 (69) \pi x) - 1 / 71 \cos(2 (71) \pi x) + 1 / 73 \cos(2 (73) \pi x) - 1 / 75 \cos(2 (75) \pi x) + 1 / 77 \cos(2 (77) \pi x) - 1 / 79 \cos(2 (79) \pi x) + 1 / 81 \cos(2 (81) \pi x) - 1 / 83 \cos(2 (83) \pi x) + 1 / 85 \cos(2 (85) \pi x) - 1 / 87 \cos(2 (87) \pi x) + 1 / 89 \cos(2 (89) \pi x) - 1 / 91 \cos(2 (91) \pi x) + 1 / 93 \cos(2 (93) \pi x) - 1 / 95 \cos(2 (95) \pi x) + 1 / 97 \cos(2 (97) \pi x) - 1 / 99 \cos(2 (99) \pi x))$

OTenda(x) = -Tenda(x)

Tenda(x) = $0.25 + 4 / \pi^2 (\cos(2\pi x) + 2 / 2^2 \cos(2 (2) \pi x) + 1 / 3^2 \cos(2 (3) \pi x) + 0\cos(2 (4) \pi x) + 1 / 5^2 \cos(2 (5) \pi x) + 2 / 6^2 \cos(2 (6) \pi x) + 1 / 7^2 \cos(2 (7) \pi x) + 0\cos(2 (8) \pi x) + 1 / 9^2 \cos(2 (9) \pi x) + 2 / 10^2 \cos(2 (10) \pi x) + 1 / 11^2 \cos(2 (11) \pi x) + 0\cos(2 (12) \pi x) + 1 / 13^2 \cos(2 (13) \pi x) + 2 / 14^2 \cos(2 (14) \pi x) + 1 / 15^2 \cos(2 (15) \pi x) + 0\cos(2 (16) \pi x) + 1 / 17^2 \cos(2 (17) \pi x) + 2 / 18^2 \cos(2 (18) \pi x) + 1 / 19^2 \cos(2 (19) \pi x) + 0\cos(2 (20) \pi x) + 1 / 21^2 \cos(2 (21) \pi x) + 2 / 22^2 \cos(2 (22) \pi x) + 1 / 23^2 \cos(2 (23) \pi x) + 0\cos(2 (24) \pi x) + 1 / 25^2 \cos(2 (25) \pi x) + 2 / 26^2 \cos(2 (26) \pi x) + 1 / 27^2 \cos(2 (27) \pi x) + 0\cos(2 (28) \pi x) + 1 / 29^2 \cos(2 (29) \pi x) + 2 / 30^2 \cos(2 (30) \pi x) + 1 / 31^2 \cos(2 (31) \pi x) + 0\cos(2 (32) \pi x) + 1 / 33^2 \cos(2 (33) \pi x) + 2 / 34^2 \cos(2 (34) \pi x) + 1 / 35^2 \cos(2 (35) \pi x) + 0\cos(2 (36) \pi x) + 1 / 37^2 \cos(2 (37) \pi x) + 2 / 38^2 \cos(2 (38) \pi x) + 1 / 39^2 \cos(2 (39) \pi x) + 0\cos(2 (40) \pi x) + 1 / 41^2 \cos(2 (41) \pi x) + 2 / 42^2 \cos(2 (42) \pi x) + 1 / 43^2 \cos(2 (43) \pi x) + 0\cos(2 (44) \pi x) + 1 / 45^2 \cos(2 (45) \pi x) + 2 / 46^2 \cos(2 (46) \pi x) + 1 / 47^2 \cos(2 (47) \pi x) + 0\cos(2 (48) \pi x) + 1 / 49^2 \cos(2 (49) \pi x) + 2 / 50^2 \cos(2 (50) \pi x) + 1 / 51^2 \cos(2 (51) \pi x) + 0\cos(2 (52) \pi x))$

Guglie(x) = QuadraPari(x) + OTenda(x

Le guglie del castello scaligero di Malcesine

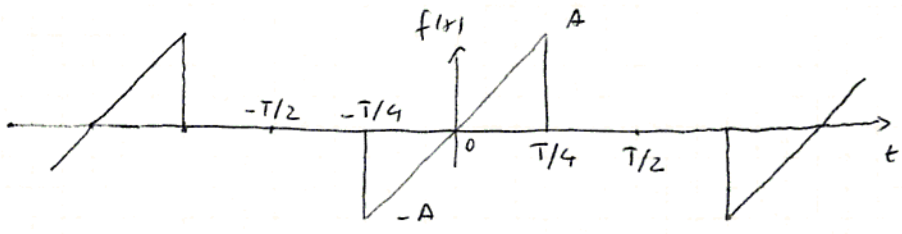


QuadraPari(x) $= 4 / \pi (\cos(2\pi x) - 1 / 3 \cos(2 (3) \pi x) + 1 / 5 \cos(2 (5) \pi x) - 1 / 7 \cos(2 (7) \pi x) + 1 / 9 \cos(2 (9) \pi x) - 1 / 11 \cos(2 (11) \pi x) + 1 / 13 \cos(2 (13) \pi x) - 1 / 15 \cos(2 (15) \pi x) + 1 / 17 \cos(2 (17) \pi x) - 1 / 19 \cos(2 (19) \pi x) + 1 / 21 \cos(2 (21) \pi x) - 1 / 23 \cos(2 (23) \pi x) + 1 / 25 \cos(2 (25) \pi x) - 1 / 27 \cos(2 (27) \pi x) + 1 / 29 \cos(2 (29) \pi x) - 1 / 31 \cos(2 (31) \pi x) + 1 / 33 \cos(2 (33) \pi x) - 1 / 35 \cos(2 (35) \pi x) + 1 / 37 \cos(2 (37) \pi x) - 1 / 39 \cos(2 (39) \pi x) + 1 / 41 \cos(2 (41) \pi x) - 1 / 43 \cos(2 (43) \pi x) + 1 / 45 \cos(2 (45) \pi x) - 1 / 47 \cos(2 (47) \pi x) + 1 / 49 \cos(2 (49) \pi x) - 1 / 51 \cos(2 (51) \pi x) + 1 / 53 \cos(2 (53) \pi x) - 1 / 55 \cos(2 (55) \pi x) + 1 / 57 \cos(2 (57) \pi x) - 1 / 59 \cos(2 (59) \pi x) + 1 / 61 \cos(2 (61) \pi x) - 1 / 63 \cos(2 (63) \pi x) + 1 / 65 \cos(2 (65) \pi x) - 1 / 67 \cos(2 (67) \pi x) + 1 / 69 \cos(2 (69) \pi x) - 1 / 71 \cos(2 (71) \pi x) + 1 / 73 \cos(2 (73) \pi x) - 1 / 75 \cos(2 (75) \pi x) + 1 / 77 \cos(2 (77) \pi x) - 1 / 79 \cos(2 (79) \pi x) + 1 / 81 \cos(2 (81) \pi x) - 1 / 83 \cos(2 (83) \pi x) + 1 / 85 \cos(2 (85) \pi x) - 1 / 87 \cos(2 (87) \pi x) + 1 / 89 \cos(2 (89) \pi x) - 1 / 91 \cos(2 (91) \pi x) + 1 / 93 \cos(2 (93) \pi x) - 1 / 95 \cos(2 (95) \pi x) + 1 / 97 \cos(2 (97) \pi x) - 1 / 99 \cos(2 (99) \pi x))$

QuadraPariM(x) $= 1 / 2 (1 + \text{QuadraPari}(x))$

Tenda(x) $= 0.25 + 4 / \pi^2 (\cos(2\pi x) + 2 / 2^2 \cos(2 (2) \pi x) + 1 / 3^2 \cos(2 (3) \pi x) + 0\cos(2 (4) \pi x) + 1 / 5^2 \cos(2 (5) \pi x) + 2 / 6^2 \cos(2 (6) \pi x) + 1 / 7^2 \cos(2 (7) \pi x) + 0\cos(2 (8) \pi x) + 1 / 9^2 \cos(2 (9) \pi x) + 2 / 10^2 \cos(2 (10) \pi x) + 1 / 11^2 \cos(2 (11) \pi x) + 0\cos(2 (12) \pi x) + 1 / 13^2 \cos(2 (13) \pi x) + 2 / 14^2 \cos(2 (14) \pi x) + 1 / 15^2 \cos(2 (15) \pi x) + 0\cos(2 (16) \pi x) + 1 / 17^2 \cos(2 (17) \pi x) + 2 / 18^2 \cos(2 (18) \pi x) + 1 / 19^2 \cos(2 (19) \pi x) + 0\cos(2 (20) \pi x) + 1 / 21^2 \cos(2 (21) \pi x) + 2 / 22^2 \cos(2 (22) \pi x) + 1 / 23^2 \cos(2 (23) \pi x) + 0\cos(2 (24) \pi x) + 1 / 25^2 \cos(2 (25) \pi x) + 2 / 26^2 \cos(2 (26) \pi x) + 1 / 27^2 \cos(2 (27) \pi x) + 0\cos(2 (28) \pi x) + 1 / 29^2 \cos(2 (29) \pi x) + 2 / 30^2 \cos(2 (30) \pi x) + 1 / 31^2 \cos(2 (31) \pi x) + 0\cos(2 (32) \pi x) + 1 / 33^2 \cos(2 (33) \pi x) + 2 / 34^2 \cos(2 (34) \pi x) + 1 / 35^2 \cos(2 (35) \pi x) + 0\cos(2 (36) \pi x) + 1 / 37^2 \cos(2 (37) \pi x) + 2 / 38^2 \cos(2 (38) \pi x) + 1 / 39^2 \cos(2 (39) \pi x) + 0\cos(2 (40) \pi x) + 1 / 41^2 \cos(2 (41) \pi x) + 2 / 42^2 \cos(2 (42) \pi x) + 1 / 43^2 \cos(2 (43) \pi x) + 0\cos(2 (44) \pi x) + 1 / 45^2 \cos(2 (45) \pi x) + 2 / 46^2 \cos(2 (46) \pi x) + 1 / 47^2 \cos(2 (47) \pi x) + 0\cos(2 (48) \pi x) + 1 / 49^2 \cos(2 (49) \pi x) + 2 / 50^2 \cos(2 (50) \pi x) + 1 / 51^2 \cos(2 (51) \pi x) + 0\cos(2 (52) \pi x))$

Casetta(x) $= \text{QuadraPariM}(x) + \text{Tenda}(x)$



$$f(t) = \begin{cases} \dfrac{A}{T/4}t = \dfrac{4A}{T}t & -\frac{T}{4}<t<T/4 \\ 0 & -\frac{T}{2}<t<-T/4 \\ 0 & \frac{T}{4}<t<T/2 \end{cases}$$

f.d.O. DISPARI A VALOR MEDIO NULLO

$$f(t) = \sum_{1}^{\infty} b_m \sin m\omega t$$

$$b_m = \frac{2}{T}\int_T f(t)\sin m\omega t\,dt = \frac{4}{T}\int_{T/2} f(t)\sin m\omega t\,dt = \frac{4}{T}\int_0^{T/4}\frac{4A}{T}t\sin m\omega t\,dt = \frac{16A}{T^2}\int_0^{T/4} t\sin m\omega t\,dt =$$

$$\int t\sin m\omega t\,dt = -\frac{t\cos m\omega t}{m\omega} + \frac{\sin m\omega t}{(m\omega)^2}$$

$$= \frac{16A}{T^2}\left[-\frac{t\cos m\omega t}{m\omega} + \frac{\sin m\omega t}{(m\omega)^2}\right]_0^{T/4} =$$

$$= \frac{16A}{T^2}\left[-\frac{T}{4}\cdot\frac{\cos m\omega T/4}{m\omega} + \frac{\sin m\omega T/4}{(m\omega)^2}\right] =$$

$$= -4A\frac{\cos m\omega T/4}{m\omega T} + \frac{16A}{(m\omega T)^2}\sin m\omega T/4$$

$$\omega T/4 = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$$

$$\omega T = 2\pi$$

$$= -\frac{4A}{m2\pi}\cos m\frac{\pi}{2} + \frac{16A}{4m^2\pi^2}\sin m\frac{\pi}{2}$$

$$= -\frac{2A}{m\pi}\cos m\frac{\pi}{2} + \frac{4A}{m^2\pi^2}\sin m\frac{\pi}{2}$$

$$\begin{cases} b_1 = \dfrac{4A}{\pi^2} & b_2 = \dfrac{A}{\pi} \\ b_3 = -\dfrac{4A}{\pi^2}\cdot\dfrac{1}{3^2} & b_4 = -\dfrac{A}{2\pi} \\ b_5 = \dfrac{4A}{\pi^2}\cdot\dfrac{1}{5^2} & b_6 = \dfrac{A}{3\pi} \\ b_7 = -\dfrac{4A}{\pi^2}\cdot\dfrac{1}{7^2} & b_8 = -\dfrac{A}{4\pi} \end{cases}$$

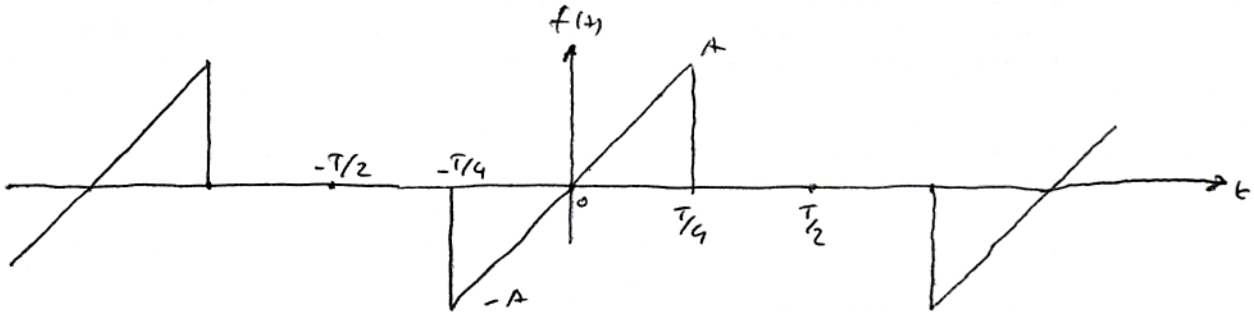
$$f(t) = \sum_1^{\infty}{}_m\left[-\frac{2A}{m\pi}\cos m\frac{\pi}{2} + \frac{4A}{m^2\pi^2}\sin m\frac{\pi}{2}\right]\sin m\omega t =$$

$$= \frac{4A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5}\sin 5\omega t - \dots\right] + \frac{A}{\pi}\left[\sin 2\omega t - \frac{1}{2}\sin 4\omega t + \frac{1}{3}\sin 6\omega t - \dots\right]$$

↑ SVILUPPO IN SERIE DI UNA f.d.O. TRIANGOLARE DISPARI DI AMPIEZZA $\frac{1}{2}$ E FREQUENZA $\frac{1}{T}$

↑ SVILUPPO IN SERIE DI UNA f.d.O A DENTE DI SEGA (1) DI AMPIEZZA $\frac{1}{2}$ E FREQUENZA $\frac{2}{T}$

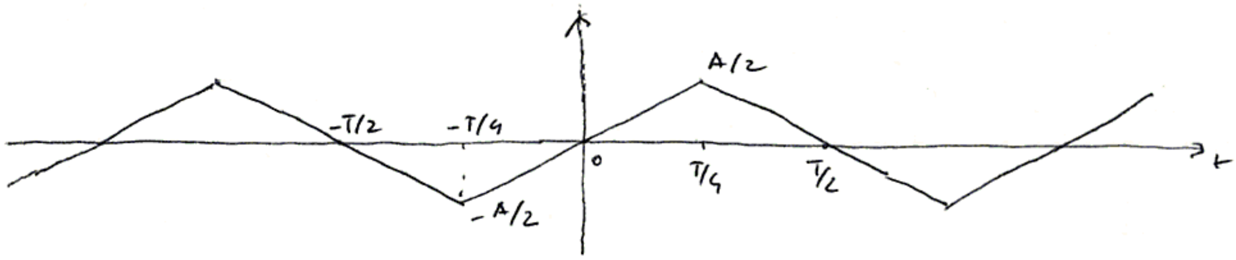
f



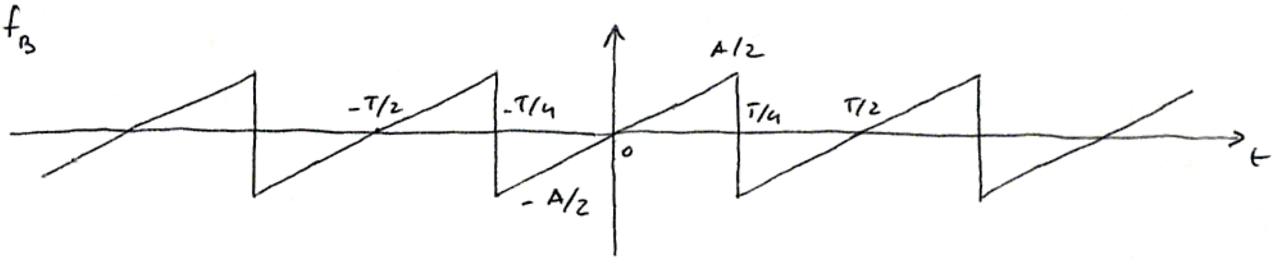
LA FORMA D'ONDA È UNA FUNZIONE PERIODICA DISPARI -

PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI 2 FUNZIONI PERIODICHE DISPARI

f_A



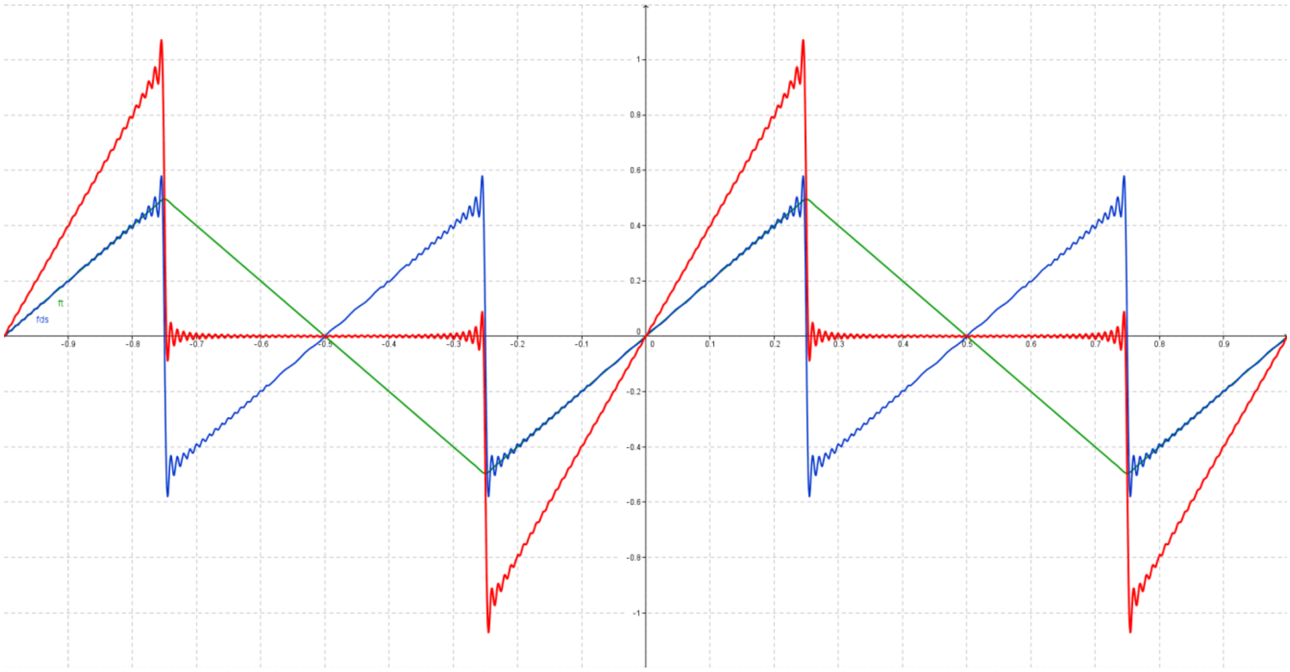
f_B



$$f = f_A + f_B$$

$$f_A(t) = \frac{4A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \frac{1}{7^2}\sin 7\omega t + \dots\right]$$

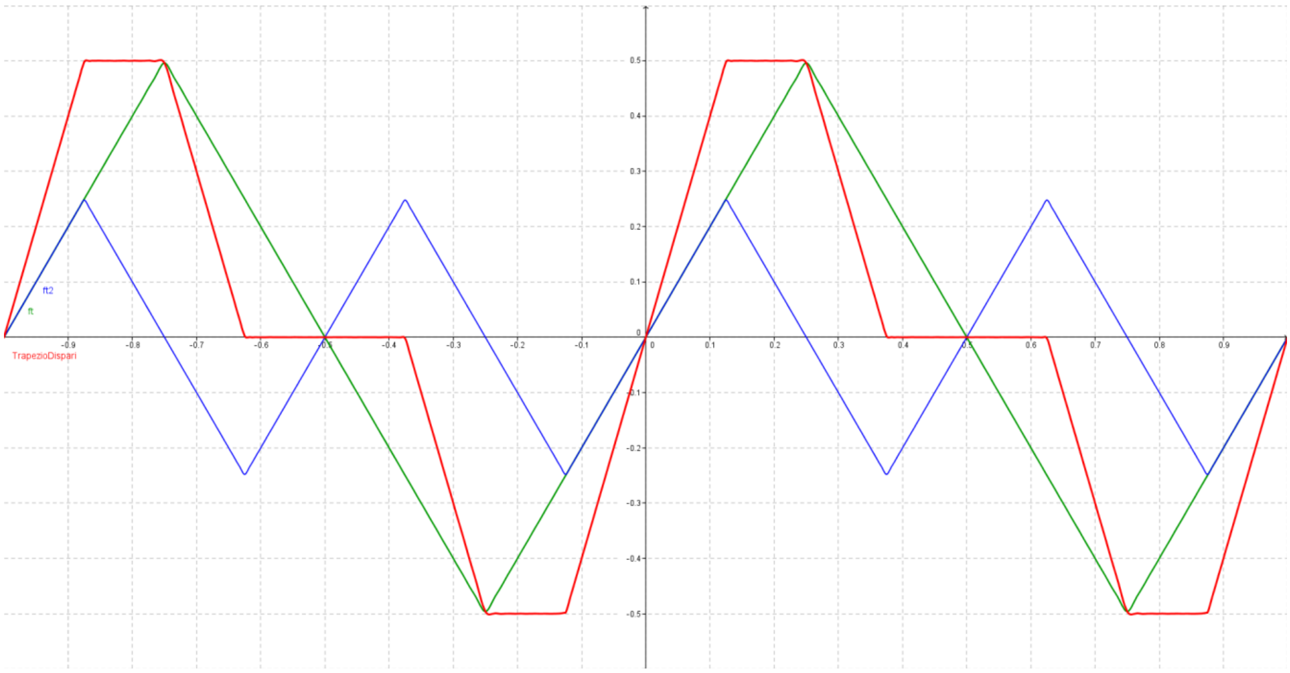
$$f_B(t) = \frac{A}{\pi}\left[\sin 2\omega t - \frac{1}{2}\sin 4\omega t + \frac{1}{3}\sin 6\omega t - \frac{1}{4}\sin 8\omega t + \dots\right]$$



$ft(x) = 4 / \pi^2 (\text{sen}(2\pi x) - 1 / 3^2 \text{sen}(2 (3) \pi x) + 1 / 5^2 \text{sen}(2 (5) \pi x) - 1 / 7^2 \text{sen}(2 (7) \pi x) + 1 / 9^2 \text{sen}(2 (9) \pi x) - 1 / 11^2 \text{sen}(2 (11) \pi x) + 1 / 13^2 \text{sen}(2 (13) \pi x) - 1 / 15^2 \text{sen}(2 (15) \pi x) + 1 / 17^2 \text{sen}(2 (17) \pi x) - 1 / 19^2 \text{sen}(2 (19) \pi x) + 1 / 21^2 \text{sen}(2 (21) \pi x) - 1 / 23^2 \text{sen}(2 (23) \pi x) + 1 / 25^2 \text{sen}(2 (25) \pi x) - 1 / 27^2 \text{sen}(2 (27) \pi x) + 1 / 29^2 \text{sen}(2 (29) \pi x) - 1 / 31^2 \text{sen}(2 (31) \pi x) + 1 / 33^2 \text{sen}(2 (33) \pi x) - 1 / 35^2 \text{sen}(2 (35) \pi x) + 1 / 37^2 \text{sen}(2 (37) \pi x) - 1 / 39^2 \text{sen}(2 (39) \pi x) + 1 / 41^2 \text{sen}(2 (41) \pi x) - 1 / 43^2 \text{sen}(2 (43) \pi x) + 1 / 45^2 \text{sen}(2 (45) \pi x) - 1 / 47^2 \text{sen}(2 (47) \pi x) + 1 / 49^2 \text{sen}(2 (49) \pi x))$

$fds(x) = 1/ \pi (\text{sen}(2\pi x/0.5) - 1 / 2 \text{sen}(2 (2) \pi x/0.5) + 1 / 3 \text{sen}(3 (2) \pi x/0.5) - 1 / 4 \text{sen}(4 (2) \pi x/0.5) + 1 / 5 \text{sen}(5 (2) \pi x/0.5) - 1 / 6 \text{sen}(6 (2) \pi x/0.5) + 1 / 7 \text{sen}(7 (2) \pi x/0.5) - 1 / 8 \text{sen}(8 (2) \pi x/0.5) + 1 / 9 \text{sen}(9 (2) \pi x/0.5) - 1 / 10 \text{sen}(10 (2) \pi x/0.5) + 1 / 11 \text{sen}(11 (2) \pi x/0.5) - 1 / 12 \text{sen}(12 (2) \pi x/0.5) + 1 / 13 \text{sen}(13 (2) \pi x/0.5) - 1 / 14 \text{sen}(14 (2) \pi x/0.5) + 1 / 15 \text{sen}(15 (2) \pi x/0.5) - 1 / 16 \text{sen}(16 (2) \pi x/0.5) + 1 / 17 \text{sen}(17 (2) \pi x/0.5) - 1 / 18 \text{sen}(18 (2) \pi x/0.5) + 1 / 19 \text{sen}(19 (2) \pi x/0.5) - 1 / 20 \text{sen}(20 (2) \pi x/0.5) + 1 / 21 \text{sen}(21 (2) \pi x/0.5) - 1 / 22 \text{sen}(22 (2) \pi x/0.5) + 1 / 23 \text{sen}(23 (2) \pi x/0.5) - 1 / 24 \text{sen}(24 (2) \pi x/0.5) + 1 / 25 \text{sen}(25 (2) \pi x/0.5) - 1 / 26 \text{sen}(26 (2) \pi x/0.5) + 1 / 27 \text{sen}(27 (2) \pi x/0.5) - 1 / 28 \text{sen}(28 (2) \pi x/0.5) + 1 / 29 \text{sen}(29 (2) \pi x/0.5) - 1 / 30 \text{sen}(30 (2) \pi x/0.5) + 1 / 31 \text{sen}(31 (2) \pi x/0.5) - 1 / 32 \text{sen}(32 (2) \pi x/0.5) + 1 / 33 \text{sen}(33 (2) \pi x/0.5) - 1 / 34 \text{sen}(34 (2) \pi x/0.5) + 1 / 35 \text{sen}(35 (2) \pi x/0.5) - 1 / 36 \text{sen}(36 (2) \pi x/0.5) + 1 / 37 \text{sen}(37 (2) \pi x/0.5) - 1 / 38 \text{sen}(38 (2) \pi x/0.5) + 1 / 39 \text{sen}(39 (2) \pi x/0.5) - 1 / 40 \text{sen}(40 (2) \pi x/0.5) + 1 / 41 \text{sen}(41 (2) \pi x/0.5) - 1 / 42 \text{sen}(42 (2) \pi x/0.5) + 1 / 43 \text{sen}(43 (2) \pi x/0.5) - 1 / 44 \text{sen}(44 (2) \pi x/0.5) + 1 / 45 \text{sen}(45 (2) \pi x/0.5) - 1 / 46 \text{sen}(46 (2) \pi x/0.5) + 1 / 47 \text{sen}(47 (2) \pi x/0.5) - 1 / 48 \text{sen}(48 (2) \pi x/0.5) + 1 / 49 \text{sen}(49 (2) \pi x/0.5) - 1 / 50 \text{sen}(50 (2) \pi x/0.5))$

$dispari(x) = ft(x) + fds(x)$



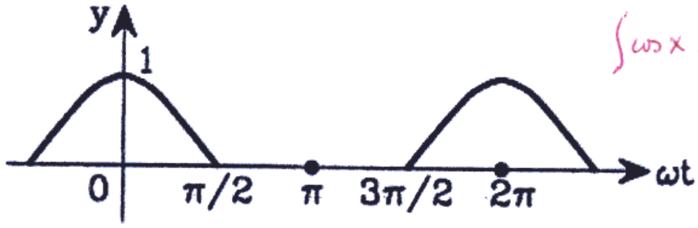
$ft(x) = 4 / \pi^2 (sen(2\pi x) - 1 / 3^2 sen(2 (3) \pi x) + 1 / 5^2 sen(2 (5) \pi x) - 1 / 7^2 sen(2 (7) \pi x) + 1 / 9^2 sen(2 (9) \pi x) - 1 / 11^2 sen(2 (11) \pi x) + 1 / 13^2 sen(2 (13) \pi x) - 1 / 15^2 sen(2 (15) \pi x) + 1 / 17^2 sen(2 (17) \pi x) - 1 / 19^2 sen(2 (19) \pi x) + 1 / 21^2 sen(2 (21) \pi x) - 1 / 23^2 sen(2 (23) \pi x) + 1 / 25^2 sen(2 (25) \pi x) - 1 / 27^2 sen(2 (27) \pi x) + 1 / 29^2 sen(2 (29) \pi x) - 1 / 31^2 sen(2 (31) \pi x) + 1 / 33^2 sen(2 (33) \pi x) - 1 / 35^2 sen(2 (35) \pi x) + 1 / 37^2 sen(2 (37) \pi x) - 1 / 39^2 sen(2 (39) \pi x) + 1 / 41^2 sen(2 (41) \pi x) - 1 / 43^2 sen(2 (43) \pi x) + 1 / 45^2 sen(2 (45) \pi x) - 1 / 47^2 sen(2 (47) \pi x) + 1 / 49^2 sen(2 (49) \pi x))$

$ft2(x) = 4 / \pi^2 (sen(2\pi x/0.5) - 1 / 3^2 sen(2 (3) \pi x/0.5) + 1 / 5^2 sen(2 (5) \pi x/0.5) - 1 / 7^2 sen(2 (7) \pi x/0.5) + 1 / 9^2 sen(2 (9) \pi x/0.5) - 1 / 11^2 sen(2 (11) \pi x/0.5) + 1 / 13^2 sen(2 (13) \pi x/0.5) - 1 / 15^2 sen(2 (15) \pi x/0.5) + 1 / 17^2 sen(2 (17) \pi x/0.5) - 1 / 19^2 sen(2 (19) \pi x/0.5) + 1 / 21^2 sen(2 (21) \pi x/0.5) - 1 / 23^2 sen(2 (23) \pi x/0.5) + 1 / 25^2 sen(2 (25) \pi x/0.5) - 1 / 27^2 sen(2 (27) \pi x/0.5) + 1 / 29^2 sen(2 (29) \pi x/0.5) - 1 / 31^2 sen(2 (31) \pi x/0.5) + 1 / 33^2 sen(2 (33) \pi x/0.5) - 1 / 35^2 sen(2 (35) \pi x/0.5) + 1 / 37^2 sen(2 (37) \pi x/0.5) - 1 / 39^2 sen(2 (39) \pi x/0.5) + 1 / 41^2 sen(2 (41) \pi x/0.5) - 1 / 43^2 sen(2 (43) \pi x/0.5) + 1 / 45^2 sen(2 (45) \pi x/0.5) - 1 / 47^2 sen(2 (47) \pi x/0.5) + 1 / 49^2 sen(2 (49) \pi x/0.5))$

$TrapezioDispari(x) = ft(x) + ft2(x)$

$$\int \cos x \cos(nx)\,dx = \frac{\sin((n-1)x)}{2(n-1)} + \frac{\sin((n+1)x)}{2(n+1)}$$

$$\int \cos x \sin(nx)\,dx = \frac{-\cos((n-1)x)}{2(n-1)} - \frac{\cos((n+1)x)}{2(n+1)}$$



$$a_n = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x)\cdot\cos(nx)\cdot dx$$

da cui:

$$a_n = \begin{cases} \dfrac{1}{\pi}\left[\dfrac{\sin(n-1)\frac{\pi}{2}}{n-1} + \dfrac{\sin(n+1)\frac{\pi}{2}}{n+1}\right] & \forall n \neq 1 \\ \dfrac{1}{2} & n = 1 \end{cases}$$

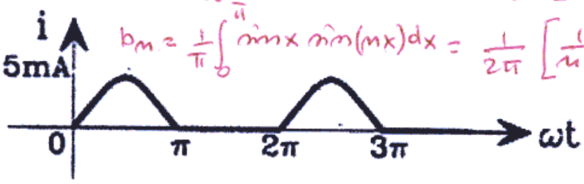
$$\frac{1}{\pi}\left[\frac{1}{n+1} - \frac{1}{n-1}\right]\cos\left(n\frac{\pi}{2}\right) = -\frac{2}{\pi}\frac{1}{(n-1)(n+1)}\cdot\cos\left(n\frac{\pi}{2}\right)$$

$$y(t) = \frac{1}{\pi}\left\{1 + \frac{\pi}{2}\cos(\omega t) + \frac{2}{3}\cos(2\omega t) - \frac{2}{15}\cos(4\omega t) + \frac{2}{35}\cos(6\omega t) - \ldots\right\}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \sin x \cos(nx)\,dx = \frac{1}{2\pi}\left[\frac{1}{n+1} - \frac{1}{n-1}\right](1 + \cos n\pi) = \frac{1}{2\pi}\left[\frac{-2}{(n+1)(n-1)}\right]\cdot 2 \quad n \text{ pari}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \sin x \sin(nx)\,dx = \frac{1}{2\pi}\left[\frac{1}{n+1} - \frac{1}{n-1}\right]\sin n\pi \quad = 0 \text{ per } n \neq 1$$

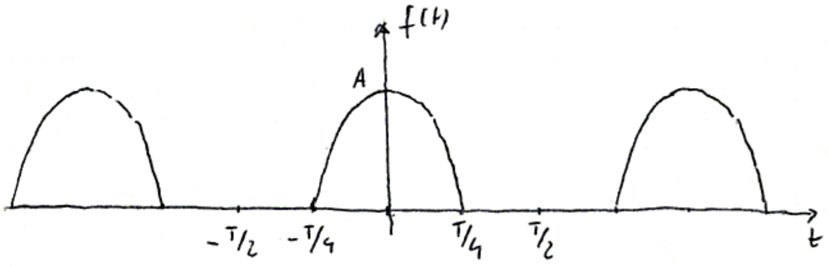
$$b_1 = \frac{1}{\pi}\int_0^{\pi} \sin^2 x\,dx = \frac{1}{2}$$



$$i(t) = \frac{5\cdot 10^{-3}}{\pi}\left\{1 + \frac{\pi}{2}\sin(\omega t) - \frac{2}{3}\cos(2\omega t) - \frac{2}{15}\cos(4\omega t) - \frac{2}{35}\cos(6\omega t) + \ldots\right\}$$

$$\int \sin x \cos(nx)\,dx = \frac{\cos((n-1)x)}{2(n-1)} - \frac{\cos((n+1)x)}{2(n+1)}$$

$$\int \sin x \sin(nx)\,dx = \frac{\sin((n-1)x)}{2(n-1)} - \frac{\sin((n+1)x)}{2(n+1)}$$



$$f(t) = \begin{cases} 0 & -\frac{T}{2} < t < -\frac{T}{4} \\ A \cos \omega t & -\frac{T}{4} < t < \frac{T}{4} \\ 0 & \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$\omega = \frac{2\pi}{T}$$

FUNZIONE PERIODICA PARI

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t + \sum_{1}^{\infty} b_n \sin n\omega t \qquad b_n = 0$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\int_{-T/4}^{T/4} f(t)\,dt = \frac{2}{T}\int_0^{T/4} f(t)\,dt = \frac{2}{T}\int_0^{T/4} A\cos\omega t\,dt =$$

$$= \frac{2A}{\omega T}\sin\omega t\Big|_0^{T/4} = \frac{2A}{\frac{2\pi}{T}\cdot T}\cdot\left[\sin\frac{2\pi}{T}\cdot\frac{T}{4} - \sin\frac{2\pi}{T}\cdot 0\right] = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\int_{-T/4}^{T/4} A\cos\omega t\cos n\omega t\,dt = \qquad \left[\cos\omega t\cos n\omega t = \frac{1}{2}\cos(n-1)\omega t + \frac{1}{2}\cos(n+1)\omega t\right]$$

$$= \frac{2A}{T}\int_{-T/4}^{T/4}\left(\frac{1}{2}\cos(n-1)\omega t + \frac{1}{2}\cos(n+1)\omega t\right)dt = \frac{2A}{T}\int_0^{T/4}\left(\cos(n-1)\omega t + \cos(n+1)\omega t\right)dt =$$

$$= \frac{2A}{(n-1)\omega T}\sin(n-1)\omega t\Big|_0^{T/4} + \frac{2A}{(n+1)\omega T}\sin(n+1)\omega t\Big|_0^{T/4} =$$

$$= \frac{2A}{\frac{2\pi}{T}\cdot T}\left[\frac{1}{n-1}\sin(n-1)\cdot\frac{2\pi}{T}\cdot\frac{T}{4} + \frac{1}{n+1}\sin(n+1)\cdot\frac{2\pi}{T}\cdot\frac{T}{4}\right] =$$

$$= \frac{A}{\pi}\,\frac{\sin(n-1)\pi/2}{n-1} + \frac{A}{\pi}\,\frac{\sin(n+1)\pi/2}{n+1} = \frac{A\sin(n-1)\pi/2}{\pi(n-1)\cdot\frac{2}{2}} + \frac{A\sin(n+1)\pi/2}{\pi(n+1)\cdot\frac{2}{2}} =$$

$$= \frac{A}{2}\left[\frac{\sin(n-1)\pi/2}{(n-1)\pi/2} + \frac{\sin(n+1)\pi/2}{(n+1)\pi/2}\right]$$

$$a_1 = \frac{A}{2}\left[\frac{\sin 0}{0} + \frac{\sin 2\pi/2}{2\pi/2}\right] = \frac{A}{2} \qquad a_2 = \frac{A}{\pi}\left[\frac{\sin\pi/2}{1} + \frac{\sin 3\pi/2}{3}\right] = \frac{A}{\pi}\left[1 - \frac{1}{3}\right] = \frac{2A}{3\pi}$$

$$a_3 = \frac{A}{\pi}\left[\frac{\sin 2\pi/2}{2} + \frac{\sin 4\pi/2}{4}\right] = 0 \qquad a_4 = \frac{A}{\pi}\left[\frac{\sin 3\pi/2}{3} + \frac{\sin 5\pi/2}{5}\right] = \frac{A}{\pi}\left[-\frac{1}{3} + \frac{1}{5}\right] = \frac{A}{\pi}\left(-\frac{2}{3\cdot 5}\right)$$

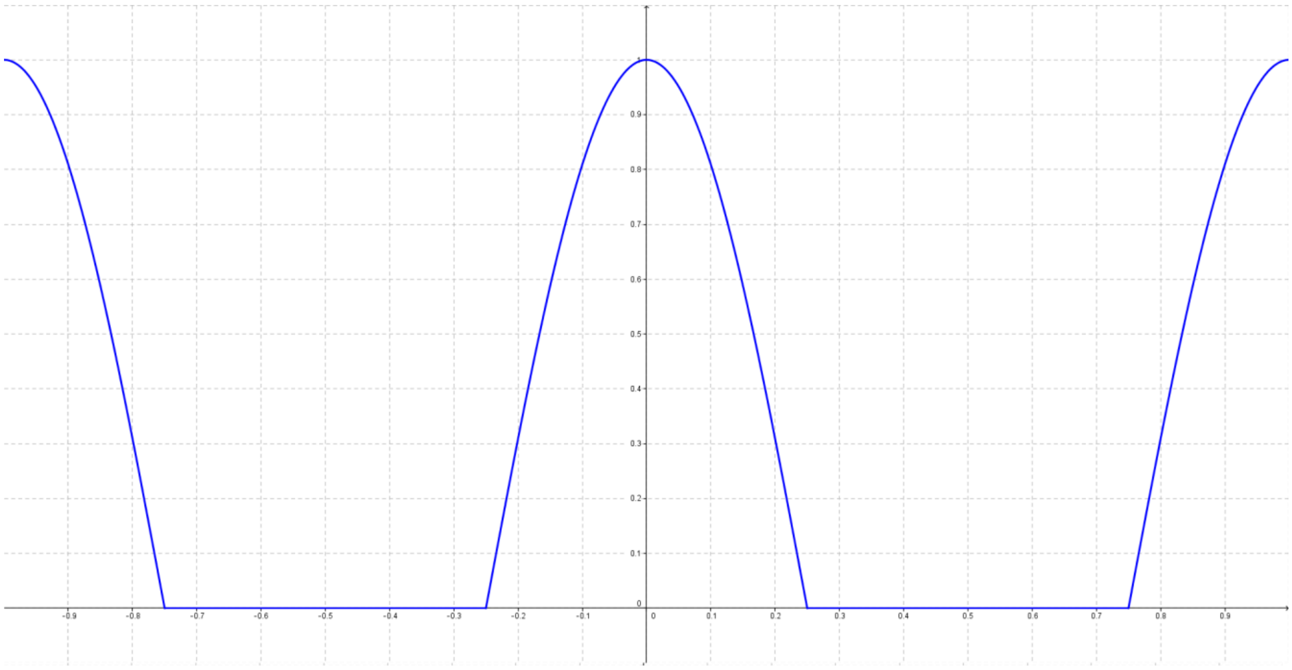
$$a_5 = \frac{A}{\pi}\left[\frac{\sin 4\pi/2}{4} + \frac{\sin 6\pi/2}{6}\right] = 0 \qquad a_6 = \frac{A}{\pi}\left[\frac{\sin 5\pi/2}{5} + \frac{\sin 7\pi/2}{7}\right] = \frac{A}{\pi}\left[\frac{1}{5} - \frac{1}{7}\right] = \frac{A}{\pi}\left(\frac{2}{5\cdot 7}\right)$$

armoniche indice dispari tutte nulle tranne la prima: $a_1 = A/2$

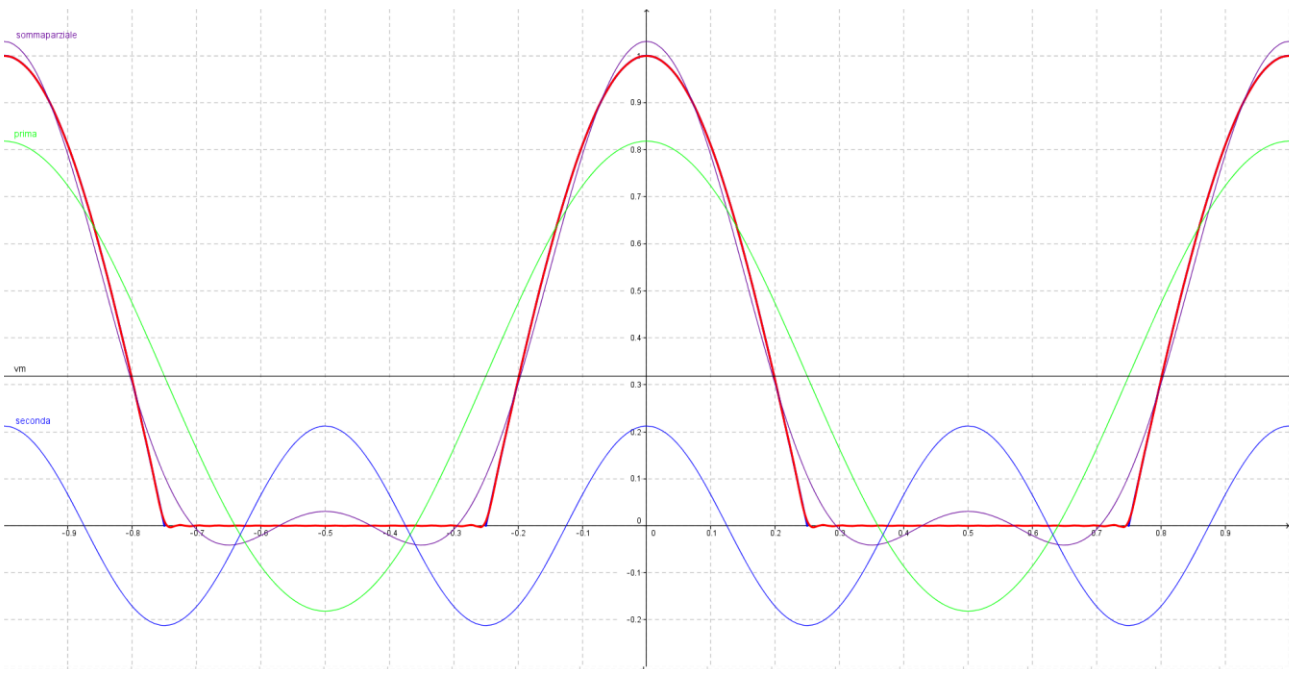
$$a_n = \frac{A}{\pi}\left[\frac{1}{n+1} - \frac{1}{n-1}\right]\cos n\frac{\pi}{2} = -\frac{2A}{\pi}\cdot\frac{1}{(n-1)(n+1)}\cos n\pi/2 \qquad \forall n \neq 1$$

$$f(t) = \frac{A}{\pi}\left[1 + \frac{\pi}{2}\cos\omega t + \frac{2}{1\cdot 3}\cos 2\omega t - \frac{2}{3\cdot 5}\cos 4\omega t + \frac{2}{5\cdot 7}\cos 6\omega t - \frac{2}{7\cdot 9}\cos 8\omega t + \dots\right]$$

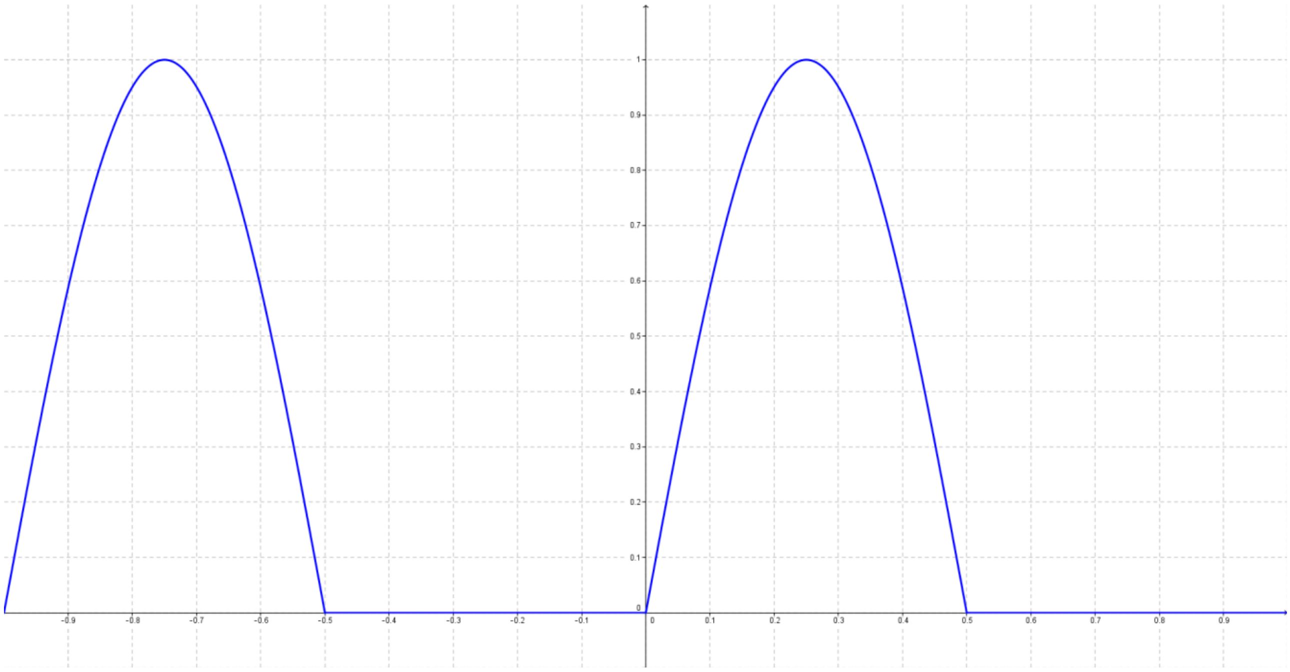
Raddrizzata semionda coseno



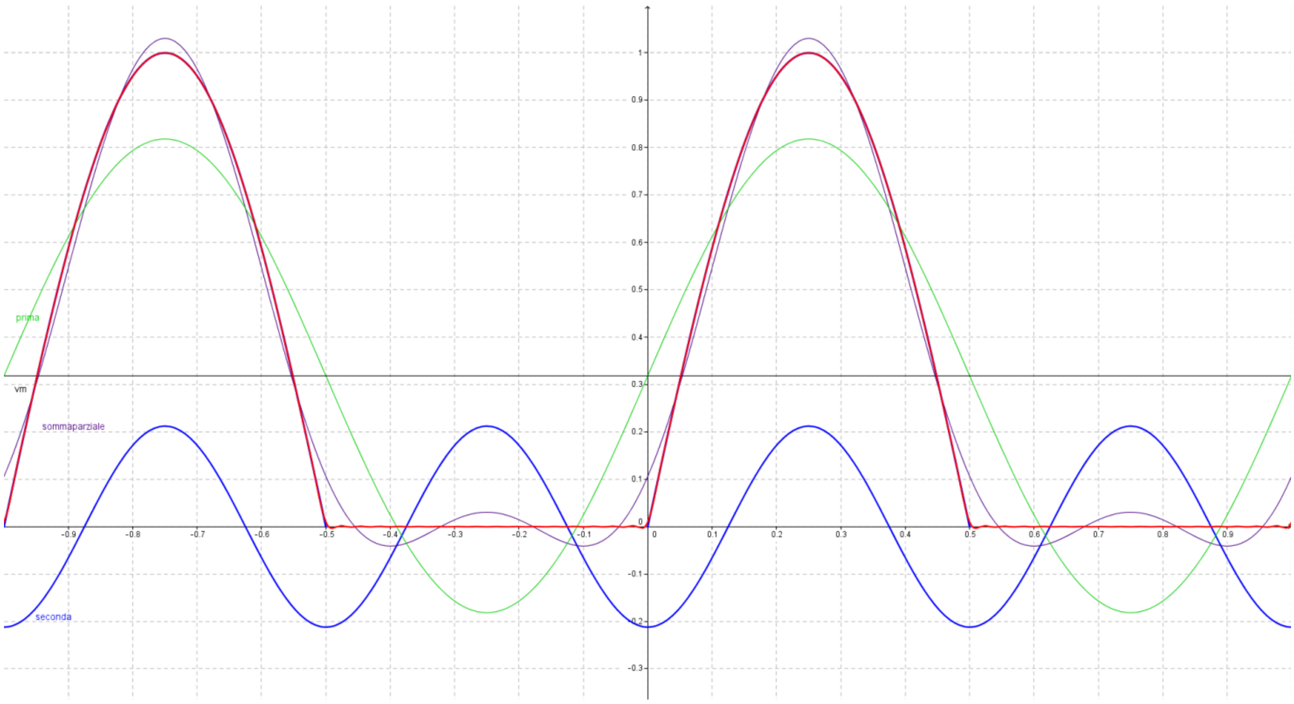
$$f(x)= 1/\pi\ (1+\pi\ /2 \cos(2\pi\ x) +2/3 \cos(2\ 2\pi\ x) - 2/(3*5) \cos(4\ 2\pi\ x) +2/(5*7) \cos(6\ 2\pi\ x) - 2/(7*9) \cos(8\ 2\pi\ x) +2/(9*11) \cos(10\ 2\pi\ x) - 2/(11*13) \cos(12\ 2\pi\ x) +2/(13*15) \cos(14\ 2\pi\ x) - 2/(15*17) \cos(16\ 2\pi\ x) +2/(17*19) \cos(18\ 2\pi\ x) - 2/(19*21) \cos(20\ 2\pi\ x) +2/(21*23) \cos(22\ 2\pi\ x) - 2/(23*25) \cos(24\ 2\pi\ x) +2/(25*27) \cos(26\ 2\pi\ x) - 2/(27*29) \cos(28\ 2\pi\ x) +2/(29*31) \cos(30\ 2\pi\ x) - 2/(31*33) \cos(32\ 2\pi\ x))$$



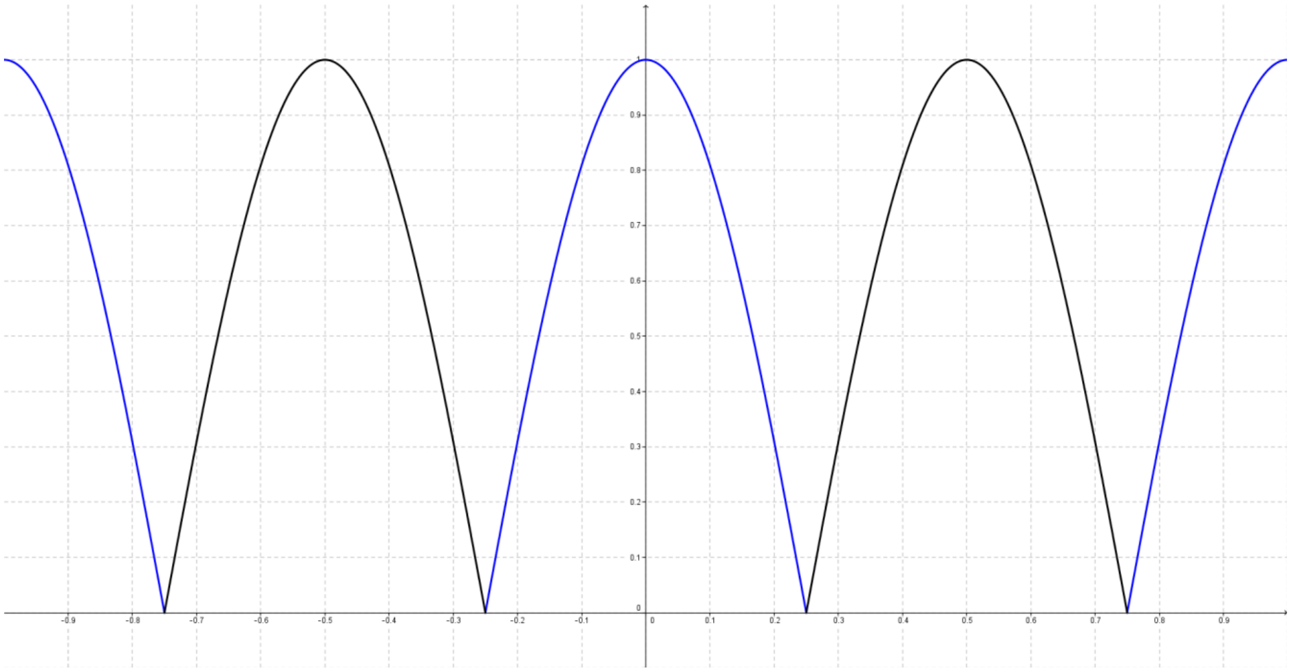
Raddrizzata semionda seno



$$f(x) = 1/\pi\,(1+\pi/2\ \mathrm{sen}(2\pi\, x) - 2/3\cos(2\ 2\pi\, x) - 2/(3*5)\cos(4\ 2\pi\, x) - 2/(5*7)\cos(6\ 2\pi\, x) - 2/(7*9)\cos(8\ 2\pi\, x) - 2/(9*11)\cos(10\ 2\pi\, x) - 2/(11*13)\cos(12\ 2\pi\, x) - 2/(13*15)\cos(14\ 2\pi\, x) - 2/(15*17)\cos(16\ 2\pi\, x) - 2/(17*19)\cos(18\ 2\pi\, x) - 2/(19*21)\cos(20\ 2\pi\, x) - 2/(21*23)\cos(22\ 2\pi\, x) - 2/(23*25)\cos(24\ 2\pi\, x) - 2/(25*27)\cos(26\ 2\pi\, x) - 2/(27*29)\cos(28\ 2\pi\, x) - 2/(29*31)\cos(30\ 2\pi\, x) - 2/(31*33)\cos(32\ 2\pi\, x))$$

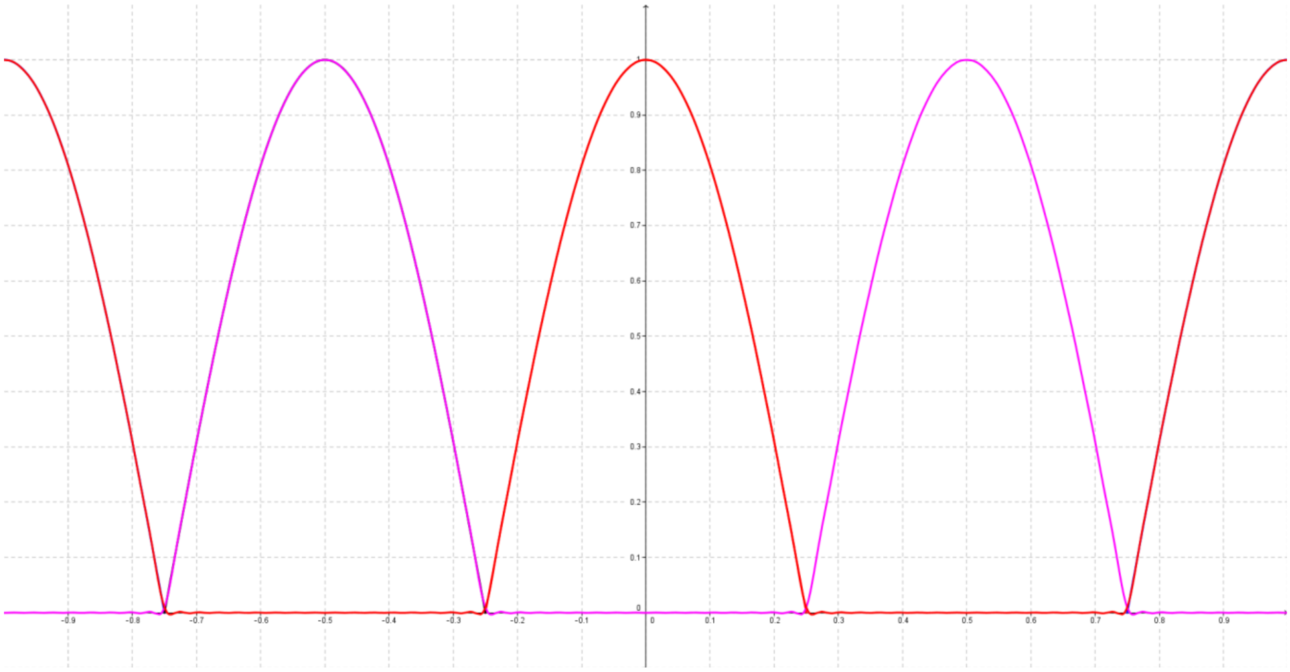


Raddrizzata doppia semionda coseno



f(x)= 1/π (1+π /2 cos(2π x) +2/3 cos(2 2π x) - 2/(3*5) cos(4 2π x) +2/(5*7) cos(6 2π x) - 2/(7*9) cos(8 2π x) +2/(9*11) cos(10 2π x) - 2/(11*13) cos(12 2π x) +2/(13*15) cos(14 2π x) - 2/(15*17) cos(16 2π x) +2/(17*19) cos(18 2π x) - 2/(19*21) cos(20 2π x) +2/(21*23) cos(22 2π x) - 2/(23*25) cos(24 2π x) +2/(25*27) cos(26 2π x) - 2/(27*29) cos(28 2π x) +2/(29*31) cos(30 2π x) - 2/(31*33) cos(32 2π x))

ftraslata(x)= 1/π (1+π /2 cos(2π (x-1/2)) +2/3 cos(2 2π (x-1/2)) - 2/(3*5) cos(4 2π (x-1/2)) +2/(5*7) cos(6 2π (x-1/2)) - 2/(7*9) cos(8 2π (x-1/2)) +2/(9*11) cos(10 2π (x-1/2)) - 2/(11*13) cos(12 2π (x-1/2)) +2/(13*15) cos(14 2π (x-1/2)) - 2/(15*17) cos(16 2π (x-1/2)) +2/(17*19) cos(18 2π (x-1/2)) - 2/(19*21) cos(20 2π (x-1/2)) +2/(21*23) cos(22 2π (x-1/2)) - 2/(23*25) cos(24 2π (x-1/2)) +2/(25*27) cos(26 2π (x-1/2)) - 2/(27*29) cos(28 2π (x-1/2)) +2/(29*31) cos(30 2π (x-1/2)) - 2/(31*33) cos(32 2π (x-1/2)))

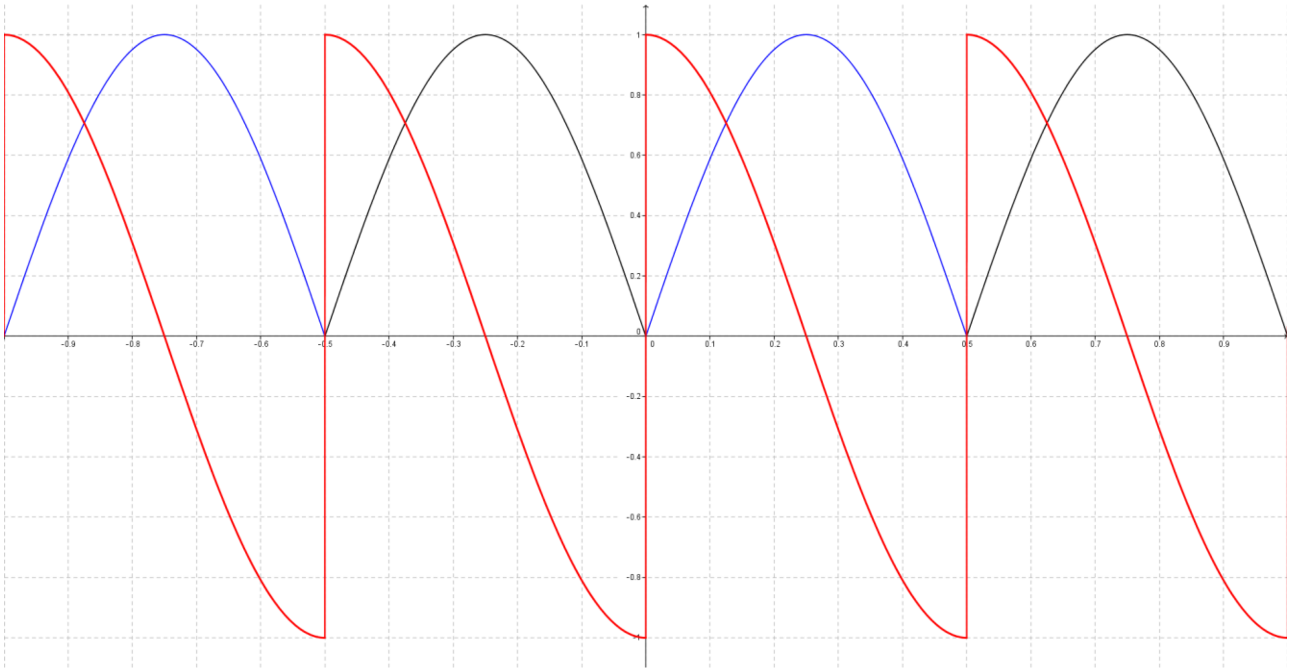


Raddrizzata doppia semionda seno

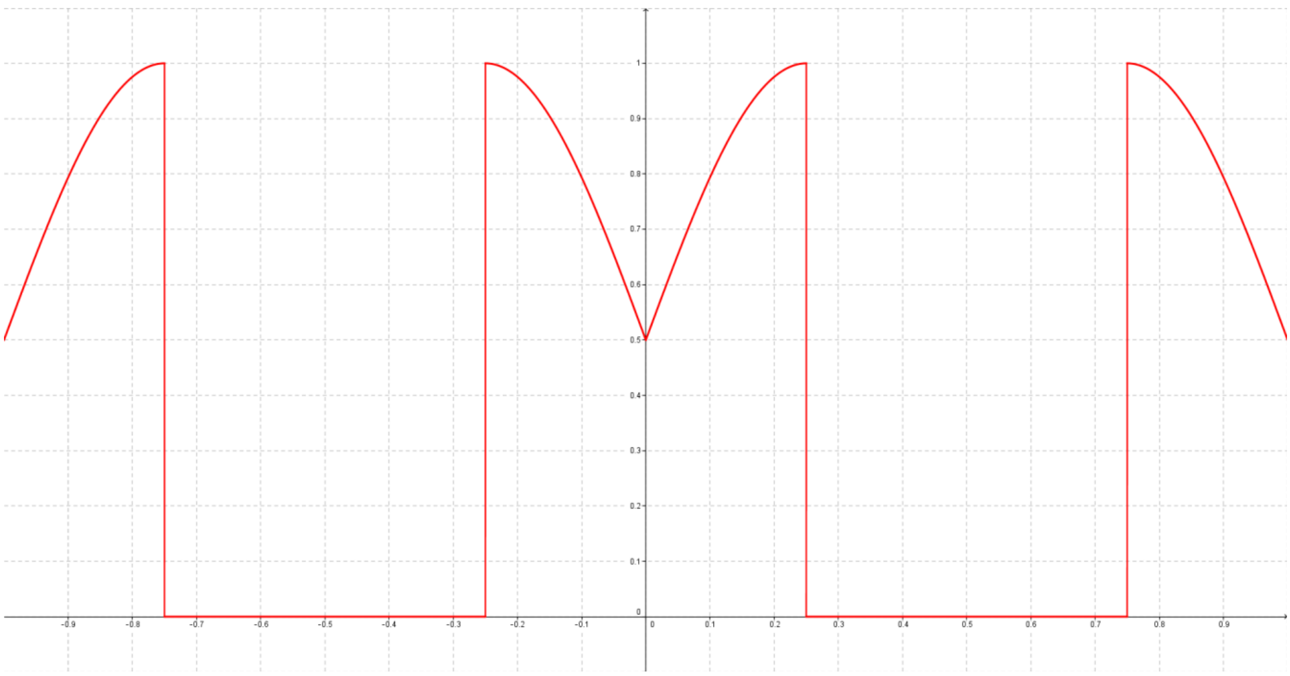
f(x)= 1/π (1+π /2 sen(2π x) - 2/3 cos(2 2π x) - 2/(3*5) cos(4 2π x) - 2/(5*7) cos(6 2π x) - 2/(7*9) cos(8 2π x) - 2/(9*11) cos(10 2π x) - 2/(11*13) cos(12 2π x) - 2/(13*15) cos(14 2π x) - 2/(15*17) cos(16 2π x) - 2/(17*19) cos(18 2π x) - 2/(19*21) cos(20 2π x) - 2/(21*23) cos(22 2π x) - 2/(23*25) cos(24 2π x) - 2/(25*27) cos(26 2π x) - 2/(27*29) cos(28 2π x) - 2/(29*31) cos(30 2π x) - 2/(31*33) cos(32 2π x))

ftraslata(x)= 1/π (1+π /2 sen(2π (x-1/2)) - 2/3 cos(2 2π (x-1/2)) - 2/(3*5) cos(4 2π (x-1/2)) - 2/(5*7) cos(6 2π (x-1/2)) - 2/(7*9) cos(8 2π (x-1/2)) -2/(9*11) cos(10 2π (x-1/2)) - 2/(11*13) cos(12 2π (x-1/2)) - 2/(13*15) cos(14 2π (x-1/2)) - 2/(15*17) cos(16 2π (x-1/2)) - 2/(17*19) cos(18 2π (x-1/2)) - 2/(19*21) cos(20 2π (x-1/2)) - 2/(21*23) cos(22 2π (x-1/2)) - 2/(23*25) cos(24 2π (x-1/2)) - 2/(25*27) cos(26 2π (x-1/2)) - 2/(27*29) cos(28 2π (x-1/2)) - 2/(29*31) cos(30 2π (x-1/2)) - 2/(31*33) cos(32 2π (x-1/2)))

Derivata Raddrizzata doppia semionda seno

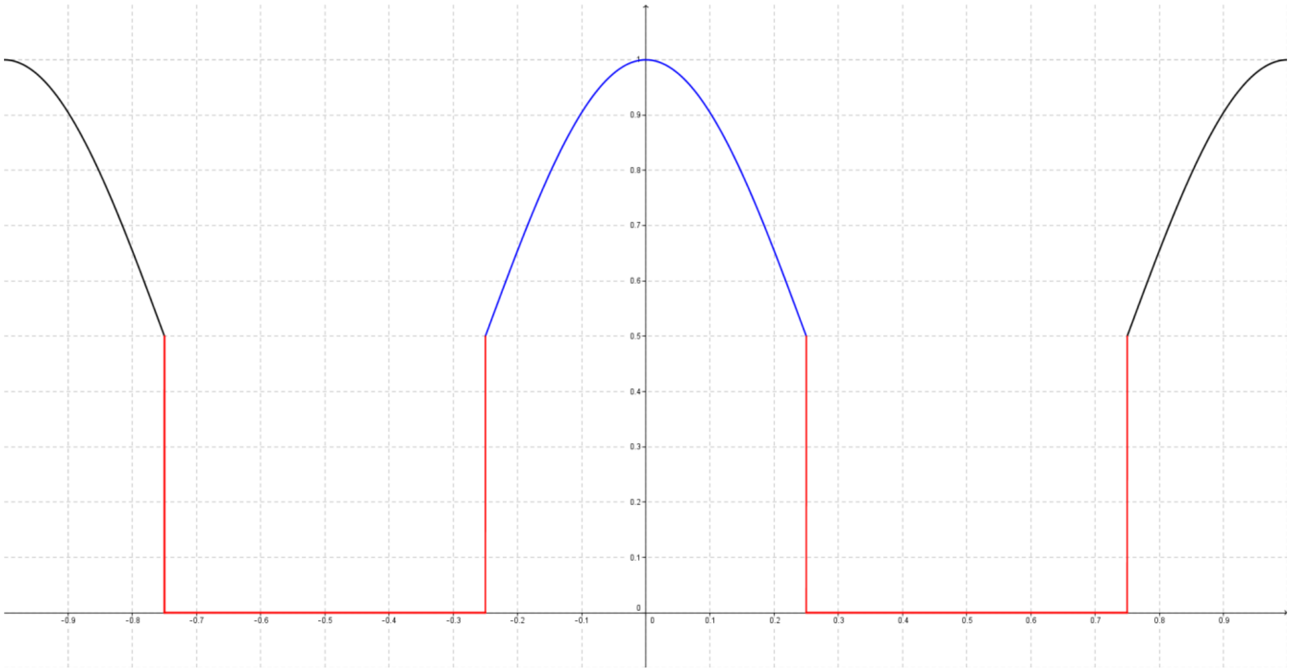


Guglie S

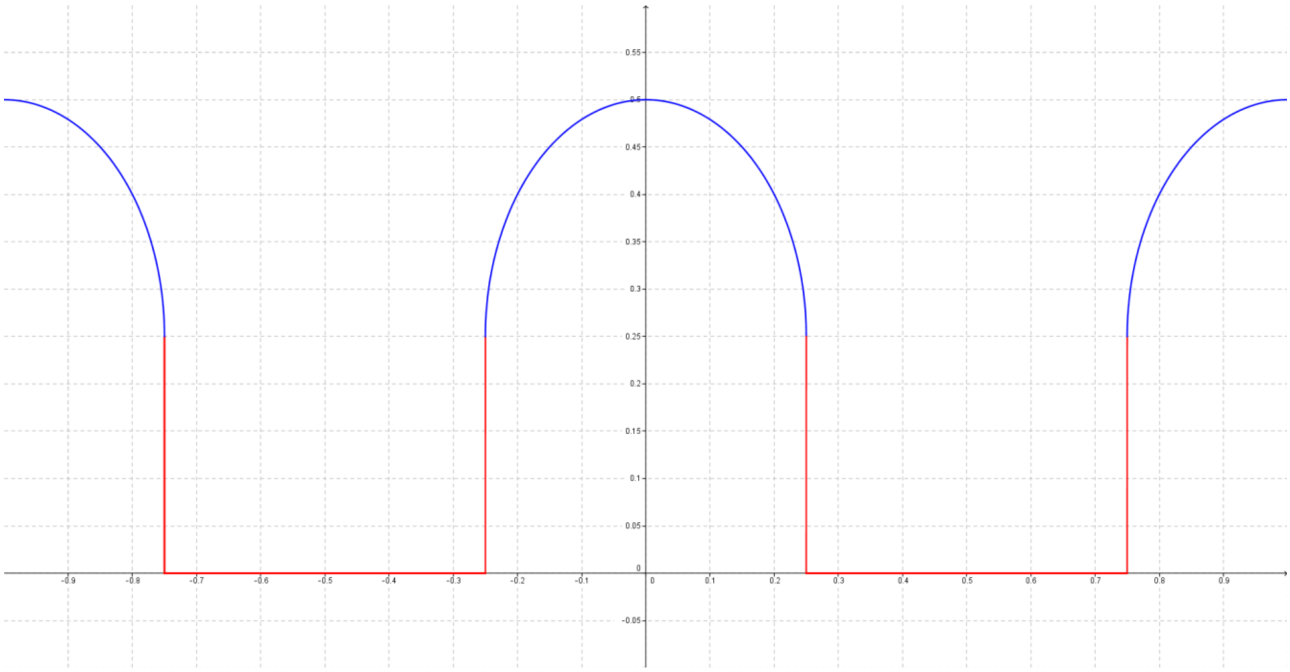


Guglie del castello scaligero di Sirmione

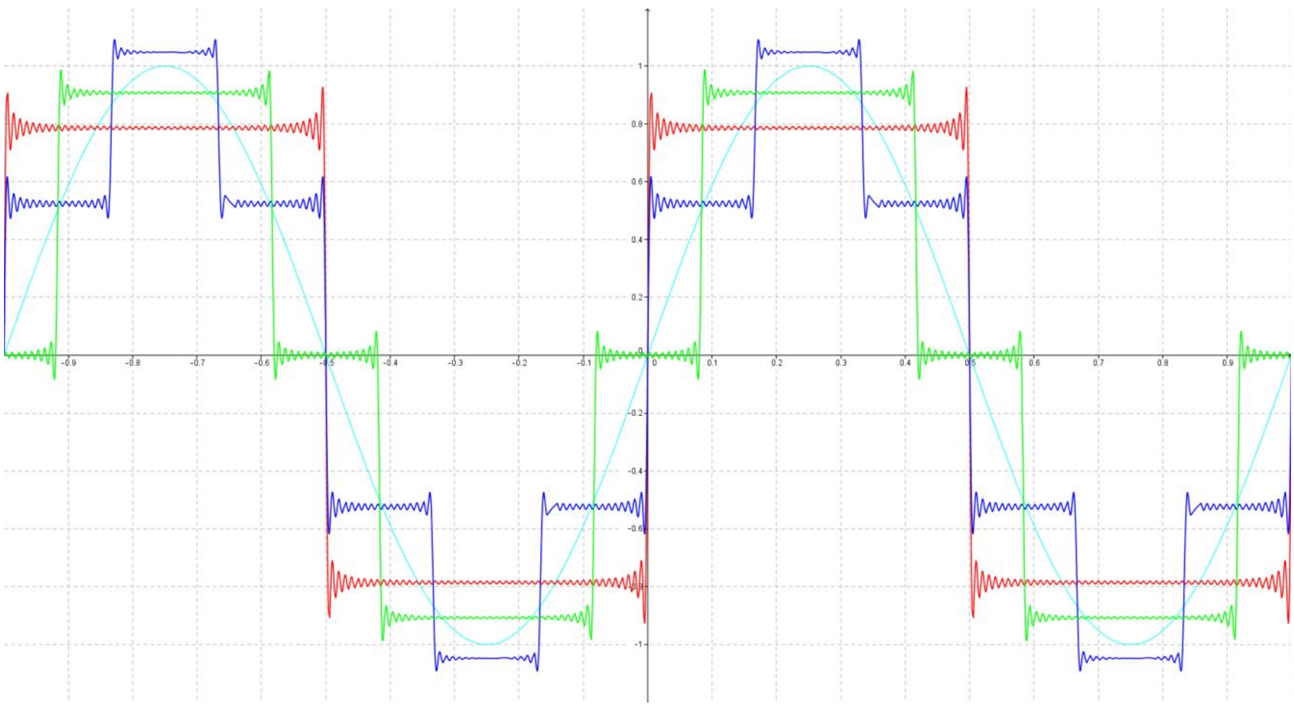
Guglie SIN



Guglie C



Confronto onda quadra / six step / quasi quadra a pari ampiezza di prima armonica



Sviluppo armonico

fq(x) = sin(2 x π) + sin(3 * 2 π x) / 3 + sin(5 * 2 π x) / 5 + sin(7 * 2 π x) / 7 + sin(9 * 2 π x) / 9 + sin(11 * 2 π x) / 11 + sin(13 * 2 π x) / 13 + sin(15 * 2 π x) / 15 + sin(17 * 2 π x) / 17 + sin(19 * 2 π x) / 19 + sin(21 * 2 π x) / 21 + sin(23 * 2 π x) / 23 + sin(25 * 2 π x) / 25 + sin(27 * 2 π x) / 27 + sin(29 * 2 π x) / 29 + sin(31 * 2 π x) / 31 + sin(33 * 2 π x) / 33 + sin(35 * 2 π x) / 35 + sin(37 * 2 π x) / 37 + sin(39 * 2 π x) / 39 + sin(41 * 2 π x) / 41 + sin(43 * 2 π x) / 43 + sin(45 * 2 π x) / 45 + sin(47 * 2 π x) / 47 + sin(49 * 2 π x) / 49 + sin(51 * 2 π x) / 51 + sin(53 * 2 π x) / 53 + sin(55 * 2 π x) / 55 + sin(57 * 2 π x) / 57 + sin(59 * 2 π x) / 59 + sin(61 * 2 π x) / 61 + sin(63 * 2 π x) / 63 + sin(65 * 2 π x) / 65 + sin(67 * 2 π x) / 67 + sin(69 * 2 π x) / 69 + sin(71 * 2 π x) / 71 + sin(73 * 2 π x) / 73 + sin(75 * 2 π x) / 75 + sin(77 * 2 π x) / 77 + sin(79 * 2 π x) / 79 + sin(81 * 2 π x) / 81 + sin(83 * 2 π x) / 83 + sin(85 * 2 π x) / 85 + sin(87 * 2 π x) / 87 + sin(89 * 2 π x) / 89 + sin(91 * 2 π x) / 91 + sin(93 * 2 π x) / 93 + sin(95 * 2 π x) / 95 + sin(97 * 2 π x) / 97 + sin(99 * 2 π x) / 99

fss(x) = sin(2 x π) + sin(5 * 2 π x) / 5 + sin(7 * 2 π x) / 7 + sin(11 * 2 π x) / 11 + sin(13 * 2 π x) / 13 + sin(17 * 2 π x) / 17 + sin(19 * 2 π x) / 19 + sin(23 * 2 π x) / 23 + sin(25 * 2 π x) / 25 + sin(29 * 2 π x) / 29 + sin(31 * 2 π x) / 31 + sin(35 * 2 π x) / 35 + sin(37 * 2 π x) / 37 + sin(41 * 2 π x) / 41 + sin(43 * 2 π x) / 43 + sin(47 * 2 π x) / 47 + sin(49 * 2 π x) / 49 + sin(53 * 2 π x) / 53 + sin(55 * 2 π x) / 55 + sin(59 * 2 π x) / 59 + sin(61 * 2 π x) / 61 + sin(65 * 2 π x) / 65 + sin(67 * 2 π x) / 67 + sin(71 * 2 π x) / 71 + sin(73 * 2 π x) / 73 + sin(77 * 2 π x) / 77 + sin(79 * 2 π x) / 79 + sin(83 * 2 π x) / 83 + sin(85 * 2 π x) / 85 + sin(89 * 2 π x) / 89 + sin(91 * 2 π x) / 91 + sin(95 * 2 π x) / 95 + sin(97 * 2 π x) / 97

fqq(x) = sin(2 x π) - sin(5 * 2 π x) / 5 - sin(7 * 2 π x) / 7 + sin(11 * 2 π x) / 11 + sin(13 * 2 π x) / 13 - sin(17 * 2 π x) / 17 - sin(19 * 2 π x) / 19 + sin(23 * 2 π x) / 23 + sin(25 * 2 π x) / 25 - sin(29 * 2 π x) / 29 - sin(31 * 2 π x) / 31 + sin(35 * 2 π x) / 35 + sin(37 * 2 π x) / 37 - sin(41 * 2 π x) / 41 - sin(43 * 2 π x) / 43 + sin(47 * 2 π x) / 47 + sin(49 * 2 π x) / 49 - sin(53 * 2 π x) / 53 - sin(55 * 2 π x) / 55 + sin(59 * 2 π x) / 59 + sin(61 * 2 π x) / 61 - sin(65 * 2 π x) / 65 - sin(67 * 2 π x) / 67 + sin(71 * 2 π x) / 71 + sin(73 * 2 π x) / 73 - sin(77 * 2 π x) / 77 - sin(79 * 2 π x) / 79 + sin(83 * 2 π x) / 83 + sin(85 * 2 π x) / 85 - sin(89 * 2 π x) / 89 - sin(91 * 2 π x) / 91 + sin(95 * 2 π x) / 95 + sin(97 * 2 π x) / 97