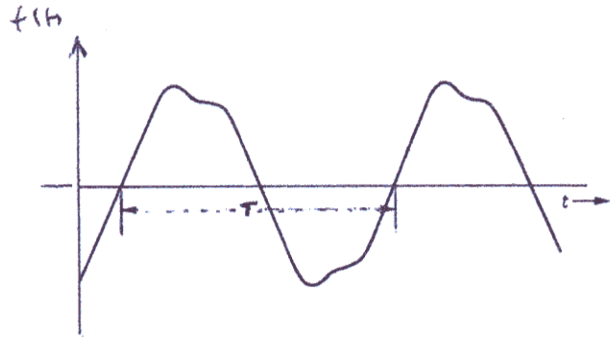
SERIE DI FOURIER

- funzione periodica del tempo t, di periodo T
- nel suo campo di definizione (t_0, t_0+T) soddisfa alle condizioni di Dirichelet:
  - la funz. è limitata
  - la funz. presenta n° finito di massimi e di minimi e di discontinuità



$$f(t) = f(t+T)$$

: S. d'F. in forma polare

$$f(t) = A_0 + \sum_{1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

Armoniche - fondamentale
n indice di armonicità
$\omega = \frac{2\pi}{T}$ pulsazione fondamentale
T periodo.

· S d'F in forma cartesiana

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t + \sum_{1}^{\infty} b_n \sin n\omega t$$

$$\begin{cases} a_n = A_n \sin \varphi_n \\ b_n = A_n \cos \varphi_n \end{cases}$$

$$\begin{cases} A_n = \sqrt{a_n^2 + b_n^2} \\ \varphi_n = \operatorname{arctg} \frac{a_n}{b_n} \end{cases}$$

$$A_0 = \frac{1}{T} \int_T f(t)\, dt$$

$$a_n = \frac{2}{T} \int_T f(t) \cos n\omega t\, dt$$

$$b_n = \frac{2}{T} \int_T f(t) \sin n\omega t\, dt$$

$$\begin{cases} \cos n\omega t = \dfrac{e^{jn\omega t} + e^{-jn\omega t}}{2} \\ \sin n\omega t = -j \dfrac{e^{jn\omega t} - e^{-jn\omega t}}{2} \end{cases}$$

1) se $f(t) = f(-t)$ FUNZ. PARI $\rightarrow b_n = 0$
2) se $f(t) = -f(-t)$ FUNZ. DISPARI $\rightarrow a_n = 0$
3) se $f(t) = -f(t+T/2)$ FUNZ. EMISIMMETRICA $\rightarrow \exists$ solo armoniche dispari.

· S d'F in forma esponenziale

$$f(t) = \sum_{-\infty}^{+\infty} \bar{C}_n e^{jn\omega t}$$

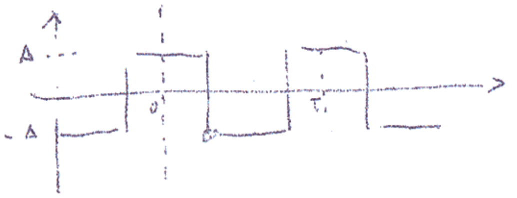
$$\bar{C}_n = \frac{1}{T} \int_T f(t) e^{-jn\omega t} dt \qquad \bar{C}_{-n} = \underline{C}_n$$

$$A_n = 2|\bar{C}_n|$$
$$C_0 = A_0$$
$$\bar{C}_n = \frac{1}{2}(a_n - j b_n)$$
$$\bar{C}_{-n} = \frac{1}{2}(a_n + j b_n)$$

1) ONDA QUADRA SIMMETRICA

$f(t) = f(-t)$ PARI

$f(t) = -f(t+T/2)$ EMISIMM.



$$f(t) = \frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t \ldots\right]$$

2) ONDA QUADRA ASIMMETRICA

$f(t) = -f(-t)$ DISPARI

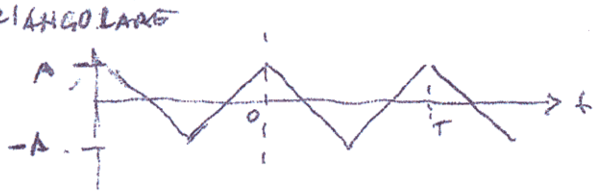
$f(t) = -f(t+T/2)$ EMISIMM.

$$f(t) = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right]$$

3) ONDA TRIANGOLARE

$f(t) = f(-t)$ PARI

$f(t) = -f(t+T/2)$ EMISIMM.

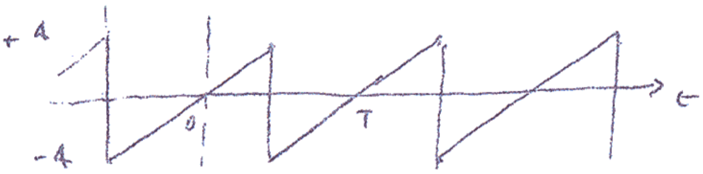


$$f(t) = \frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\cos 5\omega t + \ldots\right]$$

4) ONDA A DENTE DI SEGA

$f(t) = -f(-t)$ DISPARI

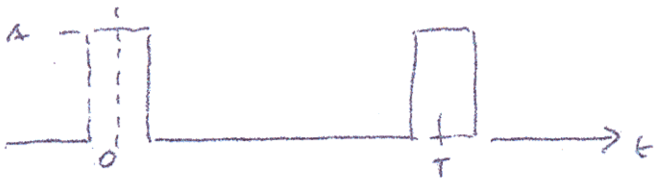
$f(t) \neq -f(t+T/2)$



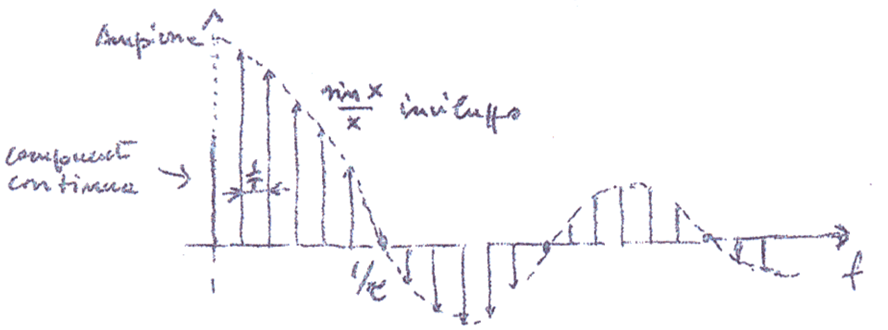
$$f(t) = \frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \ldots\right)$$

ONDA IMPULSIVA

$f(t) = f(-t)$ PARI



$$f(t) = \frac{A\tau}{T} + \frac{2A\tau}{T}\sum_{1}^{\infty}\frac{\sin(n\omega\tau/2)}{n\omega\tau/2}\cdot\cos n\omega t$$



1) ONDA QUADRA SIMMETRICA

$$a_0 = 0 \qquad b_n = 0$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t \, dt \qquad f(t)\begin{cases} -A & -T/2 - -T/4 \\ +A & -T/4 - T/4 \\ -A & T/4 - T/2 \end{cases}$$

$$a_n = \frac{8A}{n\omega T}\left[\sin n\omega T/4\right]$$

2) ONDA QUADRA ASIMMETRICA

$$b_0 = 0 \qquad a_n = 0$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t \, dt \qquad f(t) = \begin{cases} -A & -T/2 - 0 \\ A & 0 - T/2 \end{cases}$$

$$b_n = \frac{4A}{n\omega T}\left[1 - \cos n\omega T/2\right]$$

3) ONDA TRIANGOLARE

$$f(t) = \sum_{-\infty}^{+\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega t} dt \qquad f(t) = \begin{cases} \left(\frac{4t}{T} + 1\right)A & -T/2 - 0 \\ \left(-\frac{4t}{T} + 1\right)A & 0 \quad T/2 \end{cases}$$

$$C_n = \left\{\frac{8}{(n\omega T)^2}\left[1 - \cos(n\omega T/2)\right] - \frac{\sin(n\omega T/2)}{n\omega}\right\}A \qquad \omega T = 2\pi$$

$$C_n = \left\{\frac{2}{n^2\pi^2}\left[1 - \cos(n\pi)\right] - \frac{\sin n\pi}{n\omega}\right\}A$$

4) FORMA D'ONDA A DENTE DI SEGA

$$f(t) = \sum_{-\infty}^{+\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega t} dt \qquad f(t) = \frac{2At}{T}$$

$$C_n = \left[-\frac{2}{jn\omega T}\cos(n\omega T/2) + \frac{4\sin(n\omega T/2)}{jn^2\omega^2T^2}\right]A \qquad \omega T = 2\pi$$

$$C_n = \left[-\frac{\cos(n\pi)}{jn\pi} + \frac{\sin(n\pi)}{jn^2\pi^2}\right]A$$

$$C_1 = C_{-1} = \frac{1}{j\pi}A$$

$$C_2 = C_{-2} = -\frac{1}{j2\pi}A \Rightarrow f(t) = C_1 e^{jn\omega t} + C_{-1}e^{-jn\omega t} + C_2 e^{j2n\omega t} + C_{-2}e^{-j2n\omega t} \ldots$$

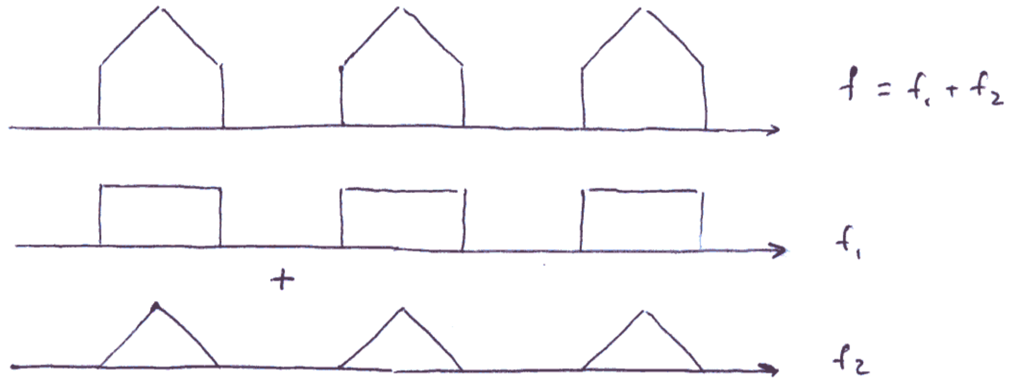
$$C_3 = C_{-3} = \frac{1}{j3\pi}A \qquad = \frac{2A}{\pi}\left[\sin \omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \ldots\right]$$

(A) LINEARITÀ DELLA SERIE

LO SVILUPPO IN SERIE DELLA SOMMA DI DUE FUNZIONI È UGUALE ALLA SOMMA DEGLI SVILUPPI IN SERIE DELLE DUE FUNZIONI

$$f = f_1 + f_2$$
$$\downarrow \quad\; \downarrow \quad \downarrow$$
$$\Sigma = \Sigma_1 + \Sigma_2$$

ES.

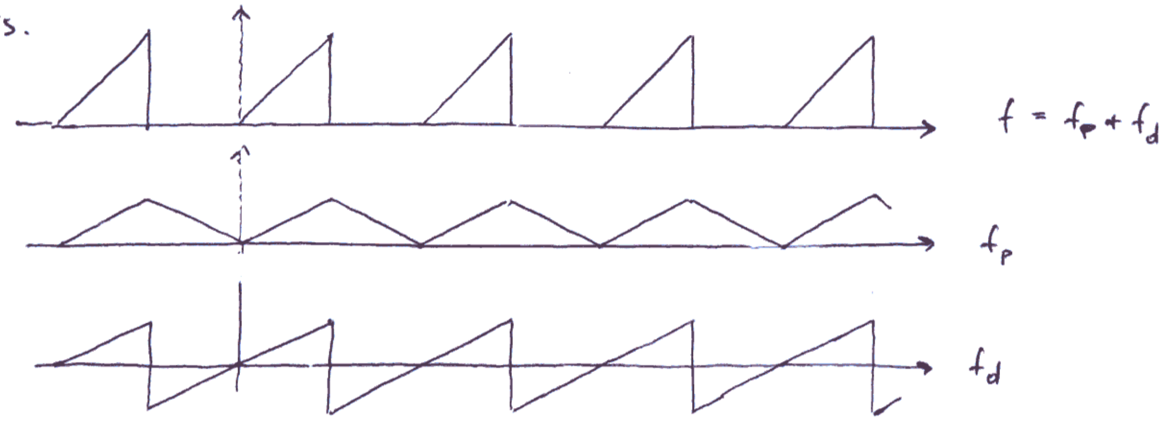


LO SVILUPPO IN SERIE DELLA FUNZIONE f È = ALLA SOMMA DEGLI SVILUPPI IN SERIE DI f_1 E DI f_2 -

(B) UNA GENERICA FUNZIONE PERIODICA PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI UNA FUNZIONE PARI + UNA FUNZIONE DISPARI

$$f(t) = f_p(t) + f_d(t) \quad \text{CON} \quad \begin{cases} f_p(t) = \dfrac{f(t) + f(-t)}{2} \\ f_d(t) = \dfrac{f(t) - f(-t)}{2} \end{cases}$$

ES.



LO SVILUPPO IN SERIE DELLA FUNZIONE $f(t)$ È LA SOMMA DEGLI SVILUPPI IN SERIE DELLA FUNZIONE PARI E DELLA FUNZIONE DISPARI

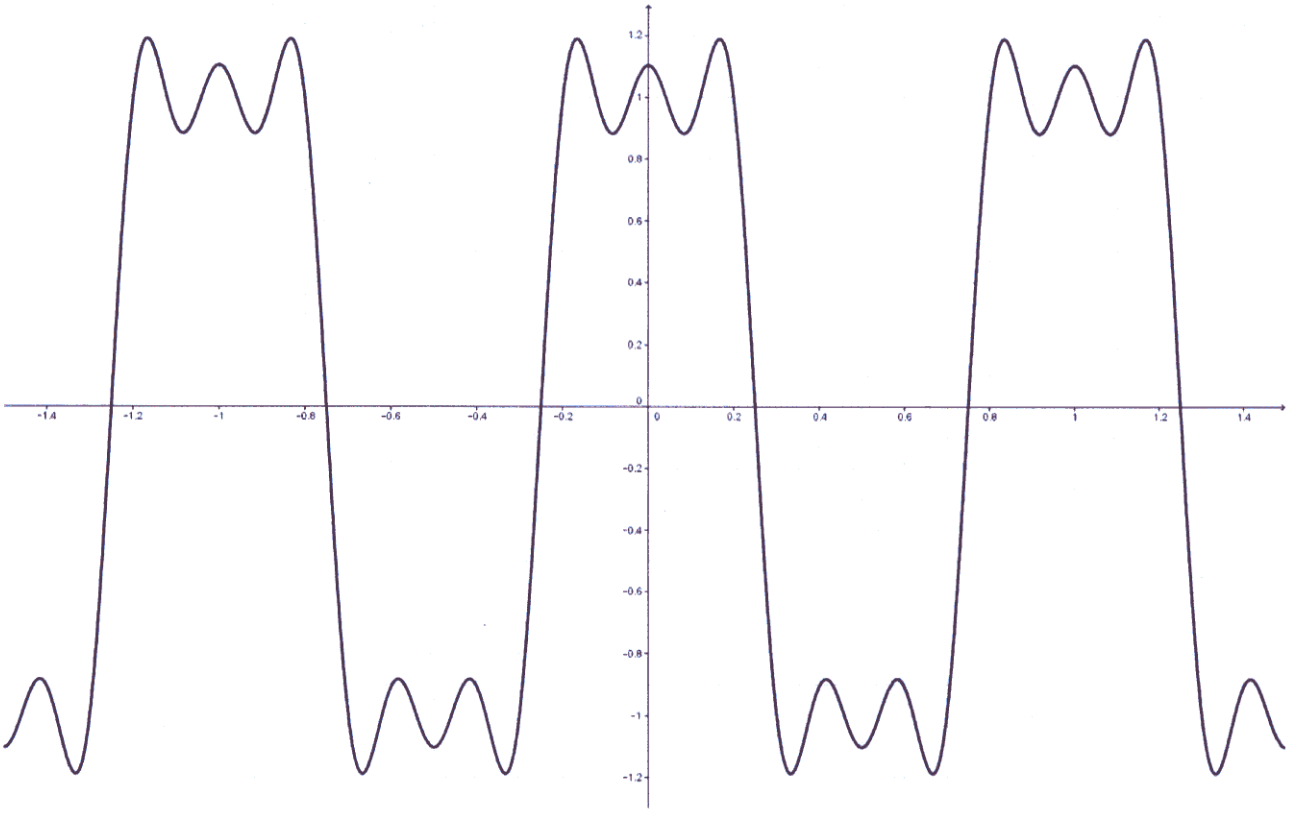
(PER RICAVARE LA FUNZ. PARI E LA FUNZ. DISPARI SI VEDANO GLI ES. RELATIVI ALLE FUNZ. 16 E 20)

(C) DATA UNA FUNZ.

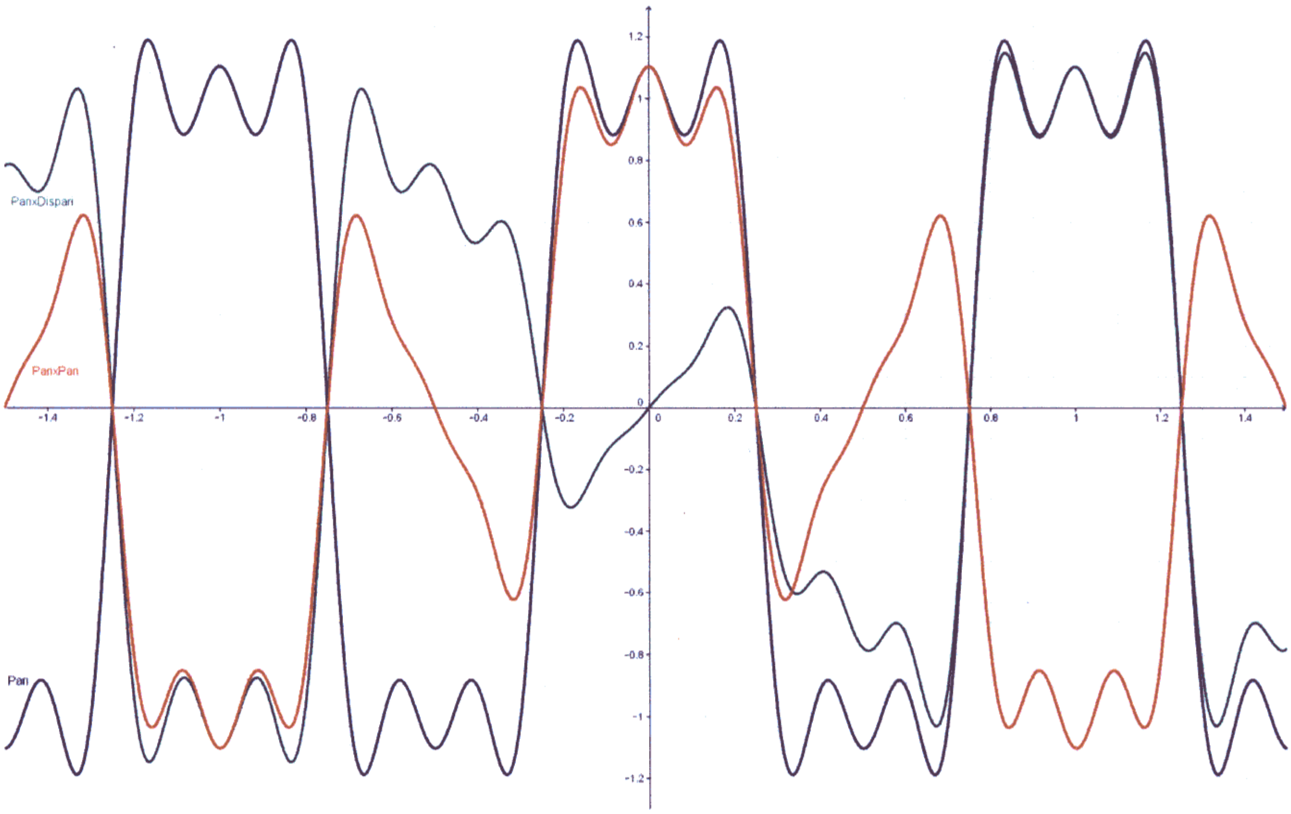
$$\begin{cases} \text{PARI} & b_m = 0 \quad \text{E} \quad a_m = \dfrac{2}{T}\displaystyle\int_T f(t)\cos m\omega t\,dt = \dfrac{4}{T}\int_{T/2} f(t)\cos m\omega t\,dt \\ \text{DISPARI} & a_m = 0 \quad \text{E} \quad b_m = \dfrac{2}{T}\displaystyle\int_T f(t)\sin m\omega t\,dt = \dfrac{4}{T}\int_{T/2} f(t)\sin m\omega t\,dt \end{cases}$$

NEL CALCOLO DI a_m BISOGNA INTEGRARE IL PRODOTTO DI FUNZIONI PARI, MENTRE NEL CALCOLO DI b_m BISOGNA INTEGRARE IL PRODOTTO DI FUNZIONI DISPARI; MA IL PRODOTTO DI 2 FUNZ. PARI È UNA FUNZ. PARI E IL PRODOTTO DI 2 FUNZ. DISPARI È ANCORA UNA FUNZ. PARI (V. FIG. ALLE PAG. SEGUENTI), QUINDI IL CALCOLO DI a_m E b_m PUÒ ESSERE FATTO SU ½ PERIODO E IL RISULTATO MOLTIPLICATO PER 2 - $\int_{-T/2}^{T/2} = 2\int_{-T/2}^{0} = 2\int_0^{T/2}$

LA PROPRIETÀ È APPLICATA NEL CALCOLO DEGLI ESEMPI SEGUENTI, QUANDO SI HA A CHE FARE CON FUNZ. PARI O DISPARI -

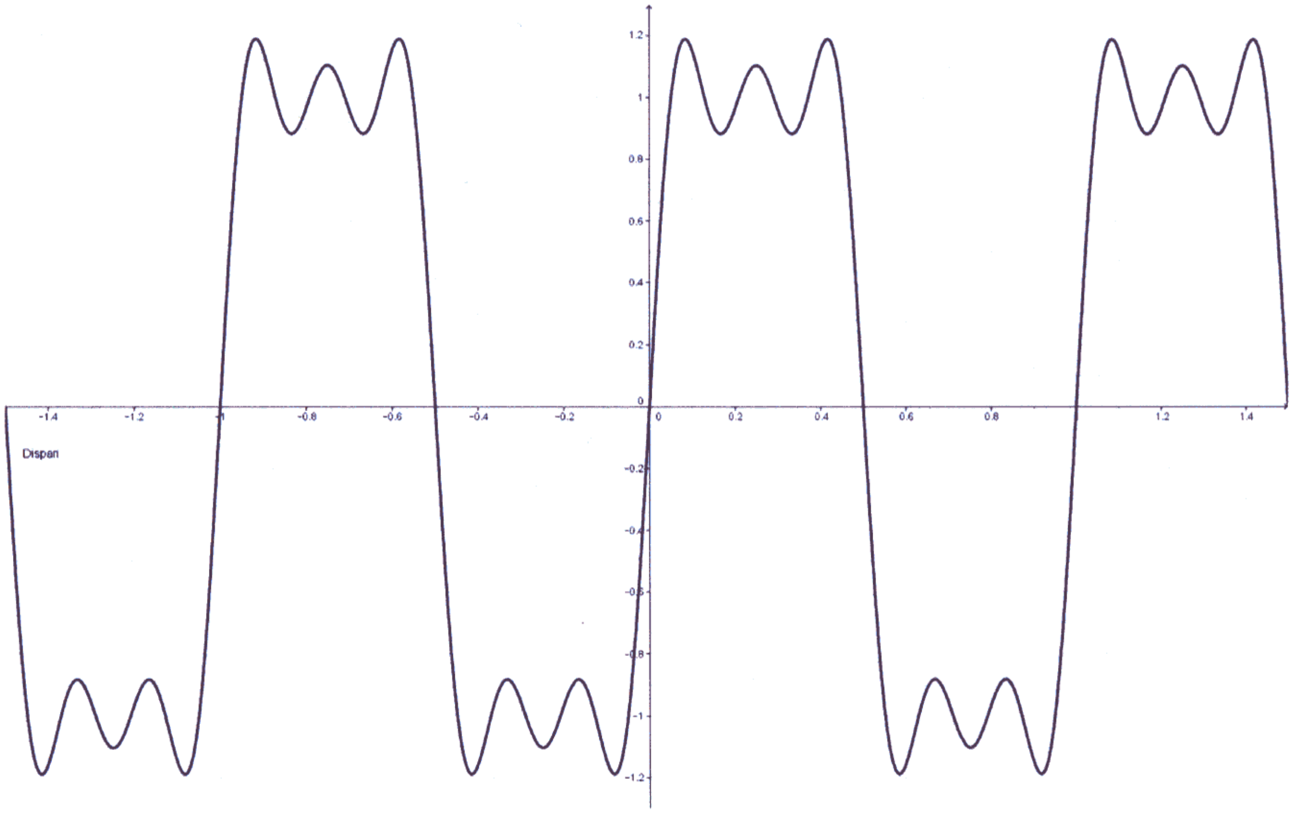


Pari(x) = 4 / π (cos(2π x) - 1 / 3 cos(6π x) + 1 / 5 cos(10π x))

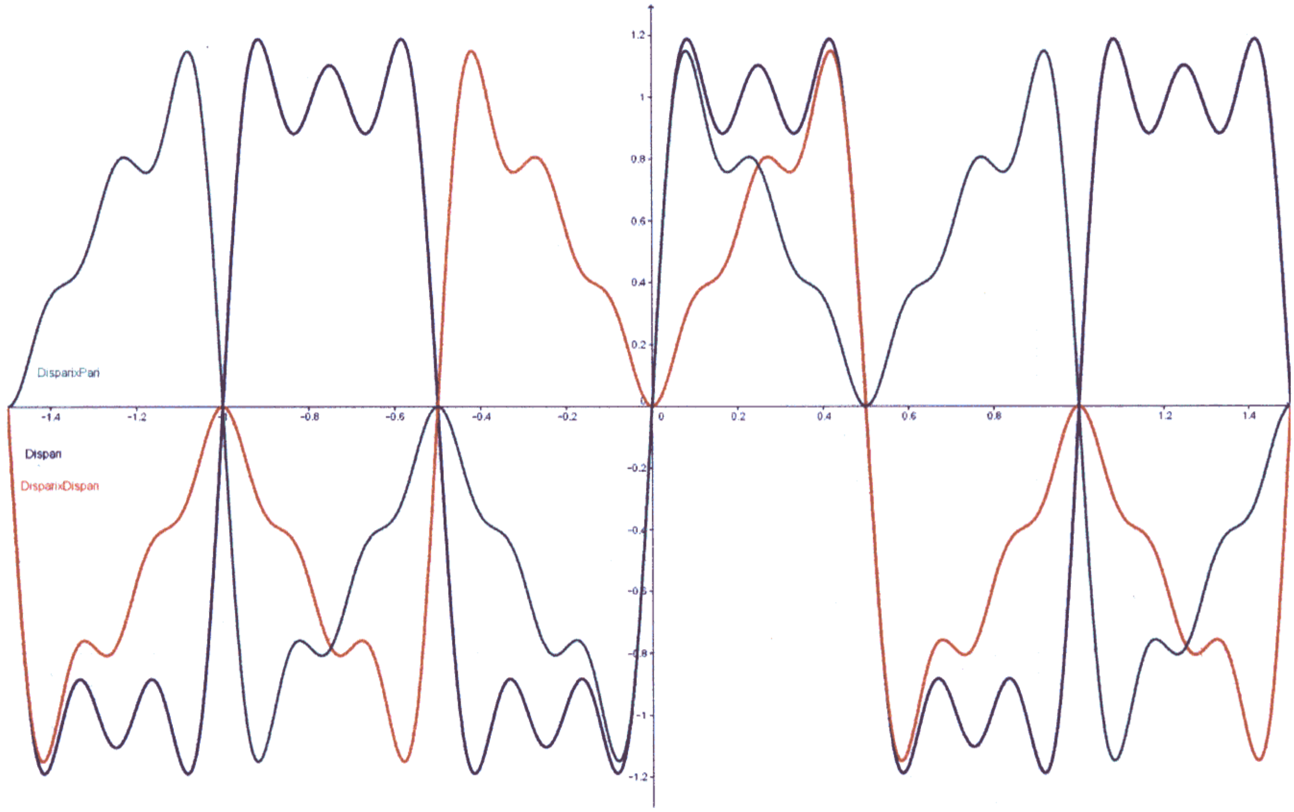


ParixPari(x) = 4 / π (cos(2π x) - 1 / 3 cos(6π x) + 1 / 5 cos(10π x)) cos(π x)

ParixDispari(x) = 4 / π (cos(2π x) - 1 / 3 cos(6π x) + 1 / 5 cos(10π x)) sen(0.5π x)



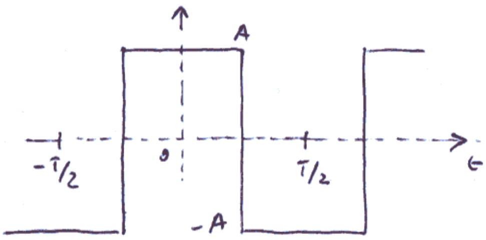
Dispari(x) = $4 / \pi$ (sen(2π x) + 1 / 3 sen(6π x) + 1 / 5 sen(10π x))



DisparixDispari(x) = $4 / \pi$ (sen(2π x) + 1 / 3 sen(6π x) + 1 / 5 sen(10π x)) sen(π x)

DisparixPari(x) = $4 / \pi$ (sen(2π x) + 1 / 3 sen(6π x) + 1 / 5 sen(10π x)) cos(π x)

1) ONDA QUADRA PARI



$$f(t) = \begin{cases} -A & \text{per } -\frac{T}{2} < t < -\frac{T}{4} \\ A & \text{per } -T/4 < t < T/4 \\ -A & \text{per } \frac{T}{4} < t < T/2 \end{cases}$$

$$f(t) = A_0 + \sum_{1}^{\infty}{}_n\, a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{2}{T}\int_0^{T/2} f(t)\,dt = \frac{2}{T}\left[\int_0^{T/4} A\,dt + \int_{T/4}^{T/2}(-A)\,dt\right] = \frac{2A}{T}\left[t\Big|_0^{T/4} - t\Big|_{T/4}^{T/2}\right] = \frac{2A}{T}\left[\frac{T}{4} - \left(\frac{T}{2}-\frac{T}{4}\right)\right] = 0$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t\,dt = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega t\,dt =$$

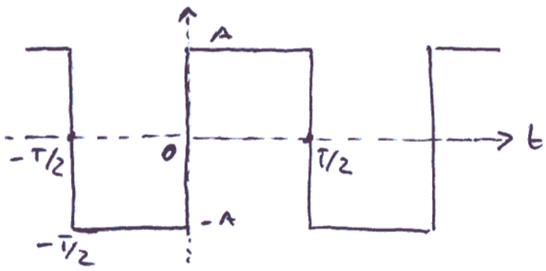
$$= \frac{4}{T}\left[\int_0^{T/4} A\cos n\omega t\,dt + \int_{T/4}^{T/2}(-A)\cos n\omega t\,dt\right] =$$

$$= \frac{4A}{T}\left[\frac{\sin n\omega t}{n\omega}\Big|_0^{T/4} - \frac{\sin n\omega t}{n\omega}\Big|_{T/4}^{T/2}\right] = \frac{4A}{n\omega T}\left(\sin n\omega\frac{T}{4} - \sin n\omega\frac{T}{2} + \sin n\omega\frac{T}{4}\right) =$$

$$= \frac{4A}{n2\pi}\left(2\sin n\omega\frac{T}{4} - \sin n\omega\frac{T}{2}\right) = \frac{2A}{n\pi}\left(2\sin n\frac{\pi}{2} - \sin n\pi\right) =$$

$$= \frac{4A}{n\pi}\sin n\frac{\pi}{2} \qquad f(t) = \frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

2) ONDA QUADRA DISPARI



$$f(t) = \begin{cases} -A & \text{per } -\frac{T}{2} < t < 0 \\ A & \text{per } 0 < t < \frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_{1}^{\infty}{}_n\, b_n \sin n\omega t$$

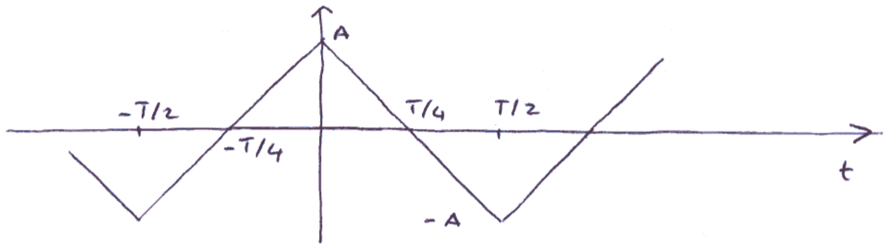
$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{2}{T}\int_{-T/4}^{T/4} f(t)\,dt = \frac{2}{T}\left[\int_{-T/4}^{0}(-A)\,dt + \int_0^{T/4} A\,dt\right] = \frac{2A}{T}\left[(-t)\Big|_{-T/4}^{0} + t\Big|_0^{T/4}\right] = \frac{2A}{T}\left[-\left(0+\frac{T}{4}\right) + \left(\frac{T}{4}-0\right)\right] = 0$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t\,dt = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\,dt = \frac{4}{T}\left[\int_0^{T/2} A\sin n\omega t\,dt\right] = -\frac{4A}{T}\,\frac{\cos n\omega t}{n\omega}\Big|_0^{T/2} =$$

$$= \frac{4A}{n\omega T}\Big|_{T/2}^{0} = \frac{4A}{n2\pi}\left(\cos 0 - \cos n\omega\frac{T}{2}\right) =$$

$$= \frac{2A}{n\pi}\left[1 - \cos^n\pi\right] \qquad f(t) = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

3) ONDA TRIANGOLARE PARI



$$f(t) = \begin{cases} \frac{4A}{T}t + A & x \quad -\frac{T}{2} < t < 0 \\ -\frac{4A}{T}t + A & x \quad 0 < t < \frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{2}{T}\int_0^{T/2}\left(-\frac{4A}{T}t + A\right)dt = \frac{2}{T}\left(-\frac{4A}{T}\frac{t^2}{2} + At\right)\Bigg|_0^{T/2} = \frac{2}{T}\left(-\frac{4A}{T}\frac{1}{2}\frac{T^2}{4} + A\frac{T}{2}\right) = 0$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t\,dt = \frac{4}{T}\left[\int_{-T/2}^{0}\left(\frac{4A}{T}t + A\right)\cos n\omega t\right] =$$

$$= \frac{16A}{T^2}\int_{-T/2}^{0} t\cos n\omega t\,dt + \frac{4A}{T}\int_{-T/2}^{0}\cos n\omega t\,dt =$$

$$= \frac{16A}{T^2}\left[t\,\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right]_{-T/2}^{0} + \frac{4A}{T}\cdot\frac{\sin n\omega t}{n\omega}\Bigg|_{-T/2}^{0} =$$

$$= \frac{16A}{T^2}\left[\frac{1}{n^2\omega^2} - \frac{\left(-\frac{T}{2}\right)}{n\omega}\sin\left(-n\omega\frac{T}{2}\right) - \frac{\cos n\omega T/2}{n^2\omega^2}\right] - \frac{4A}{T}\frac{\sin(-n\omega T/2)}{n\omega} =$$

$$n\omega T = n2\pi \qquad n\omega\frac{T}{2} = n\pi$$

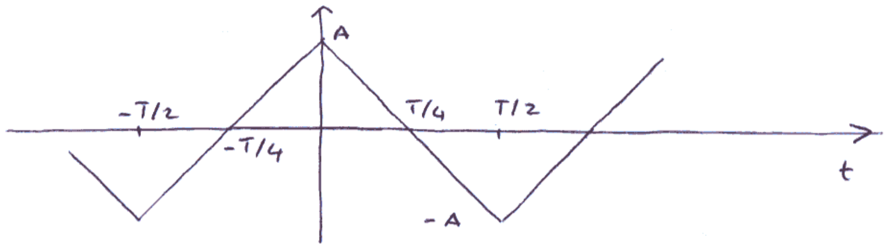
$$= \frac{16A}{T^2}\left[\frac{1}{n^2\omega^2} + \underbrace{\frac{T}{2}\cdot\frac{1}{n\omega}\sin(-n\pi)}_{0} - \frac{\cos n\pi}{n^2\omega^2}\right] - \underbrace{\frac{4A}{2n\pi}\cdot\sin(-n\pi)}_{0} =$$

$$= \frac{16A}{n^2\omega^2T^2}\left[1 - \cos n\pi\right] = \frac{16A}{(n2\pi)^2}\left[1 - \cos n\pi\right] =$$

$$= \frac{4A}{n^2\pi^2}\left[1 - \cos n\pi\right]$$

$$f(t) = \frac{8A}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right)$$

3) ONDA TRIANGOLARE PARI



$$f(t) = \begin{cases} \frac{4A}{T}t + A & x \quad -\frac{T}{2} < t < 0 \\ -\frac{4A}{T}t + A & x \quad 0 < t < \frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_{1}^{\infty}{}_n\, a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{2}{T}\int_0^{T/2}\left(-\frac{4A}{T}t + A\right)dt = \frac{2}{T}\left(-\frac{4A}{T}\frac{t^2}{2} + At\right)\Bigg|_0^{T/2} = \frac{2}{T}\left(-\frac{4A}{T}\frac{1}{2}\frac{T^2}{4} + A\frac{T}{2}\right) = 0$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t\,dt = \frac{4}{T}\left[\int_{-T/2}^{0}\left(\frac{4A}{T}t + A\right)\cos n\omega t\right] =$$

$$= \frac{16A}{T^2}\int_{-T/2}^{0} t\cos n\omega t\,dt + \frac{4A}{T}\int_{-T/2}^{0}\cos n\omega t\,dt =$$

$$= \frac{16A}{T^2}\left[t\,\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right]_{-T/2}^{0} + \frac{4A}{T}\cdot\frac{\sin n\omega t}{n\omega}\Bigg|_{-T/2}^{0} =$$

$$= \frac{16A}{T^2}\left[\frac{1}{n^2\omega^2} - \frac{\left(-\frac{T}{2}\right)}{n\omega}\sin\left(-n\omega\frac{T}{2}\right) - \frac{\cos n\omega T/2}{n^2\omega^2}\right] - \frac{4A}{T}\,\frac{\sin(-n\omega T/2)}{n\omega} =$$

$$n\omega T = n2\pi \qquad n\omega\frac{T}{2} = n\pi$$

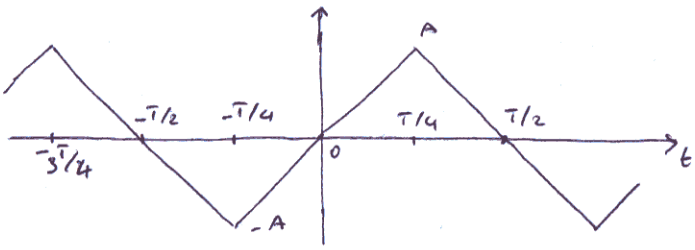
$$= \frac{16A}{T^2}\left[\frac{1}{n^2\omega^2} + \underbrace{\frac{T}{2}\cdot\frac{1}{n\omega}\sin(-n\pi)}_{0} - \frac{\cos n\pi}{n^2\omega^2}\right] - \underbrace{\frac{4A}{2n\pi}\cdot\sin(-n\pi)}_{0} =$$

$$= \frac{16A}{n^2\omega^2T^2}\left[1 - \cos n\pi\right] = \frac{16A}{(n2\pi)^2}\left[1 - \cos n\pi\right] =$$

$$= \frac{4A}{n^2\pi^2}\left[1 - \cos n\pi\right]$$

$$f(t) = \frac{8A}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right)$$

4) ONDA TRIANGOLARE DISPARI



$$f(t) = \begin{cases} -\frac{4A}{T}\left(t+\frac{T}{2}\right) = -\frac{4A}{T}t - 2A & -\frac{T}{2} < t < -\frac{T}{4} \\ \frac{4A}{T}t & -\frac{T}{4} < t < \frac{T}{4} \\ -\frac{4A}{T}\left(t-\frac{T}{2}\right) = -\frac{4A}{T}t + 2A & \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

f dispari $a_n = 0$

f. valor medio nullo $A_0 = 0$

f emisimmetrica $b_{2n} = 0$ (∃ solo armoniche dispari)

$$f(t) = \sum_{1}^{\infty}{}_n\, b_n \sin n\omega t$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \sin n\omega t\, dt = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega t\, dt =$$

$$= \frac{4}{T}\left[\int_0^{T/4} \frac{4A}{T} t \sin n\omega t\, dt + \int_{T/4}^{T/2}\left(-\frac{4A}{T}t + 2A\right)\sin n\omega t\, dt\right] =$$

$$= \frac{16A}{T^2}\left[\int_0^{T/4} t \sin n\omega t\, dt - \int_{T/4}^{T/2} t\sin n\omega t\, dt\right] + \frac{8A}{T}\int_{T/4}^{T/2}\sin n\omega t\, dt =$$

$$* \; n\omega\frac{T}{4} = n\frac{2\pi}{T}\cdot\frac{T}{4} = \frac{n\pi}{2}$$

integrando per parti $\int t \sin n\omega t\, dt = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2}$

$$= \frac{16A}{T^2}\left[\left(-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2}\right)\Bigg|_0^{T/4} - \left(-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2}\right)\Bigg|_{T/4}^{T/2}\right] + \frac{8A}{T}\left(-\frac{\cos n\omega t}{n\omega}\right)\Bigg|_{T/4}^{T/2} =$$

$$= \frac{16A}{T^2}\left[-\frac{T}{4}\frac{\cos n\omega T/4}{n\omega} + \frac{\sin n\omega T/4}{n^2\omega^2} - \left(-\frac{T}{2}\frac{\cos n\omega T/2}{n\omega} + \frac{\sin n\omega T/2}{n^2\omega^2} + \frac{T}{4}\frac{\cos n\omega T/4}{n\omega} - \frac{\sin n\omega T/4}{n^2\omega^2}\right)\right] +$$

$$+ \frac{8A}{n\omega T}\left(-\cos n\omega\frac{T}{2} + \cos n\omega\frac{T}{4}\right) = \; *$$

$$= \frac{16A}{T^2}\left[-\frac{T}{4}\frac{\cos n\pi/2}{n\omega} + \frac{\sin n\pi/2}{n^2\omega^2} + \frac{T}{2}\frac{\cos n\pi}{n\omega} - \cancel{\frac{\sin n\pi}{n^2\omega^2}}_{0} - \frac{T}{4}\frac{\cos n\pi/2}{n\omega} + \frac{\sin n\pi/2}{n^2\omega^2}\right] + \frac{8A}{n2\pi}\left(\cos n\frac{\pi}{2} - \cos n\pi\right) =$$

$$= \frac{16A}{T^2}\left[\frac{T}{2}\frac{\cos n\pi}{n\omega} - \frac{T}{2}\frac{\cos n\pi/2}{n\omega} + 2\frac{\sin n\pi/2}{n^2\omega^2}\right] + \frac{8A}{n2\pi}\cos n\frac{\pi}{2} - \frac{8A}{n2\pi}\cos n\pi =$$

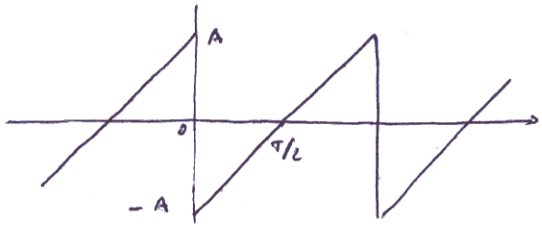
$$= \frac{8A}{n\omega T}\cos n\pi - \frac{8A}{n\omega T}\cos n\pi/2 + \frac{32A}{(n\omega T)^2}\sin n\pi/2 + \frac{8A}{n2\pi}\cos n\pi/2 - \frac{8A}{n2\pi}\cos n\pi =$$

$$= \cancel{\frac{8A}{n2\pi}\cos n\pi} - \cancel{\frac{8A}{n2\pi}\cos n\pi/2} + \frac{32A}{4n^2\pi^2}\sin n\pi/2 + \cancel{\frac{8A}{n2\pi}\cos n\pi/2} - \cancel{\frac{8A}{n2\pi}\cos n\pi} =$$

$$= \frac{8A}{n^2\pi^2}\sin n\pi/2$$

$$f(t) = \frac{8A}{\pi^2}\left(\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \frac{1}{7^2}\sin 7\omega t + \dots\right)$$

5) ONDA A DENTE DI SEGA 1



$f(t) = \frac{2A}{T}\left(t - \frac{T}{2}\right) = \frac{2A}{T}t - A$ per $0 < t < T$

f dispari $a_n = 0$
v. medio nullo $A_0 = 0$

$$f(t) = \sum_{1}^{\infty} {}_n \, b_n \sin n\omega t$$

$$b_n = \frac{2}{T}\int_0^T f(t) \sin n\omega t \, dt = \frac{2}{T}\int_0^T \left(\frac{2A}{T}t - A\right) \sin n\omega t \, dt =$$

$$= \frac{4A}{T^2}\int_0^T t \sin n\omega t \, dt - \frac{2A}{T}\int_0^T \sin n\omega t \, dt =$$

integrando x parti $\int t \sin n\omega t \, dt = -\frac{t \cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}$

$$= \frac{4A}{T^2}\left[-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_0^T - \frac{2A}{T}\left(-\frac{\cos n\omega t}{n\omega}\right)\Bigg|_0^T =$$

$n\omega T = n2\pi$

$$= \frac{4A}{T^2}\left[-\frac{T\cos n2\pi}{n\omega} + \cancel{\frac{\sin n2\pi}{n^2\omega^2}}\right] + \frac{2A}{Tn\omega}\cos n2\pi - \frac{2A}{T}\frac{\cos 0}{n\omega} =$$

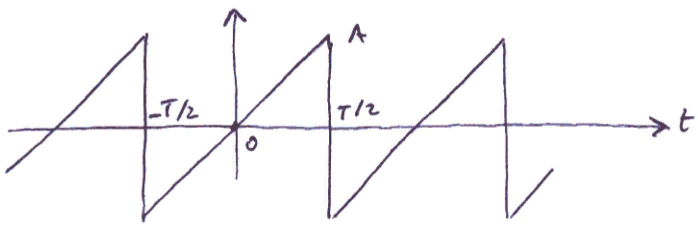
$\cos n2\pi = 1$, $\cos 0 = 1$

$$= -\frac{4A}{T}\cdot\frac{1}{n\omega} + \cancel{\frac{2A}{n\omega T}} - \cancel{\frac{2A}{n2\pi}} = -\frac{4A}{n2\pi}$$

$$= -\frac{2A}{n\pi}$$

$$f(t) = -\frac{2A}{\pi}\left(\sin \omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{4}\sin 4\omega t + \dots\right)$$

6) ONDA A DENTE DI SEGA 2



$f(t) = \frac{2A}{T}t$ per $-\frac{T}{2} < t < \frac{T}{2}$

f dispari $a_n = 0$
v. medio nullo $A_0 = 0$

$$f(t) = \sum_{1}^{\infty} {}_n \, b_n \sin n\omega t$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \sin n\omega t \, dt = \frac{4}{T}\int_0^{T/2} f(t) \sin n\omega t \, dt =$$

$$= \frac{4}{T}\int_0^{T/2} \frac{2A}{T} t \sin n\omega t \, dt = \frac{8A}{T^2}\int_0^{T/2} t \sin n\omega t \, dt =$$

$$\left(\int t \sin n\omega t \, dt = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right)$$

$$= \frac{8A}{T^2}\left(-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right)\Bigg|_0^{T/2} = \frac{8A}{T^2}\left[-\frac{T}{2}\frac{\cos n\omega T/2}{n\omega} + \frac{\sin n\omega T/2}{(n\omega)^2}\right] =$$

$n\omega\frac{T}{2} = n\pi$

$\cos n\pi \begin{cases} -1 & n \text{ disp.} \\ +1 & n \text{ pari} \end{cases}$

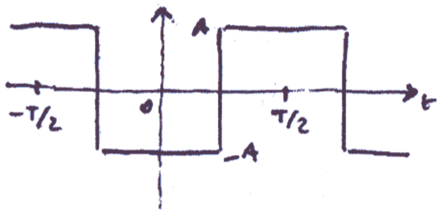
$\sin n\pi = 0$

$$= -\frac{\cancel{8}^{4}A}{T^{\cancel{2}}}\cdot\frac{\cancel{T}}{\cancel{2}_1}\frac{\cos n\pi}{n\omega} = -\frac{4A\cos n\pi}{n\omega T} = -\frac{4A}{n2\pi}\cos n\pi =$$

$$= -\frac{2A}{n\pi}\cos n\pi$$

$$f(t) = +\frac{2A}{\pi}\left(\sin \omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \dots\right)$$

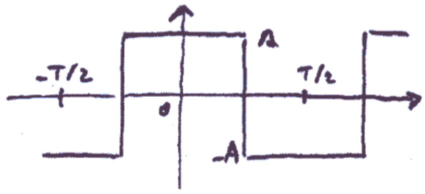
7)



ONDA QUADRA PARI

LA FUNZIONE È OPPOSTA AD f_1 IL CUI SVILUPPO IN Σ È GIÀ STATO DETERMINATO

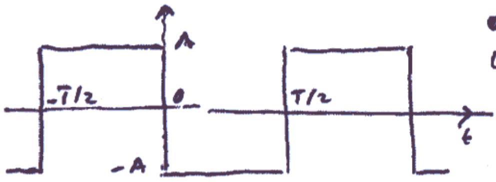
$f(t) = -f_1(t)$



$$f(t) = -\frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

$$f_1(t) = \frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

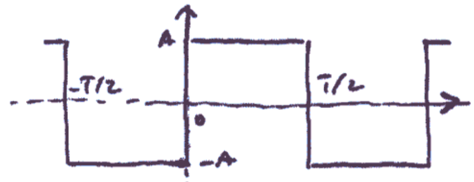
8)



ONDA QUADRA DISPARI

LA FUNZIONE È OPPOSTA AD f_2 GIÀ DETERMINATA

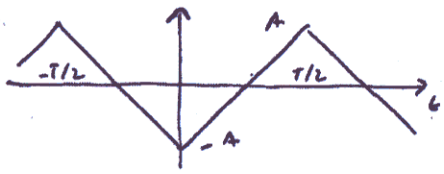
$f(t) = -f_2(t)$



$$f(t) = -\frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

$$f_2(t) = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

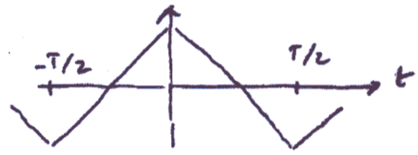
9)



ONDA TRIANGOLARE PARI

LA FUNZIONE È OPPOSTA AD f_3 GIÀ DETERMINATA

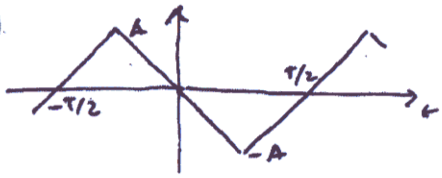
$f(t) = -f_3(t)$



$$f(t) = -\frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

$$f_3(t) = \frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

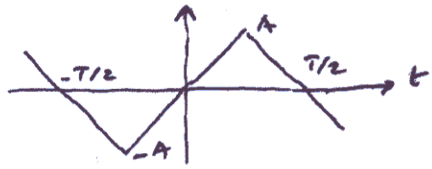
10)



ONDA TRIANGOLARE DISPARI

LA FUNZIONE È OPPOSTA AD f_4 GIÀ DETERMINATA

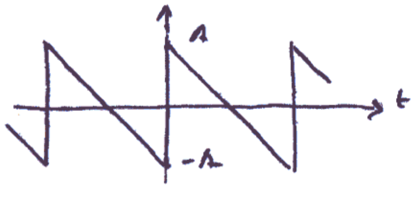
$f(t) = -f_4(t)$



$$f(t) = -\frac{8A}{\pi^2}\left(\sin\omega t - \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t - \dots\right)$$

$$f_4(t) = \frac{8A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \dots\right]$$

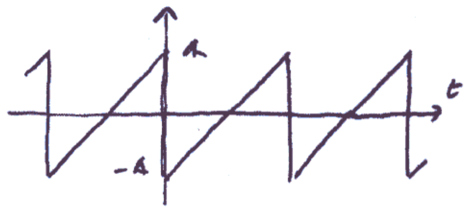
11)



ONDA A DENTE DI SEGA 3

LA FUNZIONE È OPPOSTA AD f_5 GIÀ DETERMINATA

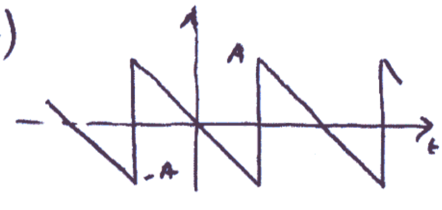
$f(t) = -f_5(t)$



$$f(t) = \frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

$$f_5(t) = -\frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

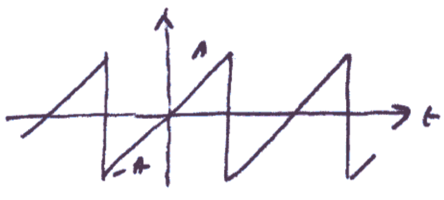
12)



ONDA A DENTE DI SEGA 4

LA FUNZIONE È OPPOSTA AD f_6 GIÀ DETERMINATA

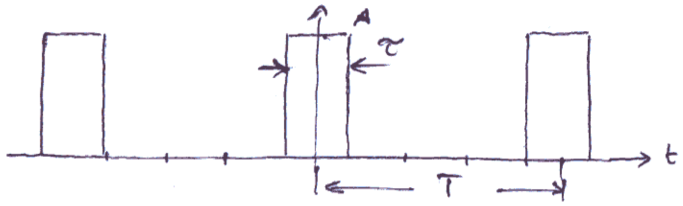
$f(t) = -f_6(t)$



$$f(t) = -\frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \dots\right)$$

$$f_6(t) = \frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \right)$$

13 a) ONDA IMPULSIVA (POSITIVA)



$$f(t) = \begin{cases} 0 & \text{per } -\frac{T}{2} < t < -\frac{\tau}{2} \\ A & \text{per } -\frac{\tau}{2} < t < \frac{\tau}{2} \\ 0 & \text{per } \frac{\tau}{2} < t < \frac{T}{2} \end{cases}$$

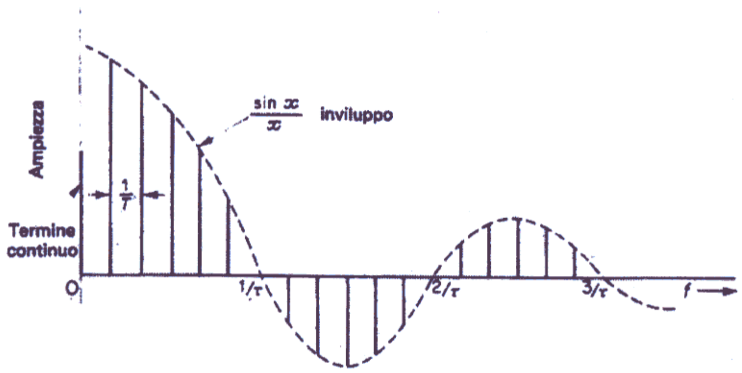
$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\int_{-\tau/2}^{\tau/2} A\,dt = \frac{2}{T}\int_0^{\tau/2} A\,dt = \frac{2A}{T}\frac{\tau}{2} = A\frac{\tau}{T} = A\delta \qquad \left[\delta = \frac{\tau}{T} \text{ duty cycle}\right]$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_{-\tau/2}^{\tau/2} A\cos n\omega t\,dt\right] = \frac{4A}{T}\int_0^{\tau/2}\cos n\omega t\,dt = \frac{4A}{T}\left.\frac{\sin n\omega t}{n\omega}\right|_0^{\tau/2} =$$

$$= \frac{4A}{n\omega T}\sin n\omega \tau/2 = \frac{4A\,\tau/2}{n\omega T\,\tau/2}\sin n\omega\tau/2 = 2A\frac{\tau}{T}\cdot\frac{\sin n\omega\tau/2}{n\omega\tau/2}$$

$$f(t) = A\delta + 2A\delta\sum_{1}^{\infty}\frac{\sin n\omega\tau/2}{n\omega\tau/2}\cos n\omega t$$



L'INVILUPPO DELLE COMPONENTI DIVERSE DA A_0 È DELLA FORMA $\frac{\sin x}{x}$ E SI HA ZERO QUANDO

$$\frac{\sin x}{x} = 0 \qquad \text{DOVE} \qquad x = \frac{n\omega\tau}{2}$$

O $$\frac{n\omega\tau}{2} = k\pi \qquad \text{DOVE} \qquad k = 1, 2, 3 \ldots$$

E $$nf = k/\tau \qquad \text{SONO MULTIPLI DI } 1/\tau$$

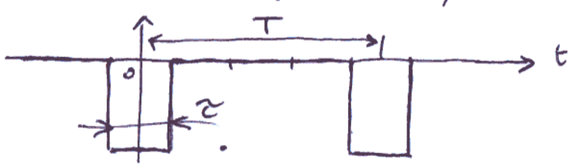
DUNQUE SI HA ZERO A $1/\tau$, $2/\tau$, ECC.

LA 1^a ARMONICA È A $f = \frac{1}{T}$, LE ALTRE A MULTIPLI INTERI DI $\frac{1}{T}$.

CON $\tau = \frac{1}{4}T \rightarrow \frac{1}{\tau} = 4\frac{1}{T} \rightarrow$ È NULLA LA 4^a ARMONICA E QUELLE MULTIPLE DI 4.

CON $\tau = \frac{3}{4}T \rightarrow \frac{1}{\tau} = \frac{4}{3}\frac{1}{T}$

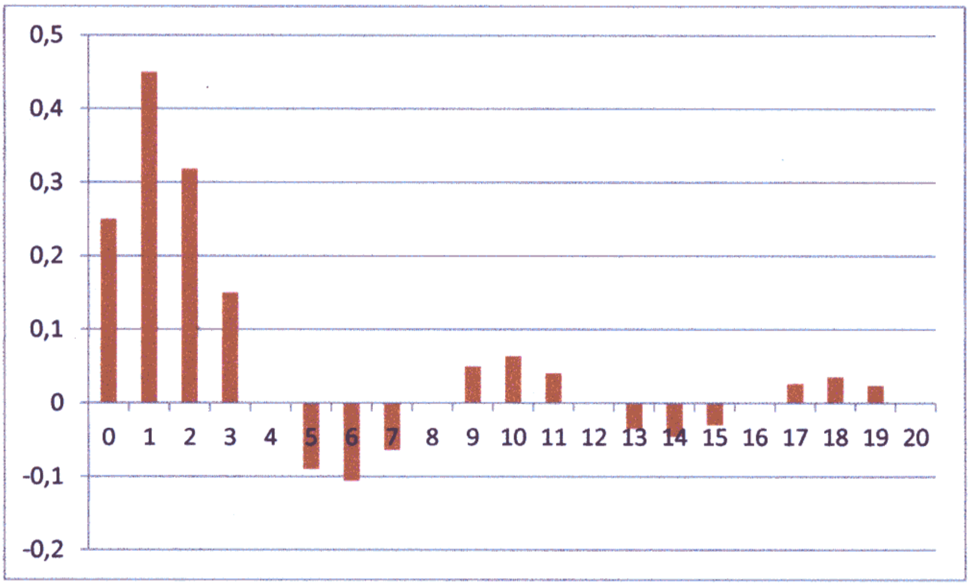
13 b) ONDA IMPULSIVA (NEGATIVA)

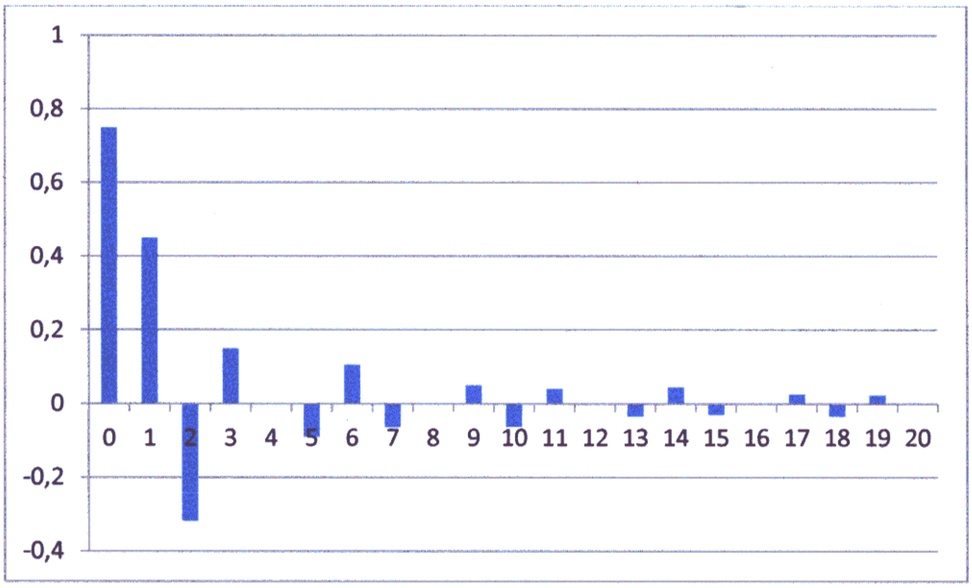


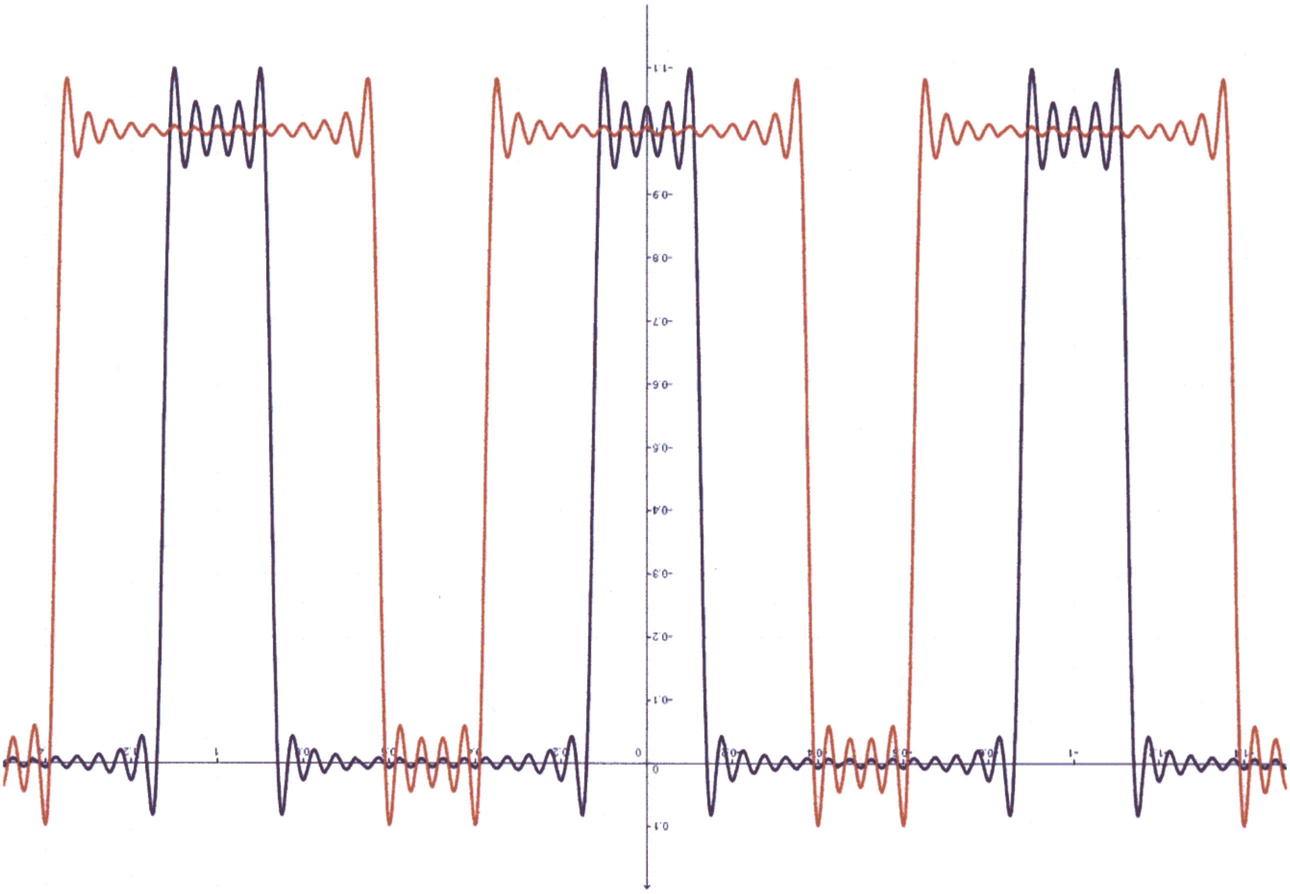
LO SVILUPPO IN SERIE È LO STESSO DELLA FUNZ. 13a) CON $A<0$

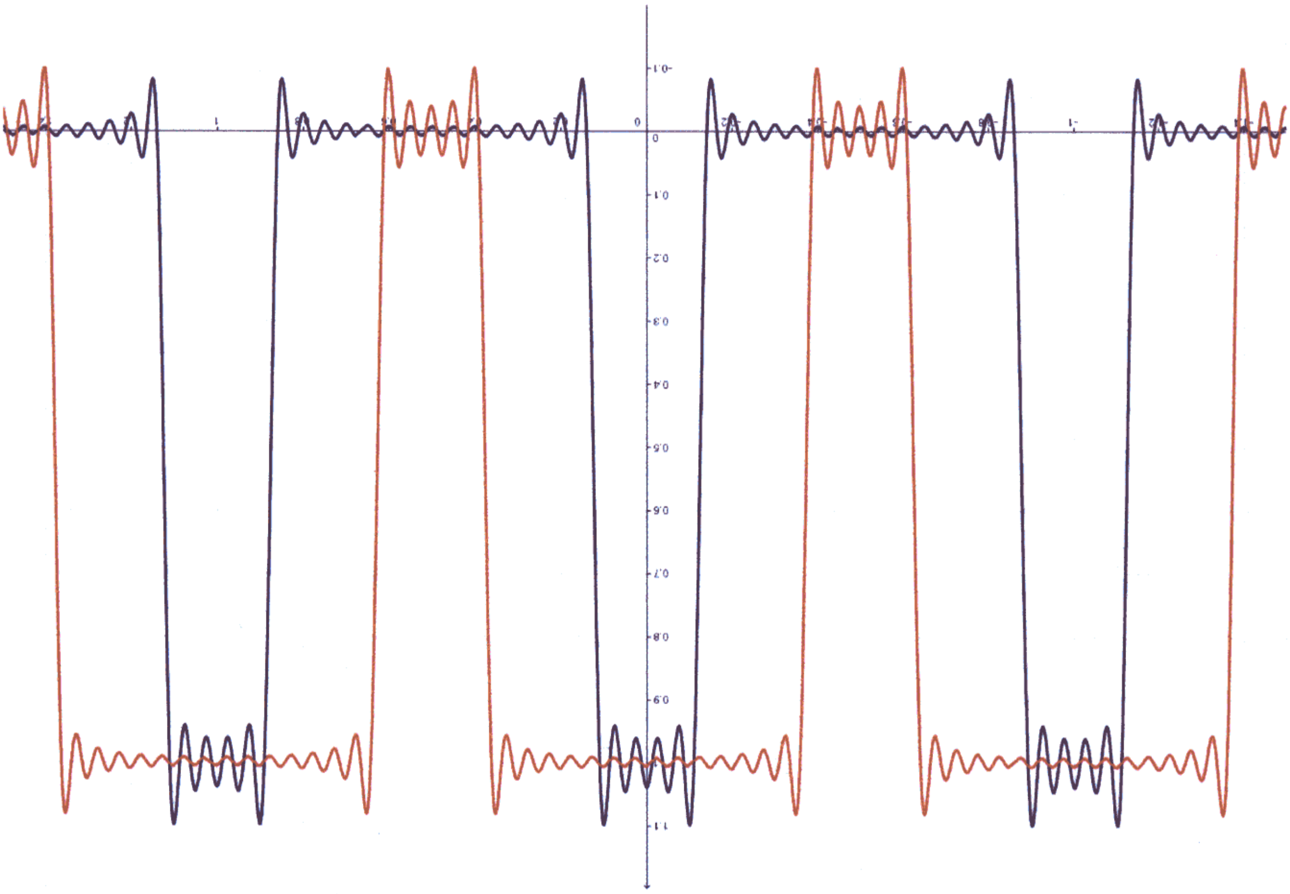
COEFF. con $\delta = 0,25$ e $\delta = 0,75$

0	0,25	0,75
1	0,450158	0,450158
2	0,31831	-0,31831
3	0,150053	0,150053
4	1,95E-17	5,85E-17
5	-0,09003	-0,09003
6	-0,1061	0,106103
7	-0,06431	-0,06431
8	-1,9E-17	-5,8E-17
9	0,050018	0,050018
10	0,063662	-0,06366
11	0,040923	0,040923
12	1,95E-17	5,85E-17
13	-0,03463	-0,03463
14	-0,04547	0,045473
15	-0,03001	-0,03001
16	-1,9E-17	-5,8E-17
17	0,02648	0,02648
18	0,035368	-0,03537
19	0,023693	0,023693
20	1,95E-17	1,72E-16

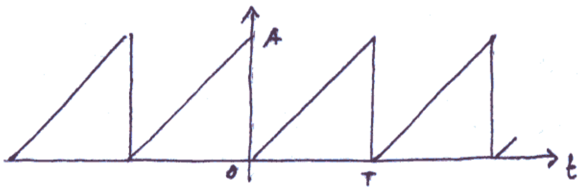








14)



$f(t) = \frac{A}{T} t$ per $0 < t < T$

$f(t) = A_0 + \sum_{1,n}^{\infty} b_n \sin n\omega t$

$$A_0 = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{T}\int_0^T \frac{A}{T} t\,dt = \frac{A}{T^2}\cdot\frac{1}{2}t^2\Big|_0^T = \frac{A}{T^2}\cdot\frac{1}{2}T^2 = \frac{A}{2}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt = \frac{2}{T}\int_0^T \frac{A}{T} t \sin n\omega t\,dt =$$

$$= \frac{2A}{T^2}\int_0^T t\sin n\omega t\,dt =$$

$$\left\{ \int t \sin n\omega t\,dt = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2} \right\} \qquad n\omega T = n\frac{2\pi}{T}\cdot T = n2\pi$$

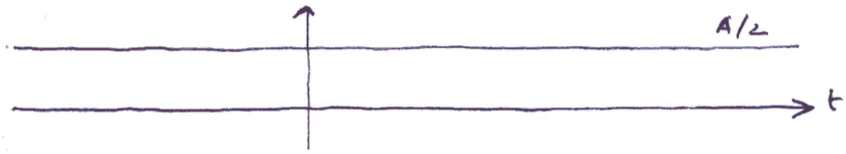
$$= \frac{2A}{T^2}\left[-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{n^2\omega^2}\right]_0^T = \frac{2A}{T^2}\left[-\frac{T\cos n2\pi}{n\omega} + \underbrace{\cancel{\frac{\sin n2\pi}{n^2\omega^2}}}_{0}\right] =$$

$$= -\frac{2A}{T^2}\frac{\cancel{T}}{n\omega}\cos n2\pi = -\frac{2A}{n2\pi} = -\frac{A}{n\pi}$$

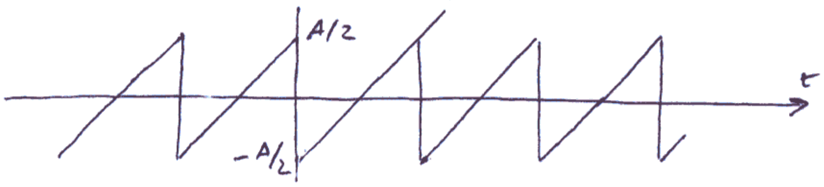
$$f(t) = \frac{A}{2} - \frac{A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

— · — · — · — · —

LA FUNZIONE PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI 2 FUNZIONI



$f_1 = A/2$

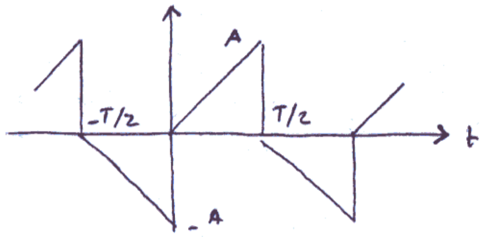


$f_2 = -\frac{2A/2}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$

PER LA PROPRIETÀ DI LINEARITÀ DELLA Σ DI FOURIER, LO SVILUPPO IN SERIE DELLA SOMMA DI 2 FUNZIONI È = ALLA SOMMA DEGLI SVILUPPI IN SERIE DELLE 2 FUNZIONI.

$$f(t) = f_1(t) + f_2(t) = \frac{A}{2} - \frac{A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

15)
(a)



$$f(t) = \begin{cases} \dfrac{2A}{T}t & \text{per} \quad 0 < t < T/2 \\ -\dfrac{2A}{T}t - A & \text{PER} \quad -\dfrac{T}{2} < t < 0 \end{cases}$$

VALOR MEDIO NULLO $A_0 = 0$

f. EMISIMMETRICA $f(t+T/2) = -f(t) \rightarrow \exists$ SOLO ARMONICHE DISPARI

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t + \sum_{1}^{\infty} b_n \sin n\omega t$$

$$A_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{1}{T}\left[\int_{-T/2}^{0}\left(-\frac{2A}{T}t - A\right)dt + \int_{0}^{T/2}\frac{2A}{T}t\,dt\right] =$$

$$= \frac{1}{T}\left[\left(-\frac{\cancel{2}A}{T}\cdot\frac{1}{\cancel{2}}t^2 - At\right)\Big|_{-T/2}^{0} + \frac{\cancel{2}A}{T}\cdot\frac{1}{\cancel{2}}t^2\Big|_{0}^{T/2}\right] = \frac{1}{T}\left[\left(\frac{A}{T}t^2 + At\right)_{0}^{-T/2} + \frac{A}{T}t^2\Big|_{0}^{T/2}\right] =$$

$$= \frac{1}{T}\left[\frac{A}{\cancel{T}}\cdot\frac{T^{\cancel{2}}}{4} - \frac{AT}{2} + \frac{A}{\cancel{T}}\cdot\frac{T^{\cancel{2}}}{4}\right] = 0$$

$$a_n = \frac{2}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega t\,dt\right] = \frac{2}{T}\left[\int_{-T/2}^{0}\left(-\frac{2A}{T}t - A\right)\cos n\omega t\,dt + \int_{0}^{T/2}\frac{2A}{T}t\cos n\omega t\,dt\right] =$$

$$= -\frac{4A}{T^2}\int_{-T/2}^{0} t\cos n\omega t\,dt - \frac{2A}{T}\int_{-T/2}^{0}\cos n\omega t\,dt + \frac{4A}{T^2}\int_{0}^{T/2} t\cos n\omega t\,dt =$$

$$\int t\cos n\omega t\,dt = \frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}$$

$$= -\frac{4A}{T^2}\left[\frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}\right]_{-T/2}^{0} - \frac{2A}{T}\frac{\sin n\omega t}{n\omega}\Big|_{-T/2}^{0} + \frac{4A}{T^2}\left[\frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}\right]_{0}^{T/2} =$$

$$= -\frac{4A}{T^2}\left[\frac{\cos 0}{(n\omega)^2} + \frac{T}{2}\cdot\frac{1}{n\omega}\sin\left(-n\omega\frac{T}{2}\right) - \frac{\cos(n\omega T/2)}{(n\omega)^2}\right] + \frac{2A}{T}\frac{\sin(-n\omega T/2)}{n\omega} + \frac{4A}{T^2}\left[\frac{T}{2}\cdot\frac{1}{n\omega}\sin n\omega\frac{T}{2} + \right.$$

$$\left. + \frac{\cos n\omega T/2}{(n\omega)^2} - \frac{1}{(n\omega)^2}\right] =$$

$$= -\frac{4A}{T^2}\cdot\frac{1}{(n\omega)^2} - \frac{\cancel{4}^{2}A\cancel{T}^{1}}{T^{\cancel{2}}\cancel{2}_{1}}\cdot\frac{1}{n\omega}\sin\left(-n\omega\frac{T}{2}\right) + \frac{4A}{n^2\omega^2T^2}\cos\left(-n\omega\frac{T}{2}\right) - \frac{2A}{n\omega T}\sin n\omega T/2 +$$

$$+ \frac{\cancel{4}^{2}A\cancel{T}^{1}}{T^{\cancel{2}}\cancel{2}n\omega}\sin n\omega\frac{T}{2} + \frac{4A\cos n\omega T/2}{T^2n^2\omega^2} - \frac{4A}{n^2\omega^2T^2} =$$

$$= -\frac{4A}{(n\omega T)^2} + \frac{2A}{n\omega T}\sin\omega\frac{T}{2} + \frac{4A}{(n\omega T)^2}\cos n\omega\frac{T}{2} - \cancel{\frac{2A}{n\omega T}\sin n\omega\frac{T}{2}} + \cancel{\frac{2A}{n\omega T}\sin n\omega\frac{T}{2}} + \frac{4A}{(n\omega T)^2}\cos n\omega\frac{T}{2} - \frac{4A}{(n\omega T)^2} =$$

$$= \frac{8A}{(n\omega T)^2}\cos n\omega\frac{T}{2} - \frac{8A}{(n\omega T)^2} + \frac{2A}{n\omega T}\sin n\omega\frac{T}{2} = \frac{8A}{(n2\pi)^2}\cos n\pi - \frac{8A}{(n2\pi)^2} + \cancel{\frac{2A}{2n\pi}\sin n\pi} =$$

$$= \frac{2A}{n^2\pi^2}(\cos n\pi - 1)$$

$$f_p(t) = -\frac{4A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

$n=1 \quad \cos\pi = -1$
$n=2 \quad \cos 2\pi = 1$
$n=3 \quad \cos 3\pi = -1$

(b)

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t\,dt = \frac{2}{T}\left[\int_{-T/2}^{0}\left(-\frac{2A}{T}t - A\right)\sin n\omega t\,dt + \int_0^{T/2}\frac{2A}{T}t\sin n\omega t\,dt\right] =$$

$$= -\frac{4A}{T^2}\int_{-T/2}^{0} t\sin n\omega t\,dt - \frac{2A}{T}\int_{-T/2}^{0}\sin n\omega t\,dt + \frac{4A}{T^2}\int_0^{T/2} t\sin n\omega t\,dt =$$

$$\int t\sin n\omega t\,dt = -t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}$$

$$= -\frac{4A}{T^2}\left[-t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_{-T/2}^{0} + \frac{2A}{T}\frac{\cos n\omega t}{n\omega}\Big|_{-T/2}^{0} + \frac{4A}{T^2}\left[-\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_0^{T/2} =$$

$$= -\frac{4A}{T^2}\left[-\frac{T}{2}\cdot\frac{1}{n\omega}\cos\left(-n\omega\frac{T}{2}\right) - \frac{\sin(-n\omega T/2)}{(n\omega)^2}\right] + \frac{2A}{n\omega T} - \frac{2A}{n\omega T}\cos\left(-n\omega\frac{T}{2}\right) - \frac{4A}{T^2}\cdot\frac{T}{2}\frac{1}{n\omega}\cos n\omega\frac{T}{2} + \frac{4A}{(n\omega T)^2}\sin n\omega\frac{T}{2} =$$

$$= \cancel{\frac{2A}{n\omega T}\cos\left(n\omega\frac{T}{2}\right)} - \cancel{\frac{4A}{(n\omega T)^2}\sin n\omega\frac{T}{2}} + \frac{2A}{n\omega T} - \frac{2A}{n\omega T}\cos n\omega\frac{T}{2} - \cancel{\frac{2A}{n\omega T}\cos n\omega\frac{T}{2}} + \cancel{\frac{4A}{(n\omega T)^2}\sin n\omega\frac{T}{2}} =$$

$$= \frac{2A}{n\omega T}\left[1 - \cos n\omega\frac{T}{2}\right] \qquad n\omega\frac{T}{2} = n\pi$$

$$= \frac{A}{n\pi}\left[1 - \cos n\pi\right] \qquad \begin{array}{ll} n=1 & b_1 = \frac{2A}{\pi} \\ n=2 & b_2 = 0 \\ n=3 & b_3 = \frac{2A}{3\pi} \end{array}$$

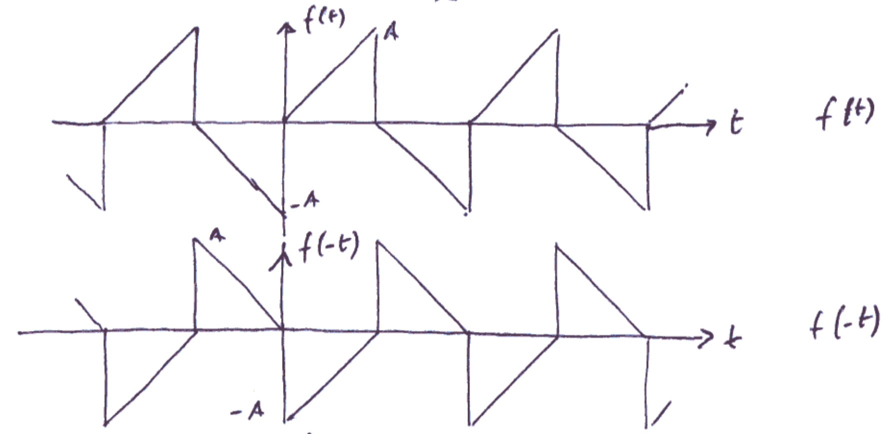
$$f_d(t) = \frac{2A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

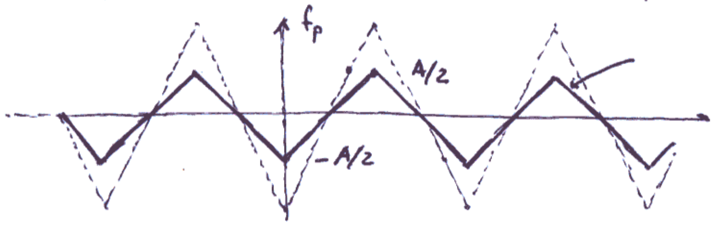
$$f(t) = f_p(t) + f_d(t) = -\frac{4A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right] + \frac{2A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

(c) LA FUNZIONE (S) PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI UNA FUNZIONE PARI E UNA FUNZIONE DISPARI

$$f(t) = f_p + f_d \qquad \text{con} \quad \begin{cases} f_p = \dfrac{f(t) + f(-t)}{2} \\[2ex] f_d = \dfrac{f(t) - f(-t)}{2} \end{cases}$$

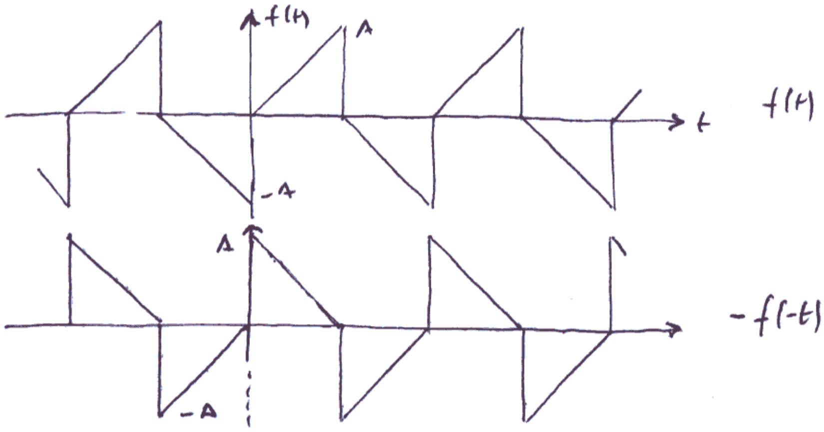
- PER RICAVARE f_p : SI TRACCIANO LE FUNZIONI $f(t)$ e $f(-t)$ [FUNZIONE SPECCHIO RISPETTO ALL'ASSE DELLE ORDINATE] E SI EFFETTUA LA SEMISOMMA DELLE 2 FUNZIONI

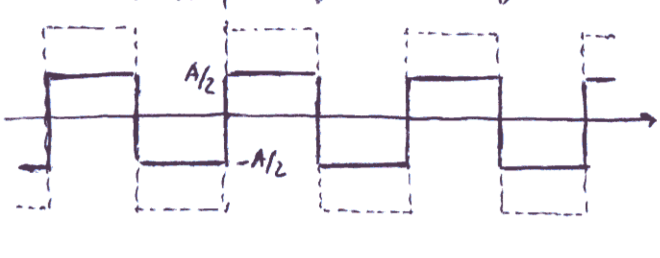




$$f_p = -\frac{8A/2}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \ldots\right]$$
$$= -\frac{4A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \ldots\right]$$

- PER RICAVARE f_d : SI TRACCIANO LE FUNZIONI $f(t)$ E $-f(-t)$ [FUNZIONE OPPOSTA DI $f(-t)$] E SI EFFETTUA LA SEMISOMMA DELLE 2 FUNZIONI

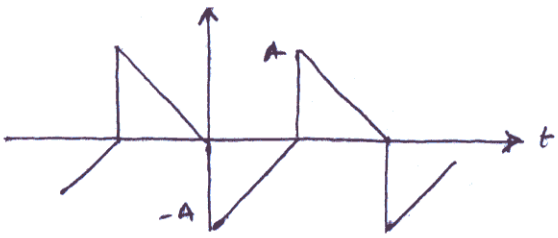




$$f_d = 4A/2\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right]$$
$$= \frac{2A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \ldots\right]$$

$$f(t) = f_p + f_d$$

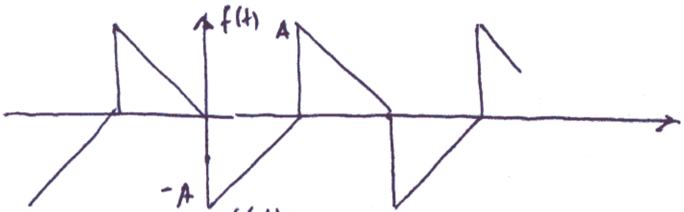
16)



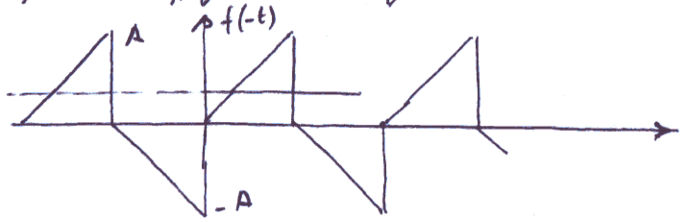
SCOMPONGO LA FUNZ. ASSEGNATA NELLA SOMMA DI 2 FUNZIONI (UNA PARI E L'ALTRA DISPARI)

$$f(t) = f_p + f_d \quad \text{CON} \quad \begin{cases} f_p = \dfrac{f(t) + f(-t)}{2} \\ f_d = \dfrac{f(t) - f(-t)}{2} \end{cases}$$

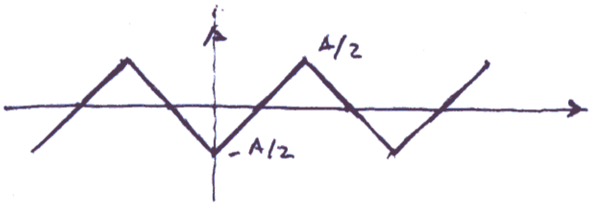
- RICAVO $f_p(t)$ TRACCIANDO $f(t)$ E $f(-t)$ E LA FUNZIONE SEMISOMMA



$f(t)$

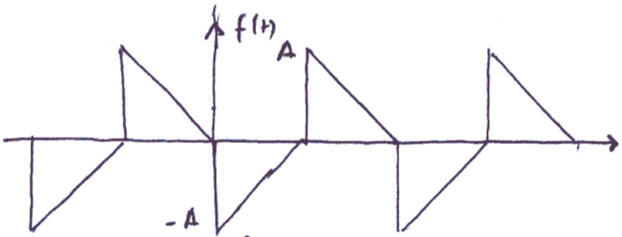


$f(-t)$

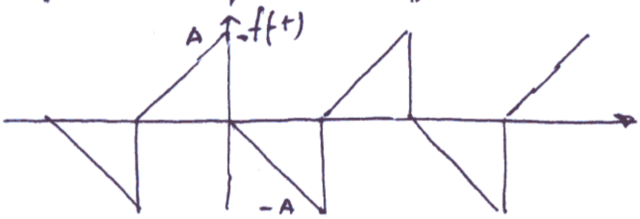


$$f_p = -\frac{4A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

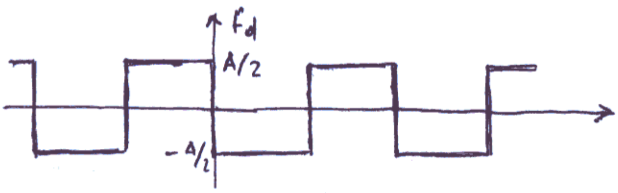
- RICAVO $f_d(t)$ TRACCIANDO $f(t)$ E $-f(-t)$ E LA FUNZIONE SEMISOMMA



$f(t)$



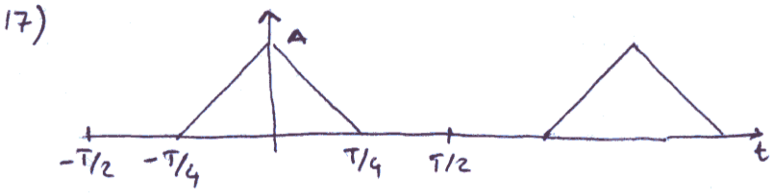
$-f(-t)$



$$f_d = -\frac{2A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

$f(t) = f_p + f_d$

17)



$$f(t) = \begin{cases} 0 & -\frac{T}{2} < t < -\frac{T}{4} \\ \frac{4A}{T}t + A & -\frac{T}{4} < t < 0 \\ -\frac{4A}{T}t + A & 0 < t < \frac{T}{4} \\ 0 & \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{2}{T}\int_0^{T/4}\left(-\frac{4A}{T}t + A\right)dt = \left(-\frac{8A}{T^2}\cdot\frac{1}{2}t^2 + \frac{2A}{T}t\right)\Bigg|_0^{T/4} = -\frac{4A}{T^2}\cdot\frac{T^2}{16} + \frac{2A}{T}\cdot\frac{T}{4} = -\frac{1}{4}A + \frac{1}{2}A = \frac{1}{4}A$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{4}{T}\int_{-T/4}^{0}\left(\frac{4A}{T}t + A\right)\cos n\omega t\,dt =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\int_{-T/4}^{0} t\cos n\omega t\,dt + \int_{-T/4}^{0} A\cos n\omega t\,dt\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(\frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right)\Bigg|_{-T/4}^{0} + A\frac{\sin n\omega t}{n\omega}\Bigg|_{-T/4}^{0}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(0 + \frac{1}{n^2\omega^2} - \frac{-T/4\,\sin n\omega(-T/4)}{n\omega} - \frac{\cos n\omega(-T/4)}{n^2\omega^2}\right) + 0 - \frac{A\sin n\omega(-T/4)}{n\omega}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(\frac{1}{n^2\omega^2} - \frac{T^2}{n8\pi}\sin n\frac{\pi}{2} - \frac{T^2}{n^2 4\pi^2}\cos n\frac{\pi}{2}\right) + \frac{AT}{n2\pi}\sin n\frac{\pi}{2}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(\frac{T^2}{4\pi^2 n^2} - \frac{T^2}{n8\pi}\sin n\frac{\pi}{2} - \frac{T^2}{n^2 4\pi^2}\cos n\frac{\pi}{2}\right) + \frac{AT}{n2\pi}\sin n\frac{\pi}{2}\right] =$$

$$= \frac{4A}{T}\left[\frac{4}{T}\cdot\frac{T^2}{4\pi^2 n^2} - \frac{4}{T}\frac{T^2}{n8\pi}\sin n\frac{\pi}{2} - \frac{4}{T}\cdot\frac{T^2}{n^2 4\pi^2}\cos n\frac{\pi}{2} + \frac{T}{2n\pi}\sin n\frac{\pi}{2}\right] =$$

$$= 4A\left[\frac{1}{n^2\pi^2} - \cancel{\frac{1}{2n\pi}\sin n\frac{\pi}{2}} - \frac{1}{n^2\pi^2}\cos n\frac{\pi}{2} + \cancel{\frac{1}{2n\pi}\sin n\frac{\pi}{2}}\right] =$$

0

$$= 4A\left[\frac{1}{n^2\pi^2} - \frac{1}{n^2\pi^2}\cos n\frac{\pi}{2}\right] =$$

$$= \frac{4A}{n^2\pi^2}\left[1 - \cos n\frac{\pi}{2}\right]$$

$$f(t) = \frac{1}{4}A + \sum_{1}^{\infty} \frac{4A}{n^2\pi^2}\left[1 - \cos n\frac{\pi}{2}\right]\cos n\omega t$$

$n=1$ $a_1 = \frac{4}{\pi^2}[1] = 4/\pi^2$

$n=2$ $a_2 = \frac{4}{4\pi^2}[1+1] = \frac{2}{\pi^2}$

$n=3$ $a_3 = \frac{4}{9\pi^2}[1] = \frac{4}{9\pi^2}$

$n=4$ $a_4 = \frac{4}{16\pi^2}[1-1] = 0$

$n=5$ $a_5 = \frac{4}{5^2\pi^2}[1]$ -

$n=6$ $a_6 = \frac{4}{6^2\pi^2}[2]$

$n=7$ $a_7 = \frac{4}{7^2\pi^2}[1]$

$n=8$ $a_8 = 0$

$a_9 = \frac{4}{9^2\pi^2}[1]$

$a_{10} = \frac{4}{10^2\pi^2}[2]$

$a_{11} = \frac{4}{11^2\pi^2}$

$a_{12} = 0$

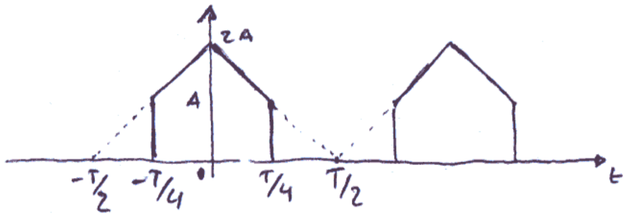
$a_{13} = \frac{4}{13^2\pi^2}$

$a_{14} = \frac{4}{14^2\pi^2}\cdot 2$

$a_{15} = \frac{4}{15^2\pi^2}$

$a_{16} = 0$

18)



$$f(t) = \begin{cases} \frac{4A}{T}t + 2A & -\frac{T}{4} < t < 0 \\ -\frac{4A}{T}t + 2A & 0 < t < \frac{T}{4} \end{cases}$$

f pari $\quad \int_{-T/2}^{T/2} = 2\int_{-T/2}^{0} = 2\int_{0}^{T/2}$

$$f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{2}{T}\int_{-T/4}^{0}\left(\frac{4A}{T}t + 2A\right)dt = \frac{8A}{T^2}\int_{-T/4}^{0} t\,dt + \frac{4A}{T}\int_{-T/4}^{0} dt = -\frac{8A}{T^2}\cdot\frac{1}{2}\cdot\frac{T^2}{16} + \frac{4A}{T}\cdot\frac{T}{4} = A\left(-\frac{1}{4}+1\right) = \frac{3}{4}A$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{4}{T}\int_{-T/4}^{0}\left(\frac{4A}{T}t + 2A\right)\cos n\omega t\,dt = \frac{4}{T}\left[\frac{4A}{T}\int_{-T/4}^{0} t\cos n\omega t\,dt + 2A\int_{-T/4}^{0}\cos n\omega t\,dt\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left\{\frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right\}_{-T/4}^{0} + 2A\left.\frac{\sin n\omega t}{n\omega}\right|_{-T/4}^{0}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(0 + \frac{1}{n^2\omega^2} - \frac{-T/4\,\sin n\omega(-T/4)}{n\omega} - \frac{\cos n\omega(-T/4)}{n^2\omega^2}\right) + 0 - 2A\frac{\sin n\omega(-T/4)}{n\omega}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\left(\frac{1}{n^2\omega^2} - \frac{T^2}{n8\pi}\sin n\frac{\pi}{2} - \frac{T^2}{n^2 4\pi^2}\cos n\frac{\pi}{2}\right) + \frac{2AT}{2n\pi}\sin n\frac{\pi}{2}\right] =$$

$$= \frac{4}{T}\left[\frac{4A}{T}\cdot\frac{T^2}{4\pi^2 n^2} - \frac{4}{T}\cdot\frac{T^2}{n8\pi}\sin n\frac{\pi}{2} - \frac{4}{T}\cdot\frac{T^2}{n^2 4\pi^2}\cos n\frac{\pi}{2} + \frac{AT}{n\pi}\sin n\frac{\pi}{2}\right] =$$

$$= \frac{4A}{T}\cdot T\left[\frac{1}{n^2\pi^2} - \frac{1}{2n\pi}\sin n\frac{\pi}{2} - \frac{1}{n^2\pi^2}\cos n\frac{\pi}{2} + \frac{1}{n\pi}\sin n\frac{\pi}{2}\right] =$$

$$= 4A\left[\frac{1}{n^2\pi^2} - \frac{1}{n^2\pi^2}\cos n\frac{\pi}{2} + \frac{1}{2n\pi}\sin n\frac{\pi}{2}\right] =$$

$$= \frac{4A}{n^2\pi^2}\left[1 - \cos n\frac{\pi}{2}\right] + \frac{2A}{n\pi}\underbrace{\sin n\frac{\pi}{2}}_{\substack{n \text{ pari} = 0 \\ n = 1,5,9\ldots = 1 \\ n = 3,7,11\ldots = -1}}$$

$$f(t) = \frac{3}{4}A + \frac{4A}{\pi^2}\sum_{1}^{\infty}\frac{1}{n^2}\left[1 - \cos n\frac{\pi}{2}\right]\cos n\omega t + \frac{2A}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin n\frac{\pi}{2}\cos n\omega t$$

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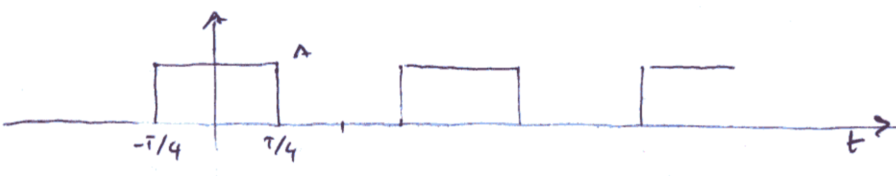
la funzione può essere scomposta nella somma di 2 funzioni di cui sono noti gli sviluppi in serie.

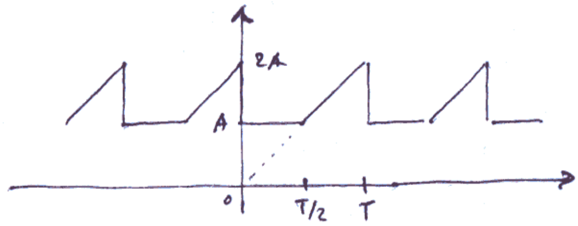


$$f_1 = \frac{A}{2} + \sum_{n=1}^{\infty}\frac{2A}{n\pi}\sin n\frac{\pi}{2}\cos n\omega t$$

$$f_2 = \frac{A}{4} + \sum_{n=1}^{\infty}\frac{4A}{n^2\pi^2}\left[1 - \cos n\frac{\pi}{2}\right]\cos n\omega t$$

$$f(t) = f_1 + f_2 =$$

19)



$$f(t) = \begin{cases} A & \text{p/ } 0<t<\frac{T}{2} \\ \frac{2A}{T}t & \text{p/ } \frac{T}{2}<t<T \end{cases}$$

$$f(t) = A_0 + \sum a_n \cos n\omega t + \sum b_n \sin n\omega t$$

- $A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} \frac{2A}{T} t\,dt\right] = \frac{A}{T}\left[\frac{T}{2} + \frac{t^2}{T}\Big|_{T/2}^{T}\right] = \frac{A}{T}\left[\frac{T}{2} + \frac{1}{T}\left(T^2 - \frac{T^2}{4}\right)\right] = \frac{A}{T}\left(\frac{T}{2} + \frac{3}{4}T\right) = \frac{5}{4}A$

- $a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega t\,dt + \int_{T/2}^{T} \frac{2A}{T} t \cos n\omega t\,dt\right] =$

$$\int t\cos n\omega t\,dt = \frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}$$

$n\omega T = n2\pi$

$n\omega\frac{T}{2} = n\pi$

$$= \frac{2A}{T}\left[\frac{\sin n\omega t}{n\omega}\Big|_0^{T/2} + \frac{2}{T}\left(\frac{t\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right)\Big|_{T/2}^{T}\right] =$$

$$= \frac{2A}{T}\left[\left(\frac{\sin n\omega T/2}{n\omega} - \frac{\sin 0}{n\omega}\right] + \frac{2}{T}\left[\frac{T\sin n\omega T}{n\omega} + \frac{\cos n\omega T}{n^2\omega^2} - \frac{T/2\,\sin n\omega T/2}{n\omega} - \frac{\cos n\omega T/2}{n^2\omega^2}\right] =$$

$$= \frac{2A}{T}\left\{\left[\underbrace{\left(\frac{\sin n\pi}{n\omega}\right.}_{=0}\right] + \frac{2}{T}\left[\underbrace{\frac{T\sin n2\pi}{n\omega}}_{=0} + \frac{1}{n^2\omega^2}\underbrace{\cos 2n\pi}_{=1} - \frac{T/2}{n\omega}\underbrace{\sin n\pi}_{=0} - \frac{\cos n\pi}{n^2\omega^2}\right]\right\}$$

$$= \frac{4A}{T^2n^2\omega^2}\left[1-\cos n\pi\right] = \frac{4A}{n^2 4\pi^2}\left[1-\cos n\pi\right] =$$

$$= \frac{A}{n^2\pi^2}\left[1-\cos n\pi\right]$$

$$f_p = A_0 + \sum a_n \cos n\omega t = \frac{5}{4}A + \frac{2A}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right)$$

- $b_n = \frac{2}{T}\left[\int_0^{T/2} A\sin n\omega t + \int_{T/2}^{T}\frac{2A}{T} t\sin n\omega t\,dt\right] = \frac{2A}{T}\left(-\frac{\cos n\omega t}{n\omega}\right)\Big|_0^{T/2} + \frac{4A}{T^2}\int_{T/2}^{T} t\sin n\omega t\,dt =$

$$\int t\sin n\omega t\,dt = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}$$

$$= -\frac{2A}{n\omega T}\left[\cos n\omega T/2 - \cos 0\right] + \frac{4A}{T^2}\left[-\frac{T\cos n\omega T}{n\omega} + \frac{\sin n\omega T}{(n\omega)^2} + \frac{T}{2}\frac{\cos n\omega T/2}{n\omega} - \frac{\sin n\omega T/2}{(n\omega)^2}\right]$$

$$= -\frac{2A}{n2\pi}\left[\cos n\pi - 1\right] - \frac{4A}{n\omega T}\underbrace{\cos n2\pi}_{=1} + \frac{4A}{(n\omega T)^2}\underbrace{\sin n2\pi}_{=0} + \frac{2A}{n\omega T}\cos n\pi - \frac{4A\,\underbrace{\sin n\pi}_{=0}}{(n\omega T)^2} =$$

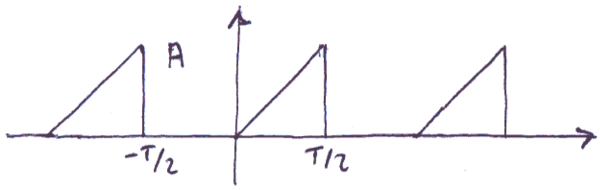
$$= \cancel{-\frac{2A}{n2\pi}\cos n\pi} + \frac{2A}{n2\pi} - \frac{4A}{n2\pi} + \cancel{\frac{2A}{n2\pi}\cos n\pi} =$$

$$= -\frac{2A}{n2\pi} = -\frac{A}{n\pi}$$

$$f_d = -\frac{A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

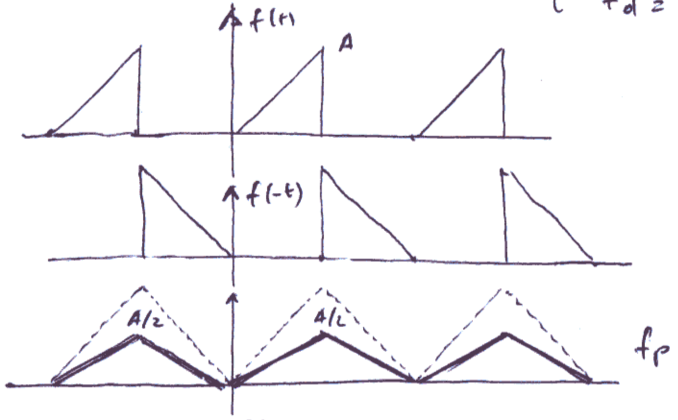
$$f(t) = f_p + f_d = \frac{5}{4}A + \frac{2A}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right) - \frac{A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

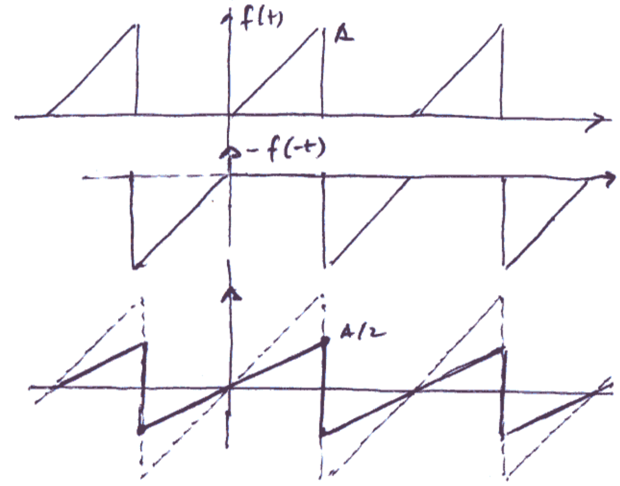
20)



LA FUNZIONE PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI UNA FUNZIONE PARI + UNA FUNZ. DISPARI

$$f(t) = f_p + f_d \qquad \text{con} \quad \begin{cases} f_p = \dfrac{f(t) + f(-t)}{2} \\ f_d = \dfrac{f(t) - f(-t)}{2} \end{cases}$$

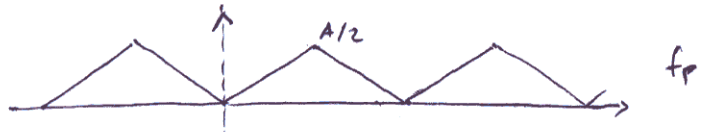


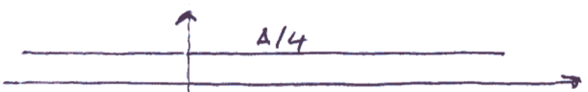


$$f_d = \frac{2\,A/2}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \frac{1}{4}\sin 4\omega t + \dots\right) \quad \text{(VEDI } f_6\text{)}$$

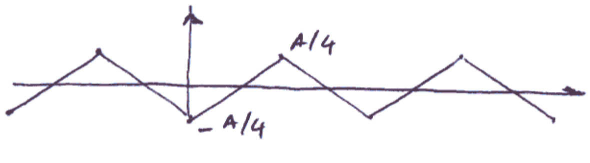
$$= \frac{A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \dots\right)$$

LA FUNZ. PARI f_p PUÒ ESSERE SCOMPOSTA NELLE SEGUENTI





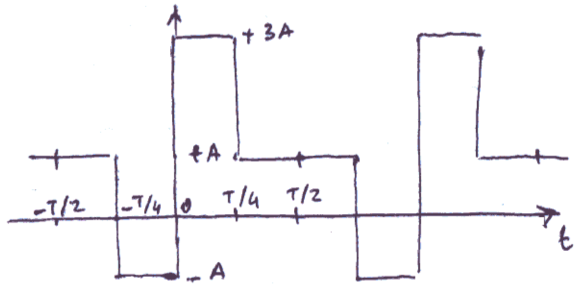
$$f_1 = \frac{1}{4}A$$



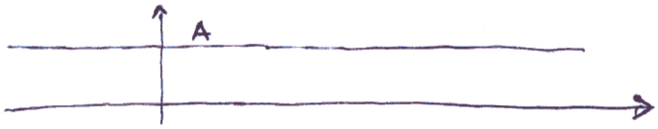
$$f_2 = -\frac{8\,A/4}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right] \quad \text{(VEDI } f_9\text{)}$$

$$f(t) = f_p + f_d = f_1 + f_2 + f_d = \frac{1}{4}A + \frac{A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \right) - \frac{2A}{\pi^2}\left(\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \dots\right)$$

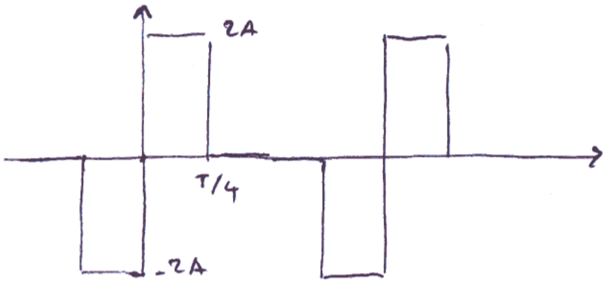
21)



LA FORMA D'ONDA PUÒ ESSERE SCOMPOSTA NELLA SOMMA DELLE SEGUENTI



$f_1 = A$



f_2 FUNZIONE DISPARI A VALOR MEDIO NULLO

$$f_2(t) = \sum_{1}^{\infty} {}_m\, b_m \sin m\omega t$$

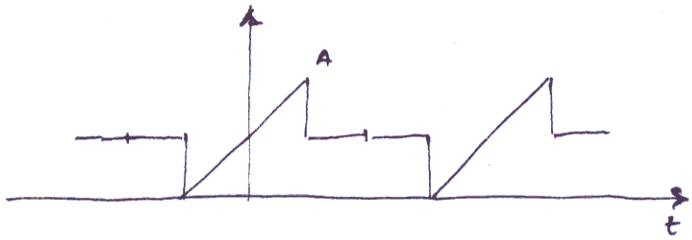
$$b_m = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin m\omega t\, dt = \frac{4}{T}\int_0^{T/2} f(t)\sin m\omega t\, dt = \frac{4}{T}\int_0^{T/4} 2A \sin m\omega t\, dt =$$

$$= \frac{8A}{T}\left(-\frac{\cos m\omega t}{m\omega}\right)\Bigg|_0^{T/4} = \frac{8A}{m\omega T}\cos m\omega t\Bigg|_{T/4}^{0} = \frac{8A}{m2\pi}\left(\cos 0 - \cos m\omega \frac{T}{4}\right) \qquad m\omega\frac{T}{4} = m\frac{2\pi}{T}\cdot\frac{T}{4} = m\frac{\pi}{2}$$

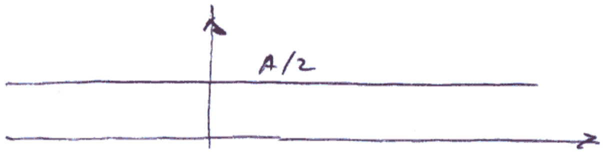
$$= \frac{8A}{m2\pi}\left(1 - \cos m\frac{\pi}{2}\right) = \frac{4A}{m\pi}\left(1 - \cos m\frac{\pi}{2}\right) \qquad f_2(t) = \sum_{1}^{\infty} {}_m \frac{4A}{m\pi}\left(1 - \cos\frac{m\pi}{2}\right)\sin m\omega t$$

$$f(t) = f_1 + f_2 = A\left[1 + \frac{4}{\pi}\sum_{1}^{\infty} {}_m \frac{1}{m}\left(1 - \cos\frac{m\pi}{2}\right)\sin m\omega t\right]$$

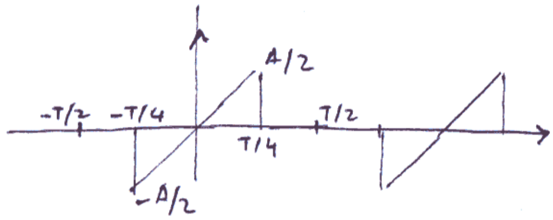
22)



LA FORMA D'ONDA PUÒ ESSERE SCOMPOSTA NELLA SOMMA DELLE SEGUENTI



$f_1 = A_0 = A/2$



$f_2 = \frac{2A}{T} t \quad$ per $-\frac{T}{4} < t < T/4$

funzione dispari a valor medio nullo

$$f_2(t) = \sum_{1}^{\infty} {}_n \, b_n \sin n\omega t$$

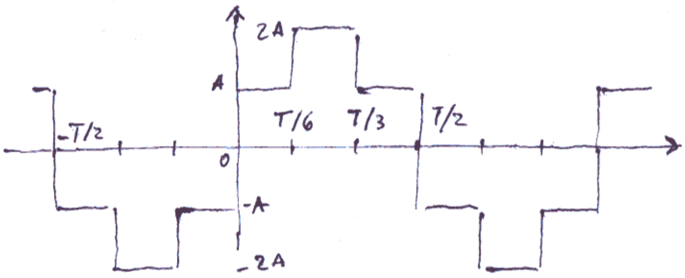
$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f_2(t) \sin n\omega t \, dt = \frac{4}{T} \int_0^{T/4} \frac{2A}{T} t \sin n\omega t \, dt =$$

$$= \frac{8A}{T^2} \int_0^{T/4} t \sin n\omega t \, dt \quad \ldots \quad ecc$$

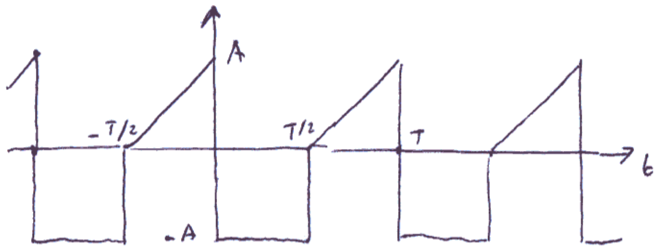
INFINE

$$f(t) = f_1 + f_2 = A/2 + \sum_{1}^{\infty} {}_n \, b_n \sin n\omega t$$

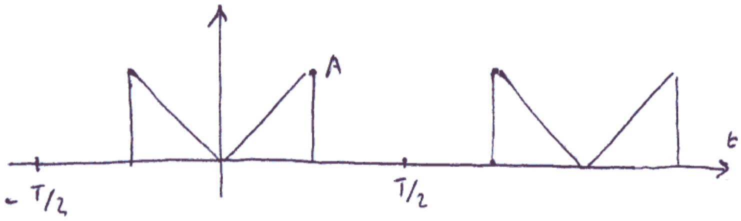
23)



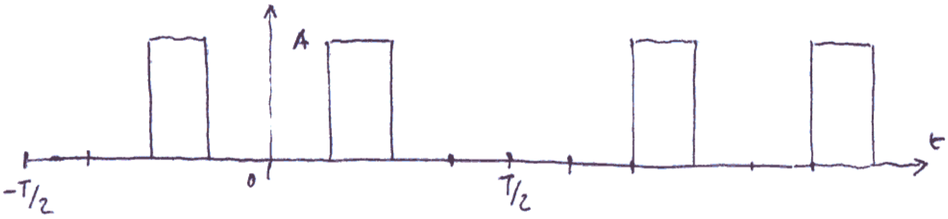
24)



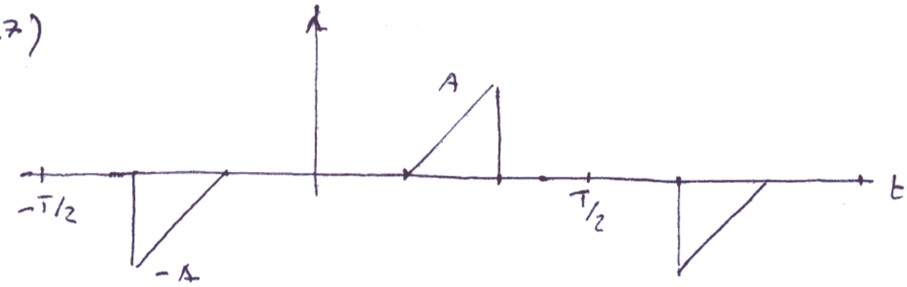
25)

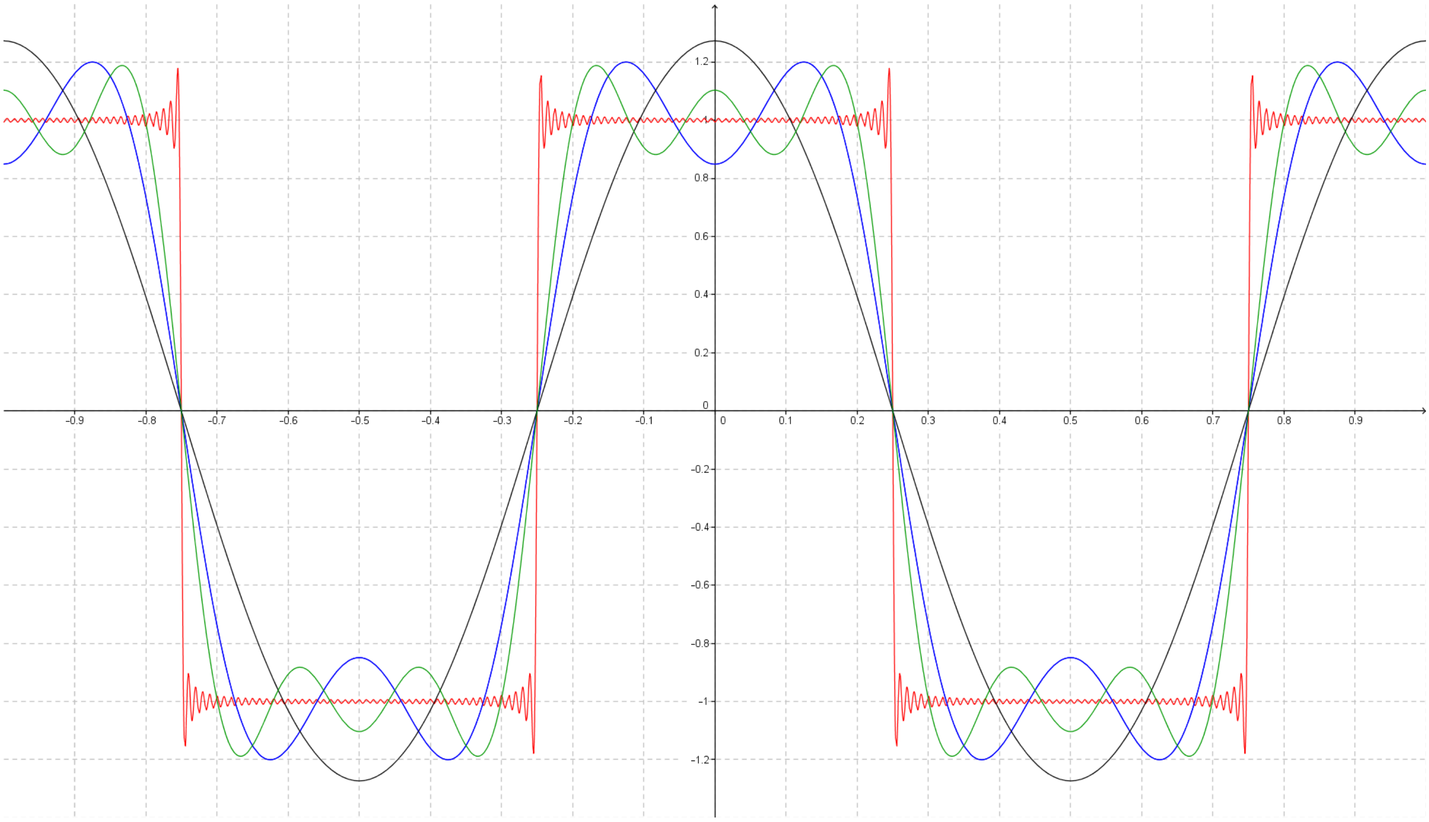


26)



27)





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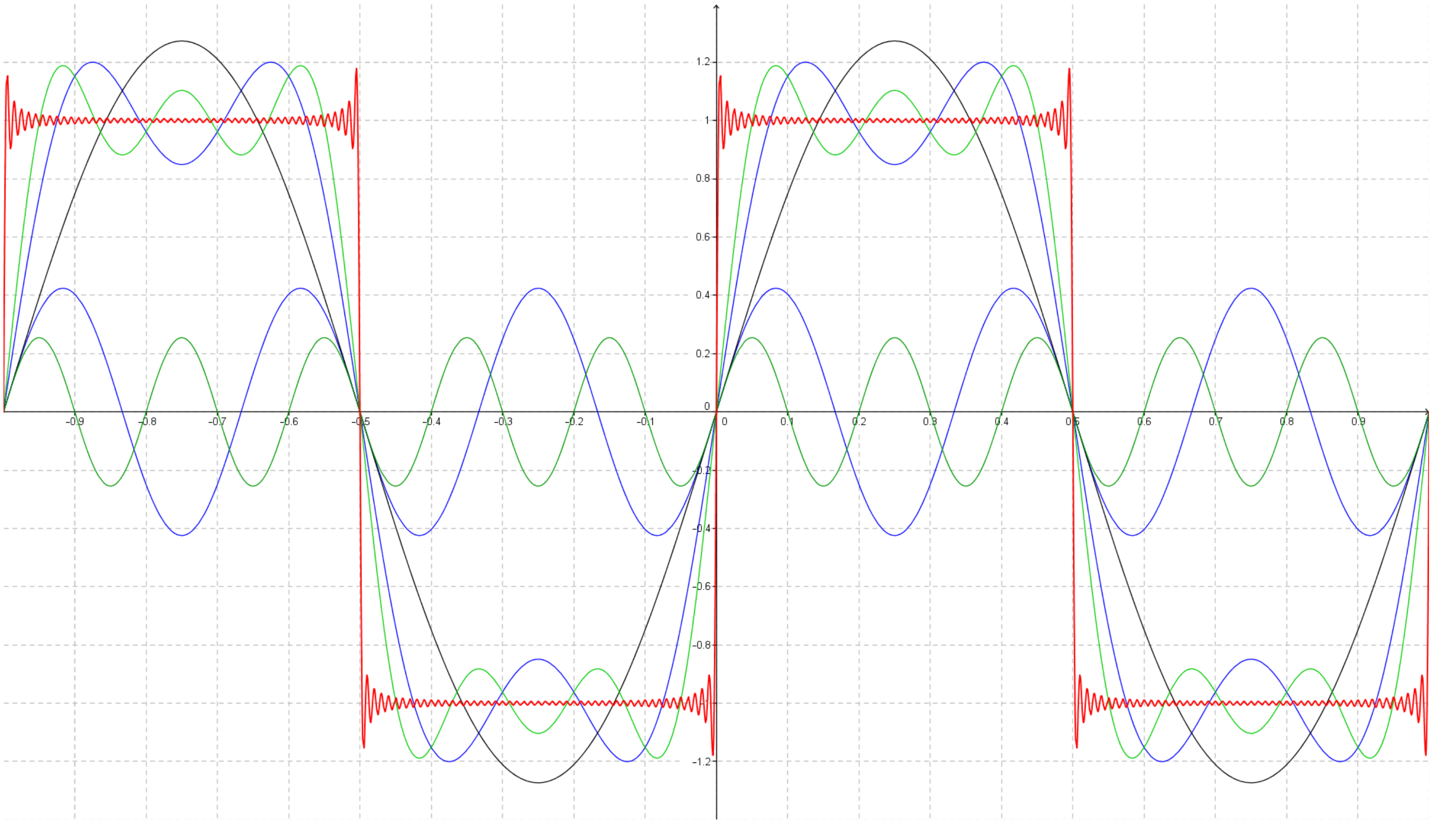

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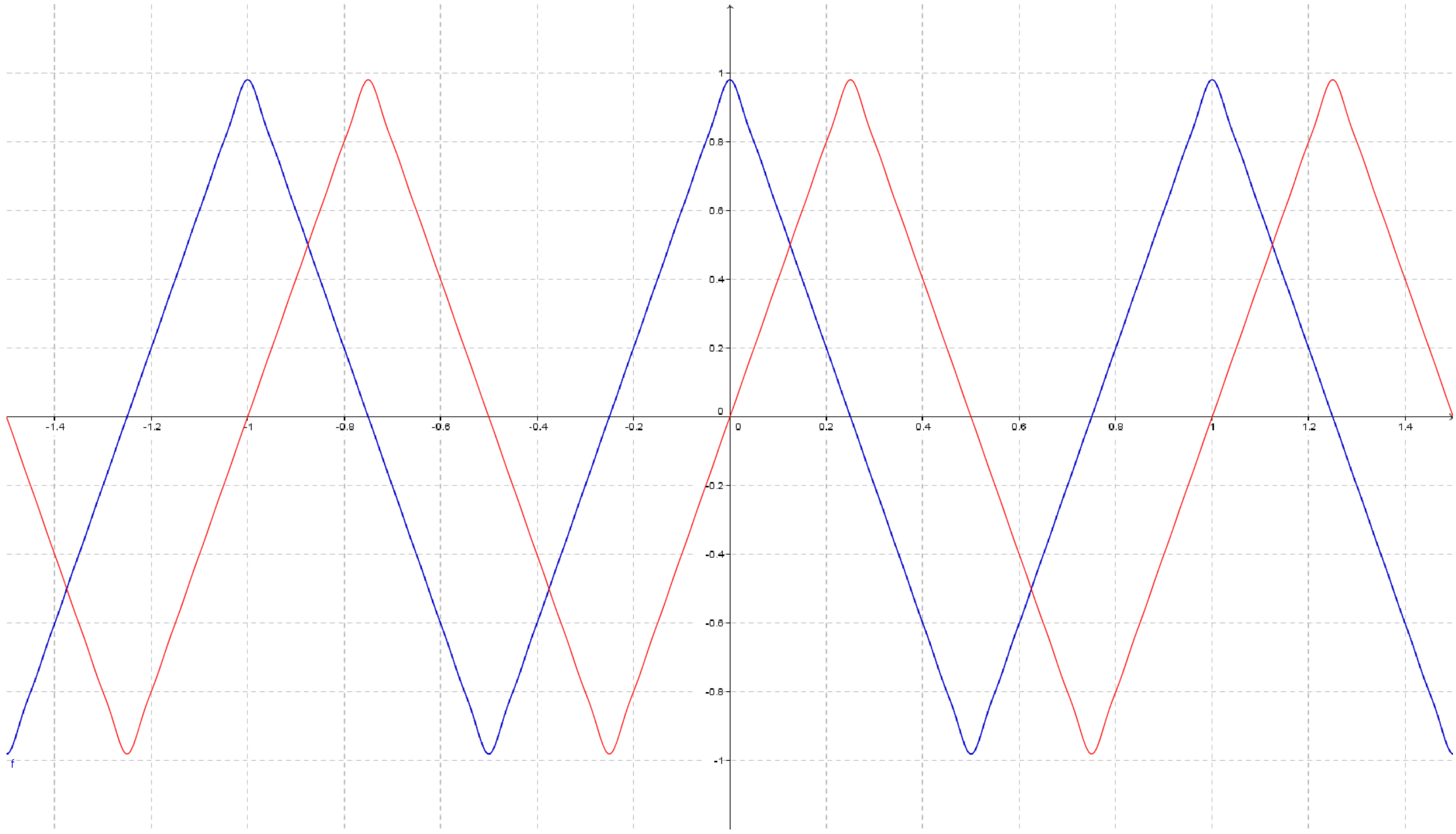
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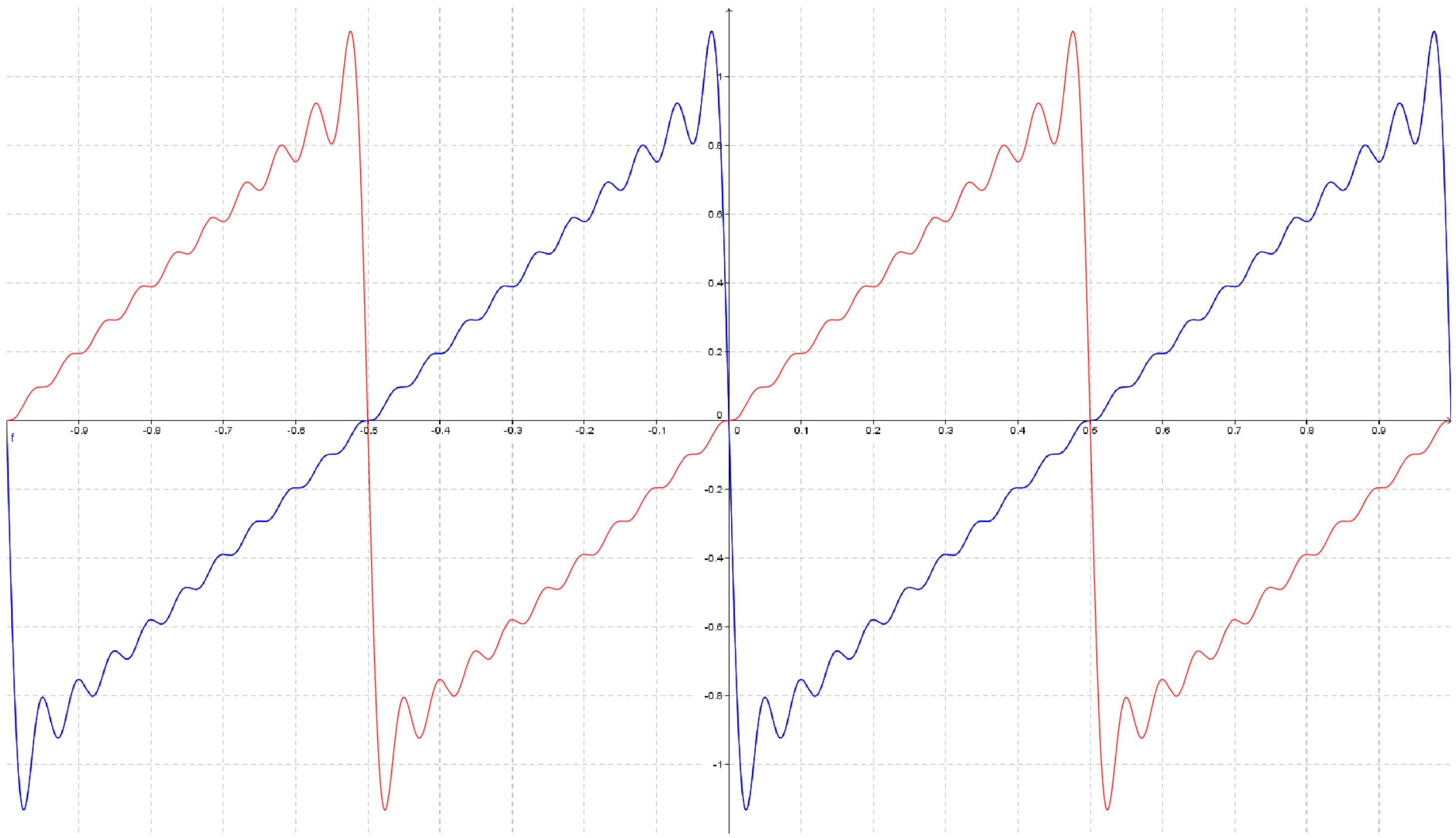
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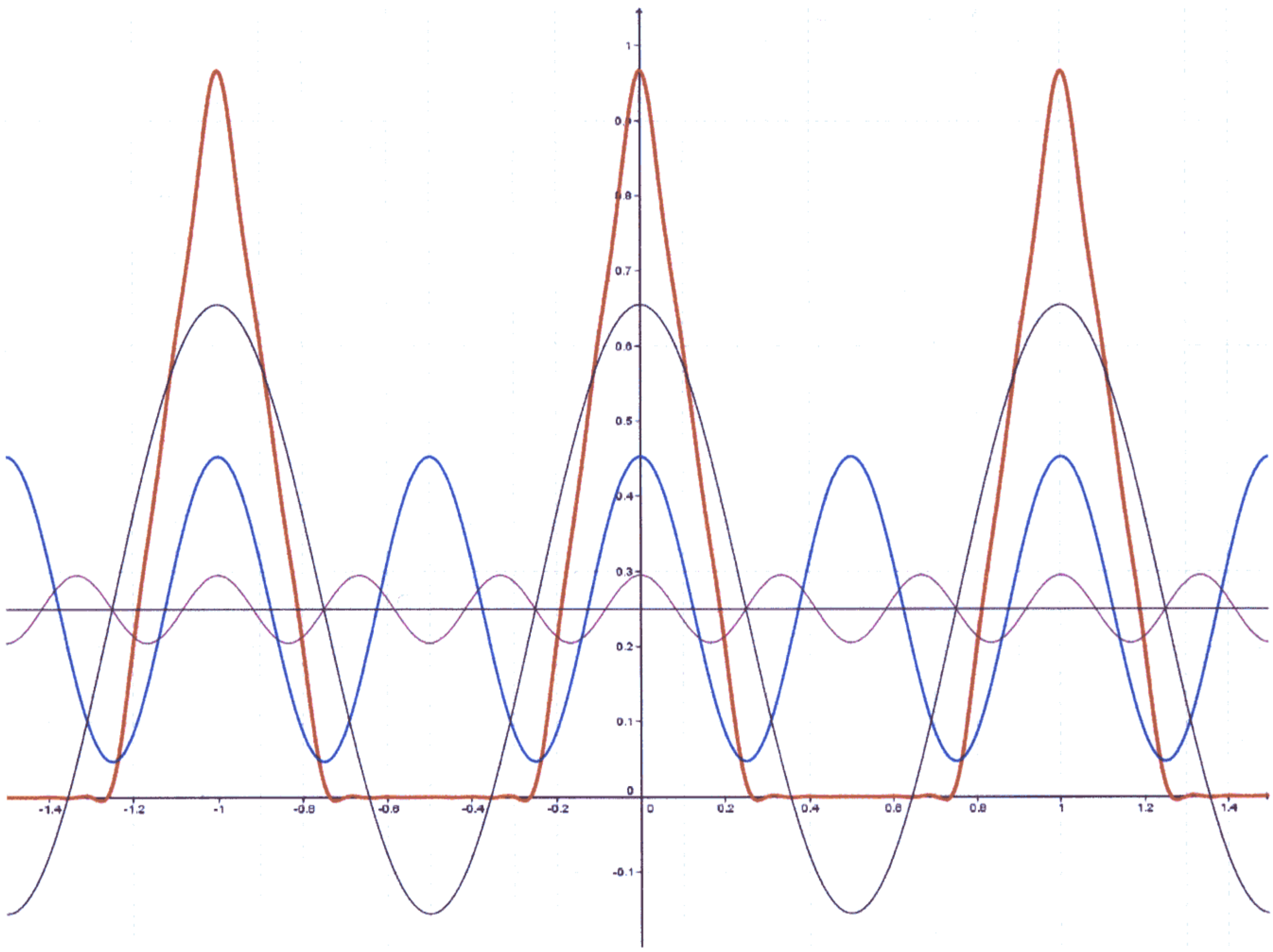

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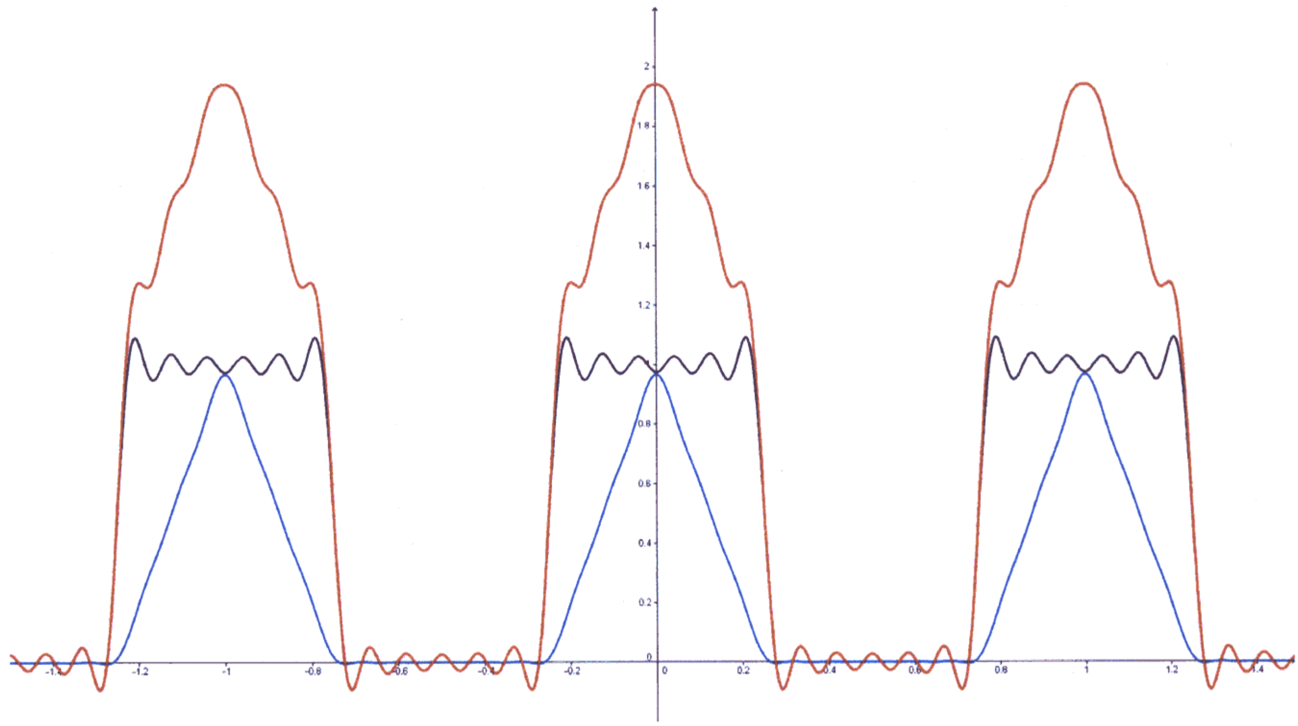

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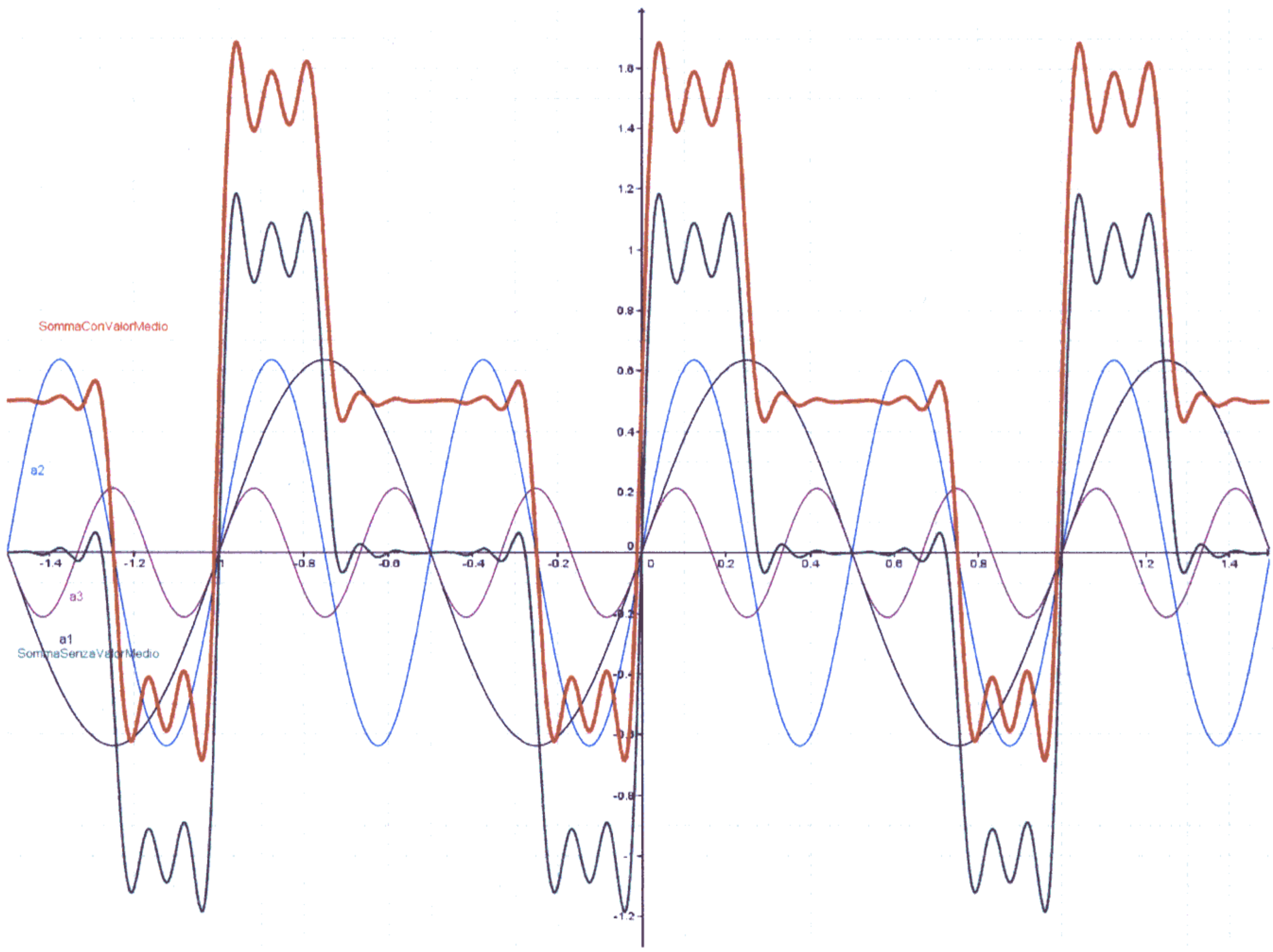
SommaConValorMedio
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a3
a1
SommaSenzaValorMedio
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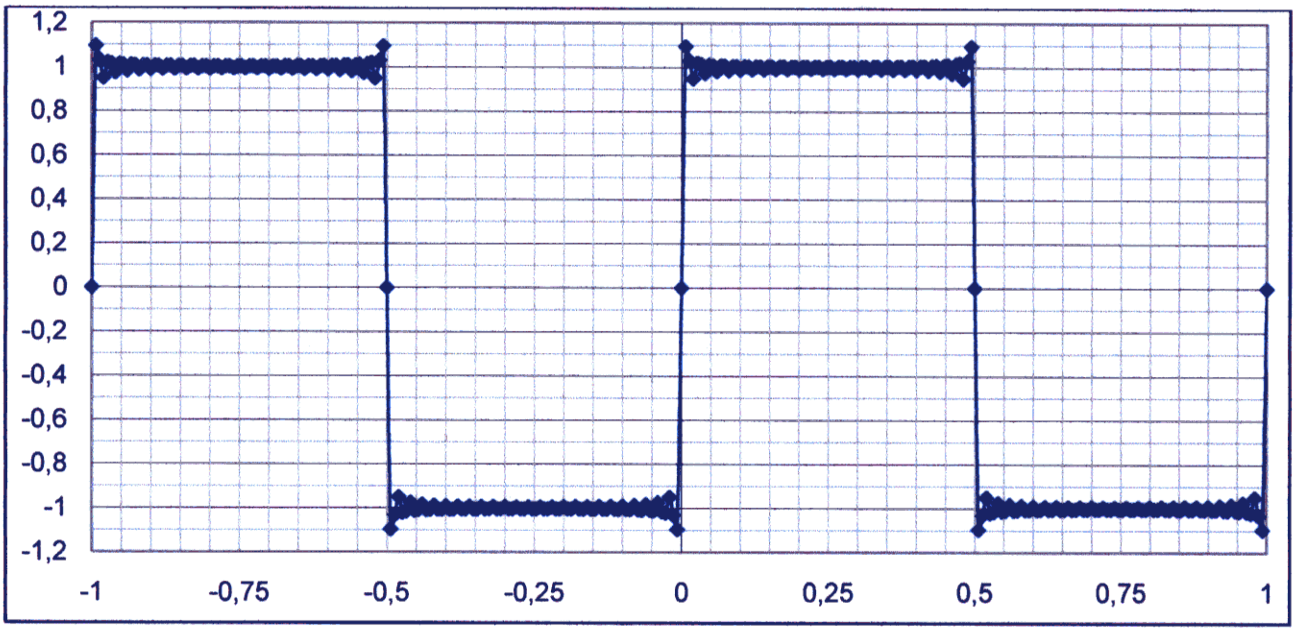
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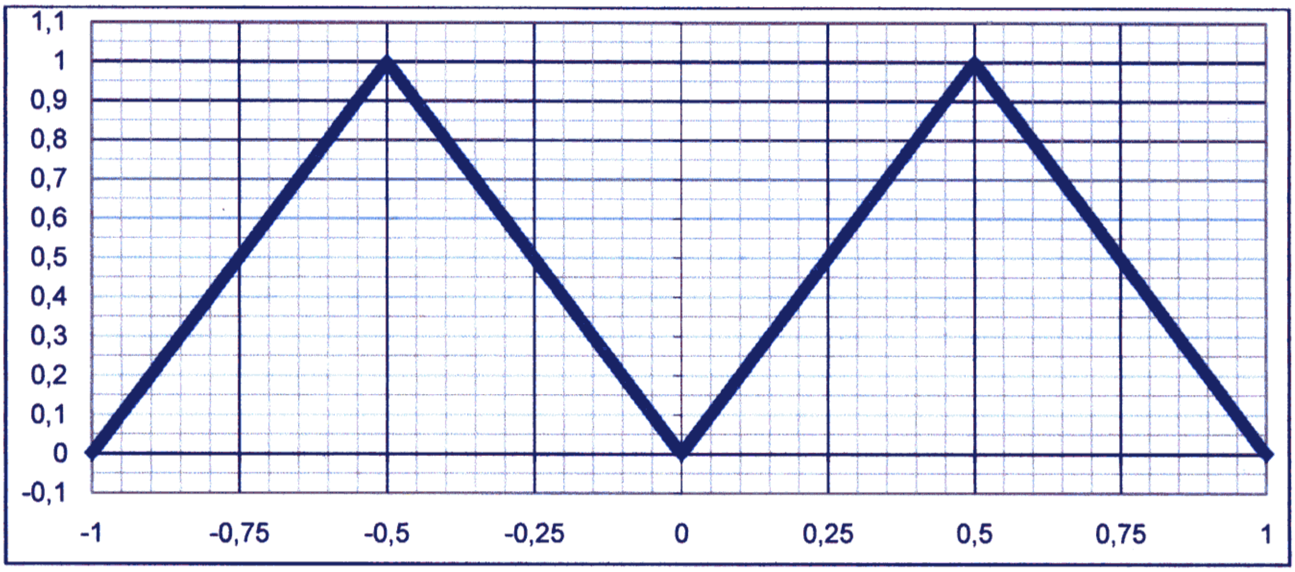
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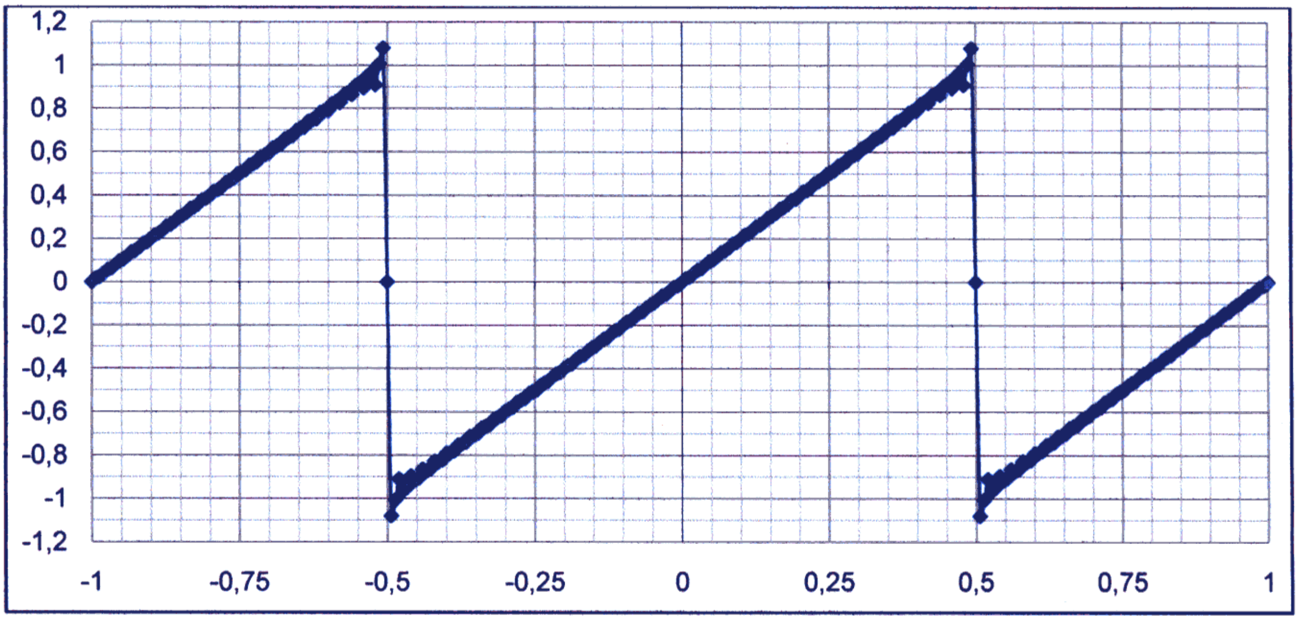
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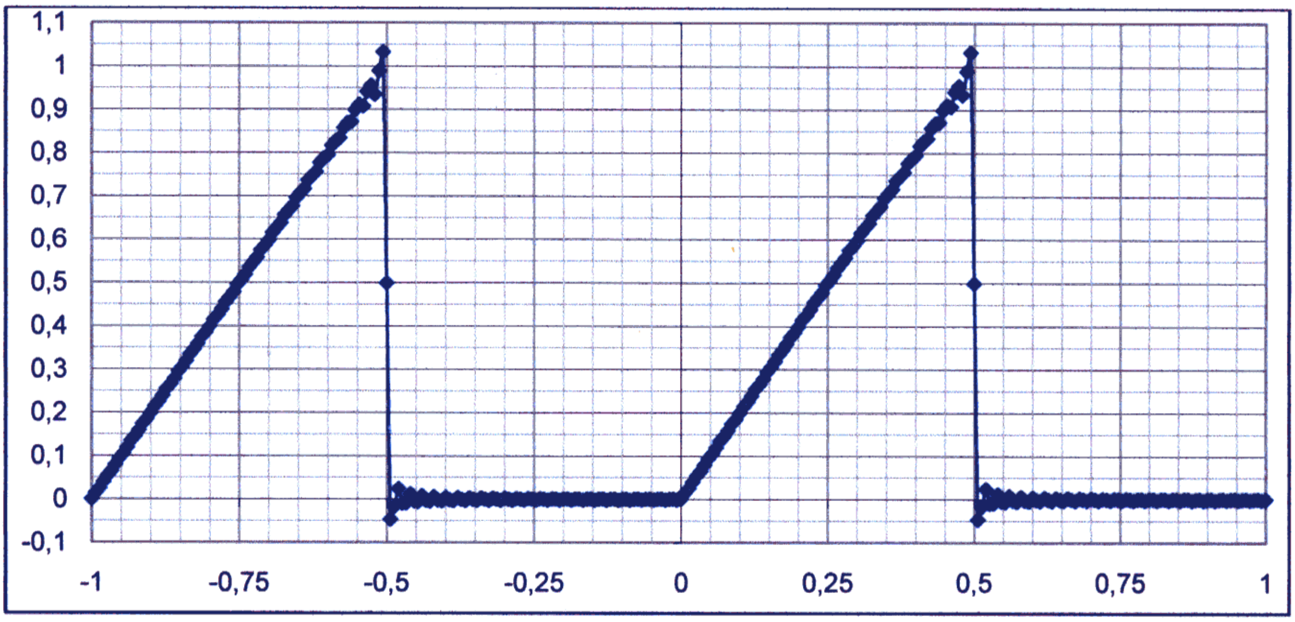
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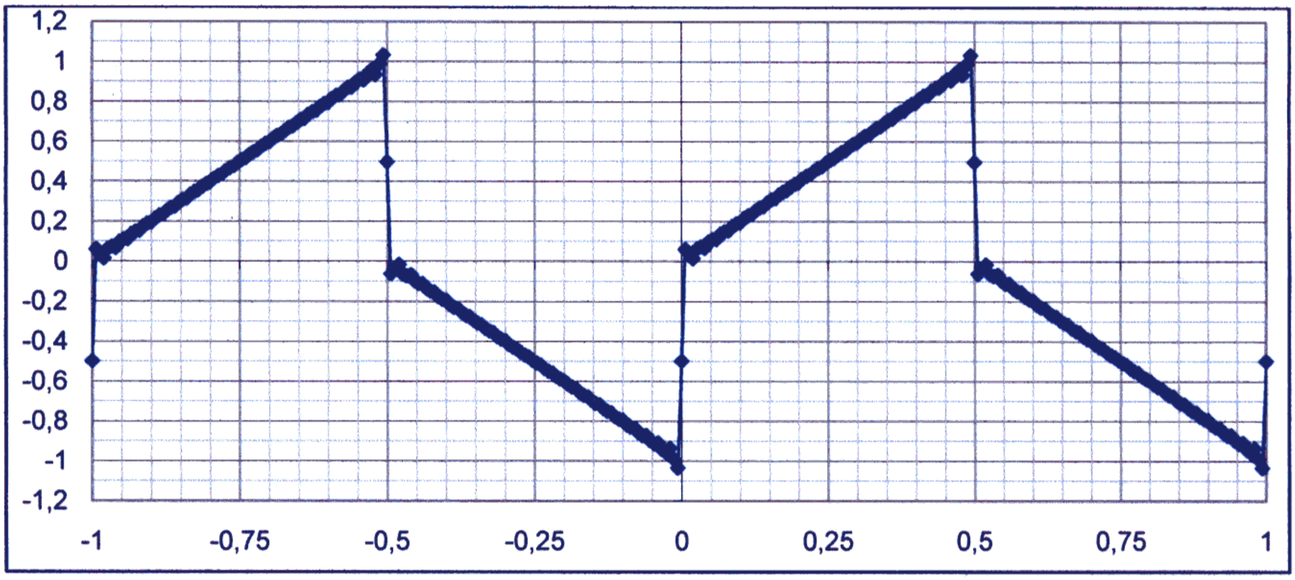
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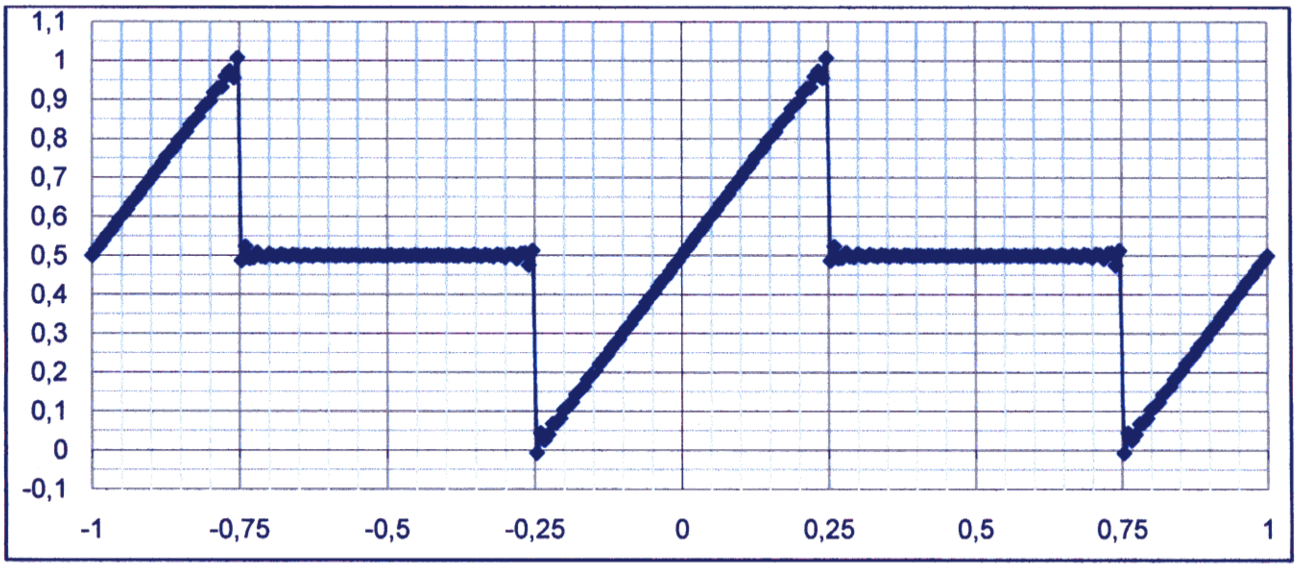
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