1

1) Nel circuito in figura determinare la corrente I_3

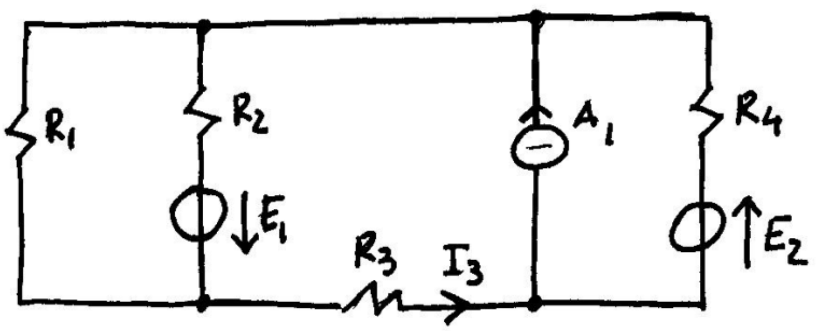
   a) Soluzione mediante applicazione del teorema di Thevenin

   b) Soluzione mediante sovrapposizione delle cause e degli effetti

   c) Soluzione mediante trasformazione di generatori equivalenti

   d) Soluzione mediante LKT e LKC

2) Verificare il bilancio energetico della rete

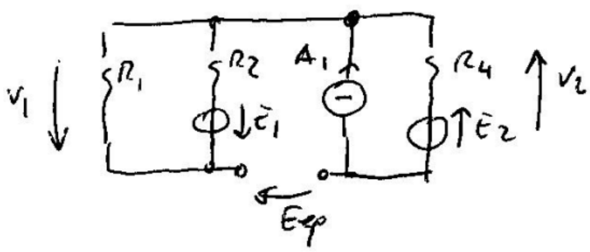


$E_1 = E_2 = 10\,V$

$A_1 = 1\,mA$

$R_1 = R_2 = R_3 = R_4 = 1\,k\Omega$

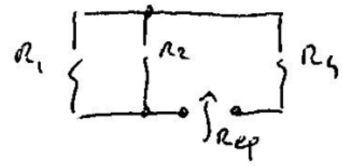
1 a)



$E_{eq} = V_1 + V_2$ $\quad V_1 = E_1 \cdot \frac{R_1}{R_1+R_2} = 10\,\frac{1}{2} = 5\,V$

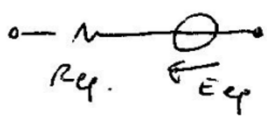
$V_2 = E_2 + R_4 A_1 = 10 + 1 = 11\,V$

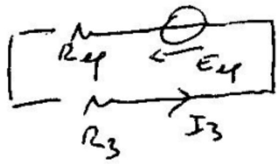
$E_{eq} = 5 + 11 = 16\,V$



$R_{eq} = R_1 /\!/ R_2 + R_4 = 1{,}5\,k\Omega$

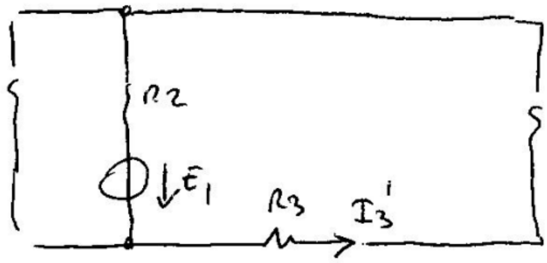
Eq. Th.



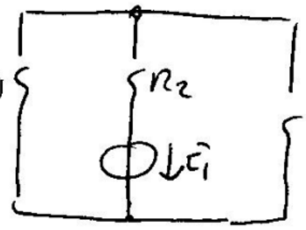


$I_3 = \frac{E_{eq}}{R_{eq} + R_3} = \frac{16\,V}{2{,}5\,k\Omega} = 6{,}4\,mA$

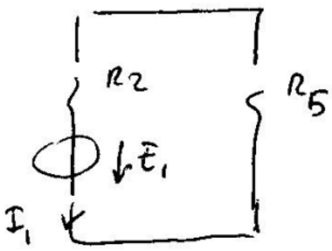
1b) ✱ $E_1 \neq 0 \quad A_1 = 0 \quad E_2 = 0$



$\Rightarrow$



$R_{34} = R_3 + R_4 = 2k\Omega$

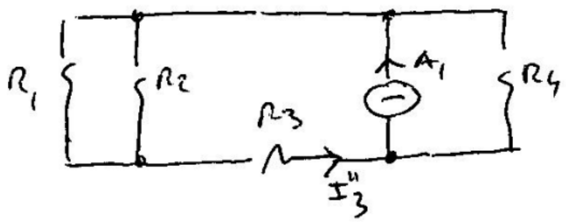


$R_5 = R_1 \| R_{34} = \frac{2}{3} k\Omega$

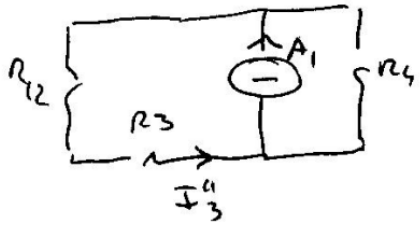
$I_1 = \frac{E_1}{R_2 + R_5} = \frac{10}{\left(1+\frac{2}{3}\right)k} = \frac{10}{\frac{5}{3}k} = \frac{30}{5k} = 6mA$

P.C. $\quad I_3' = I_1 \cdot \frac{R_1}{R_1 + R_{34}} = 6m \cdot \frac{1k}{3k} = 2mA$

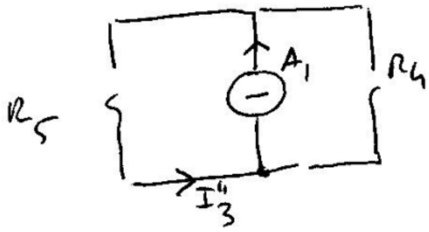
✱ $A_1 \neq 0 \quad E_1 = 0 \quad E_2 = 0$



$\Rightarrow$



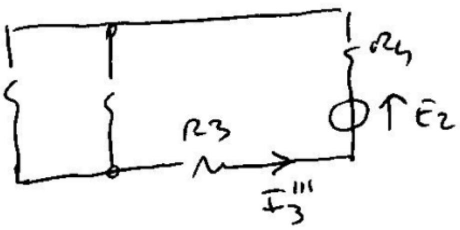
$R_{12} = R_1 \| R_2 = 0{,}5k$



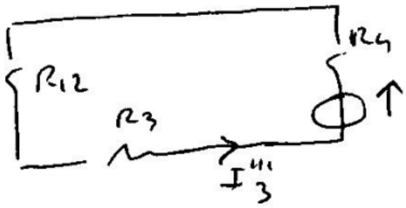
$R_5 = R_{12} + R_3 = 1{,}5k$

P.C. $\quad I_3'' = A_1 \cdot \frac{R_4}{R_5 + R_4} = 1m \cdot \frac{1}{2{,}5} = 0{,}4mA$

✱ $E_2 \neq 0 \quad A_1 = 0 \quad E_1 = 0$



$\Rightarrow$

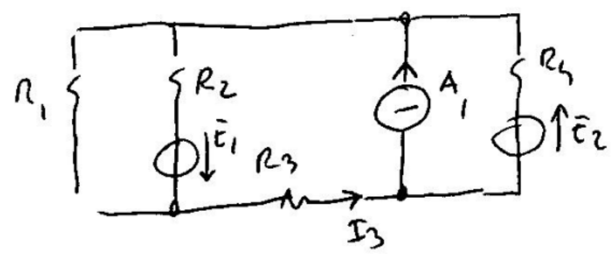


$R_{12} = R_1 \| R_2 = 0{,}5k$

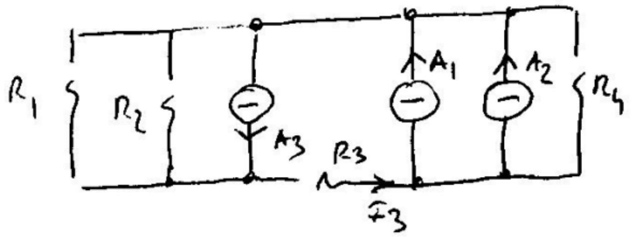
$I_3''' = \frac{E_2}{R_{12} + R_3 + R_4} = \frac{10}{2{,}5k} = 4mA$

$I_3 = I_3' + I_3'' + I_3''' = 2mA + 0{,}4mA + 4mA = 6{,}4mA$

1c)



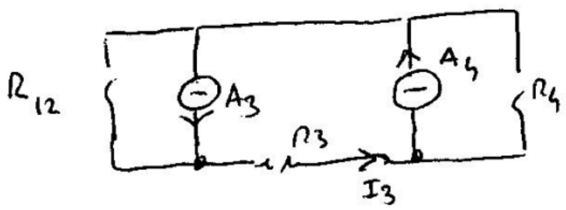
$\Rightarrow$



$$A_3 = \frac{E_1}{R_2} = \frac{10}{1k} = 10mA$$

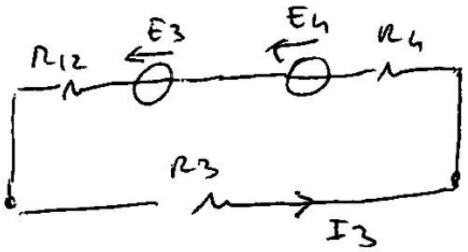
$$A_2 = \frac{E_2}{R_4} = \frac{10}{1k} = 10mA$$

$\Downarrow$



$$R_{12} = R_1 \| R_2 = 0{,}5k\Omega$$

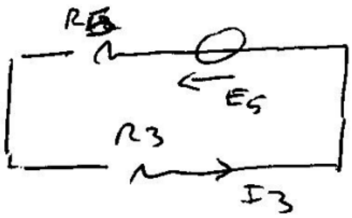
$$A_4 = A_1 + A_2 = 11 mA$$



$$E_3 = R_{12} A_3 = 0{,}5k \cdot 10m = 5V$$

$$E_4 = R_4 A_4 = 1K \cdot 11m = 11V$$

$\Downarrow$

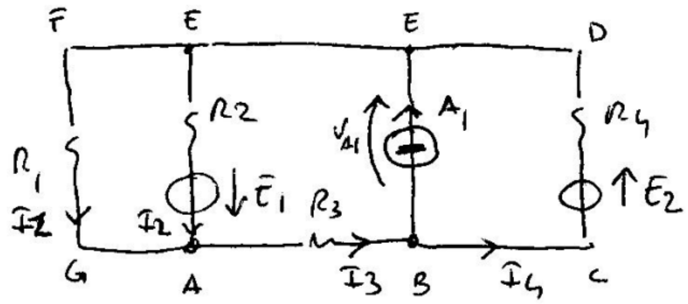


$$E_5 = E_3 + E_4 = 16V$$

$$R_5 = R_{12} + R_4 = 1{,}5V$$

$$I_3 = \frac{E_5}{R_5 + R_3} = \frac{16V}{2{,}5k\Omega} = 6{,}4 mA$$

1d)



3 NODI : A B E

3 MAGLIE INDIPENDENTI (2 senza GEN DI CORR.)

LKC NODO A $\quad I_1 + I_2 = I_3$

LKC NODO B $\quad I_3 = I_4 + A_1$

LKT MAGLIA AGFEA $\quad E_1 = R_2 I_2 - R_1 I_1$

LKT MAGLIA CDFGC $\quad E_2 = R_4 I_4 + R_1 I_1 + R_3 I_3$

4 EQ / 4 INCOGN. (I_1, I_2, I_3, I_4)

— — —

2) $I_3 = 6,4\,mA$ (RICAVATO CON METODI 1a, 1b, 1c)

DALLA 2ª EQ. DELL'ES 1d) SI RICAVA

$$I_4 = I_3 - A_1 = 6,4 - 1 = 5,4\,mA$$

$$V_{DC} = E_2 - R_4 I_4 = 10 - 5,4 = 4,6\,V$$

$$V_{AB} = R_3 I_3 = 6,4\,V$$

$$V_{FG} = V_{BA} + V_{DC} = -6,4 + 4,6 = -1,8\,V$$

$$I_1 = \frac{V_{FG}}{R_1} = -1,8\,mA$$

$$I_2 = I_3 - I_1 = 6,4 + 1,8 = 8,2\,mA$$

$$V_{A1} = V_{DC} = 4,6\,V$$

POT. DISS.

$$P(R_1) = R_1 I_1^2 = 3,24\,mW$$

$$P(R_2) = R_2 I_2^2 = 67,24\,mW$$

$$P(R_3) = R_3 I_3^2 = V_{AB} \cdot I_3 = 40,96\,mW$$

$$P(R_4) = R_4 \cdot I_4^2 = 29,16\,mW$$

$$P_{DISS} = P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = 140,6\,mW$$

P EROG.

$$P(E_1) = E_1 \cdot I_2 = 82\,mW$$

$$P(A_1) = V_{A1} \cdot A_1 = 4,6\,mW$$

$$P(E_2) = E_2 I_4 = 54\,mW$$

$$P_{EROG} = P_{E_1} + P_{A_1} + P_{E_2} = 140,6\,mW$$

1) Nel circuito in figura determinare la corrente I_3

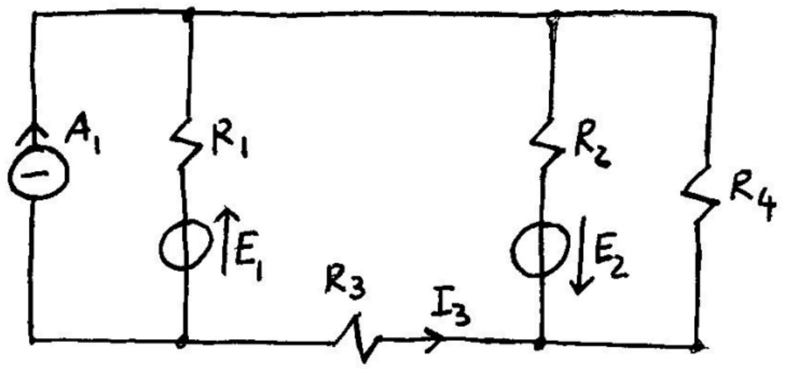
   e) Soluzione mediante applicazione del teorema di Thevenin

   f) Soluzione mediante sovrapposizione delle cause e degli effetti

   g) Soluzione mediante trasformazione di generatori equivalenti

   h) Soluzione mediante LKT e LKC

3) Verificare il bilancio energetico della rete

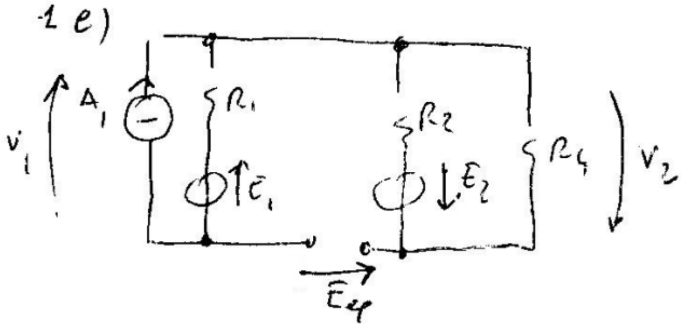


$A_1 = 2\ mA$

$E_1 = E_2 = 20V$

$R_1 = R_2 = R_3 = R_4 = 2\ K\Omega$

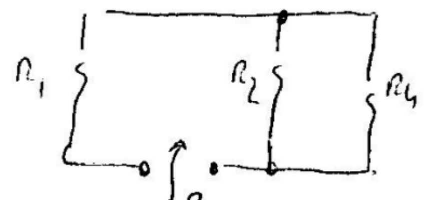
1 e)



$E_{eq} = V_1 + V_2 \qquad V_2 = E_2 \cdot \frac{R_4}{R_2+R_4} = 20\ \frac{1}{2} = 10V$

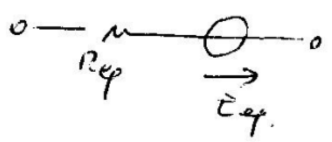
$V_1 = E_1 + R_1 A_1 = 20 + 4 = 24V$

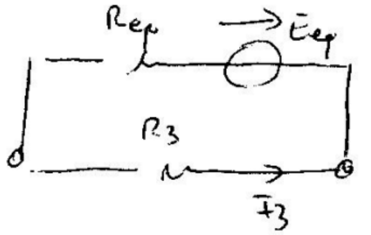
$E_{eq} = 10 + 24 = 34V$



$R_{eq} = R_1 + R_2 // R_4 = 3\ K\Omega$

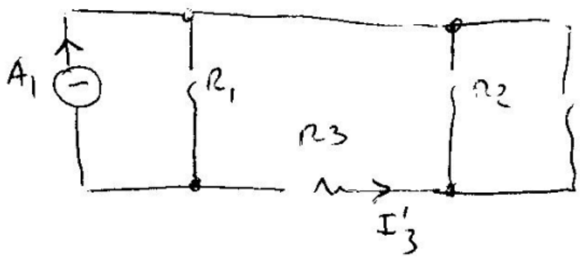
Eq. Th.



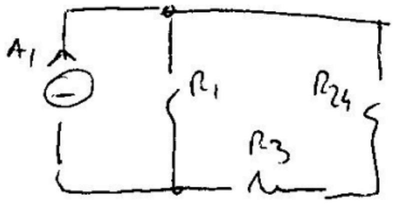


$I_3 = -\frac{E_{eq}}{R_{eq}+R_3} = -\frac{34V}{5K\Omega} = -6,8\ mA$

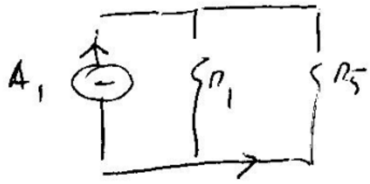
1 f) ☆ $A_1 \neq 0 \quad E_1 = 0 \quad E_2 = 0$



$\Rightarrow$



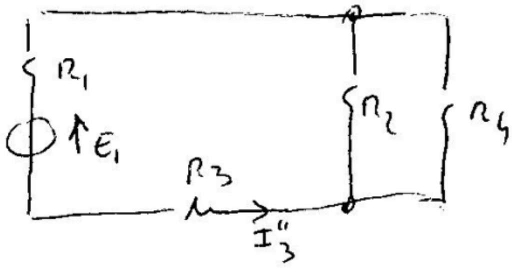
$R_{24} = R_2 \| R_4 = 1\,k\Omega$



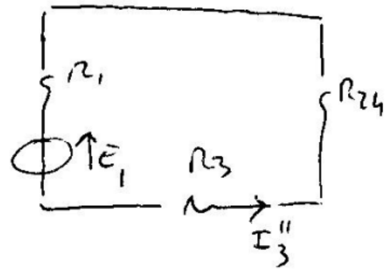
$R_5 = R_{24} + R_3 = 3\,k\Omega$

P.C. $I_3' = -A_1 \cdot \dfrac{R_1}{R_1 + R_5} = -2m\,\dfrac{2}{5} = -0{,}8\,mA$

☆ $E_1 \neq 0 \quad A_1 = 0 \quad E_2 = 0$



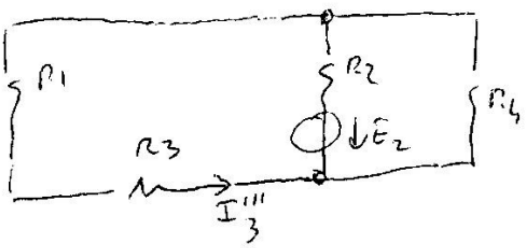
$\Rightarrow$



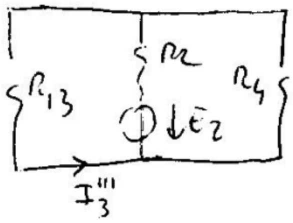
$R_{24} = R_2 \| R_4 = 1\,k$

$I_3'' = \dfrac{E_1}{R_1 + R_{24} + R_3} = -\dfrac{20}{5k} = -4\,mA$

☆ $E_2 \neq 0 \quad A_1 = 0 \quad E_1 = 0$



$\Rightarrow$



$R_{13} = R_1 + R_3 = 4\,k\Omega$

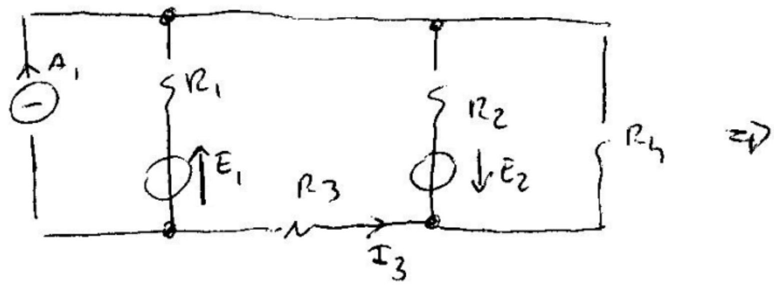
$R_5 = R_{13} \| R_4 = \dfrac{4 \cdot 2}{6}k = \dfrac{4}{3}k$

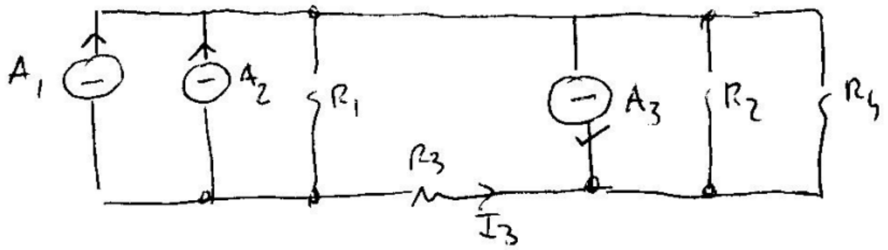
$I_2 = \dfrac{E_2}{R_2 + R_5} = \dfrac{20}{\left(2 + \frac{4}{3}\right)k} = \dfrac{20}{\frac{10}{3}k} = 6\,mA$

P.C. $I_3''' = -I_2 \cdot \dfrac{R_4}{R_4 + R_{13}} = -6m \cdot \dfrac{2k}{6k} = -2\,mA$

$I_3 = I_3' + I_3'' + I_3''' = -0{,}8\,mA - 4\,mA - 2\,mA = -6{,}8\,mA$

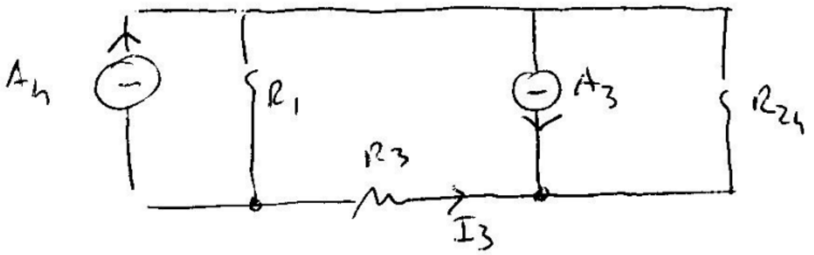
1 g)





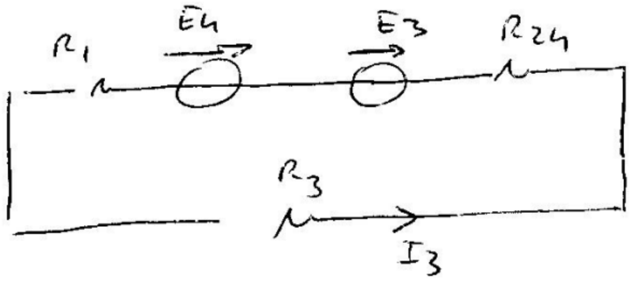
$$A_2 = \frac{E_1}{R_1} = \frac{20}{2K} = 10\,mA$$

$$A_3 = \frac{E_2}{R_2} = \frac{20}{2K} = 10\,mA.$$



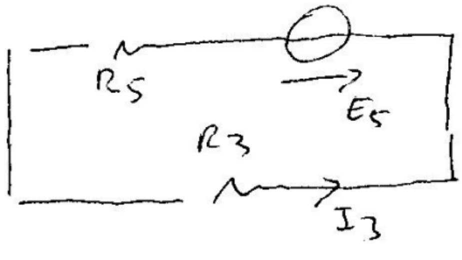
$$R_{24} = R_2 \| R_4 = 1\ K\Omega$$

$$A_4 = A_1 + A_2 = 12\,mA.$$



$$E_3 = R_{24} A_3 = 10\,V$$

$$E_4 = R_1 A_4 = 24\,V$$

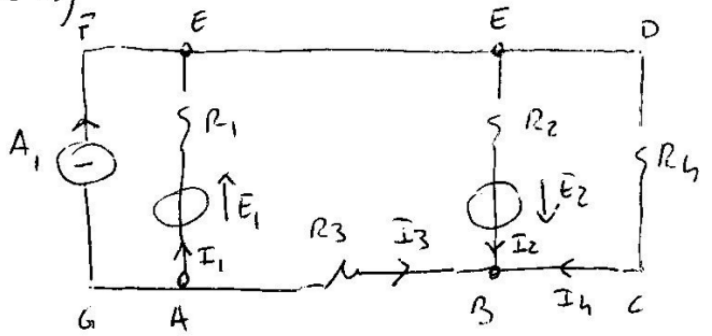


$$E_5 = E_3 + E_4 = 34\,V$$

$$R_5 = R_1 + R_{24} = 3\ K\Omega$$

$$I_3 = -\frac{E_5}{R_3 + R_5} = -\frac{34\,V}{5\,K\Omega} = -6{,}8\,mA$$

1 h)



3 NODI : A B E

3 MAGLIE INDIPENDENTI (2 SENZA GEN. DI CORR.)

LKC NODO A $\quad I_1 + I_3 = -A_1$

LKC NODO B $\quad I_3 + I_2 + I_4 = 0$

LKT MAGLIA AEDCA $\quad E_1 = R_1 I_1 + R_4 I_4 - R_3 I_3$

LKT MAGLIA BCDEB $\quad E_2 = R_2 I_2 - R_4 I_4$

— . — . —

3) $I_3 = -6{,}8\,mA$ (RICAVATO CON METODI 1e, 1f, 1h)

DALLA 1ª EQ. DELL'ES. 1h) SI RICAVA

$$I_1 = -I_3 - A_1 = +6{,}8 - 2 = 4{,}8\,mA$$
$$V_{EA} = E_1 - R_1 I_1 = 20 - 9{,}6 = 10{,}4\,V$$
$$V_{AB} = R_3 I_3 = -2k \cdot 6{,}8m = -13{,}6\,V$$
$$V_{DC} = V_{EA} + V_{AB} = 10{,}4 - 13{,}6 = -3{,}2\,V$$
$$I_4 = \frac{V_{DC}}{R_4} = -1{,}6\,mA$$
$$I_2 = -I_3 - I_4 = 6{,}8 + 1{,}6 = 8{,}4\,mA$$
$$V_{A_1} = V_{EA} = 10{,}4\,V$$

POT. DISS.

$P(R_1) = R_1 I_1^2 = 46{,}08\,mW$

$P(R_2) = R_2 I_2^2 = 141{,}12\,mW$

$P(R_3) = R_3 I_3^2 = V_{AB} \cdot I_3 = 92{,}48\,mW$

$P(R_4) = R_4 I_4^2 = 5{,}12\,mW$

$P_{EROG.}$

$P(E_1) = E_1 I_1 = 96\,mW$

$P(E_2) = E_2 I_2 = 168\,mW$

$P(A_1) = V_{A_1} \cdot A_1 = 10{,}4 \cdot 2m = 20{,}8\,mW$

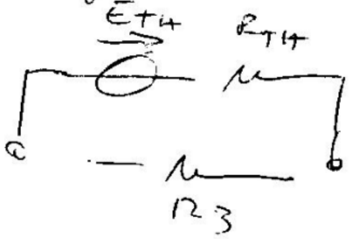
$P_{DISS} = P_{R_1} + P_{R_2} + P_{R_3} + P_{R_4} = 284{,}8\,mW$

$P_{EROG} = P_{E_1} + P_{E_2} + P_{A_1} = 284{,}8\,mW$

$$P_{ASS} = P_{DISS} = P_{EROG}$$

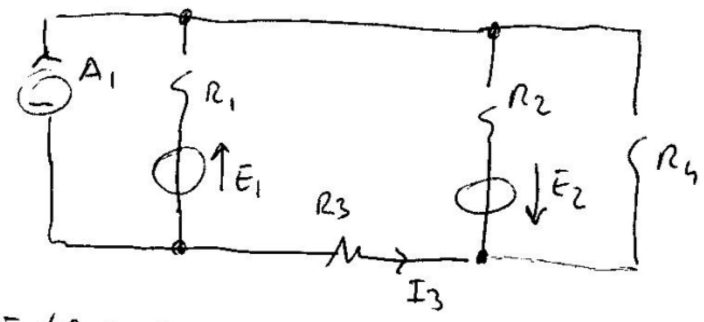
OSSERVAZIONI

ES 1e Il teorema di Thévenin richiede la determinazione del circuito equivalente serie ai capi dell'elemento circuitale attraversato dalla corrente da determinare o ai capi del quale si trova la tensione richiesta (o la potenza) -
Una volta determinato il circ. equiv., questo può essere utilizzato in sostituzione di tutta la rete collegata al modello elemento - Ma a questo punto i calcoli sono praticamente immediati -

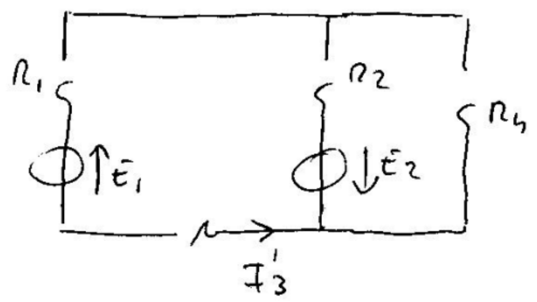


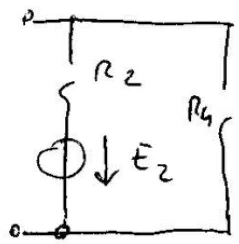
ES. 1f. Il principio di sovrapposizione degli effetti richiede di determinare "separatamente" il contributo di ciascun generatore alla grandezza da calcolare - In questo modo i circuiti "parziali" da analizzare risultano + semplici - Alla fine si tratterà di sommare i vari contributi -
In alcuni casi può essere anche conveniente considerare congiuntamente l'effetto di + generatori (soprattutto se ciò non complica troppo la topologia della rete da analizzare) l'importante è che alla fine si considerino tutti i generatori e nessuno più di una volta -

Soluzione alternativa dell'es. 1f con generatori di tensione considerati congiuntamente _



✱ $E_1 \neq 0$ $E_2 \neq 0$ $A_1 = 0$

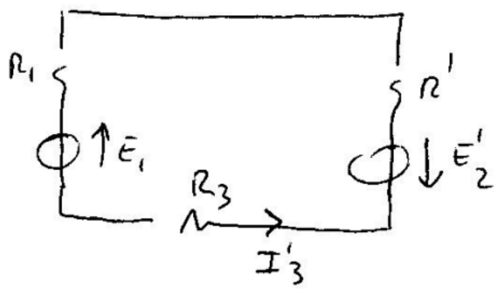




⇒

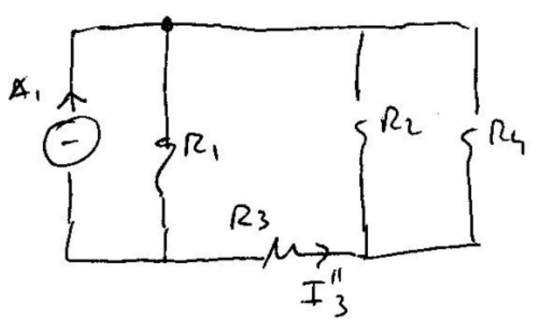
$R' = R_2 \parallel R_4 = 1K$

$E_2' = E_2 \cdot \frac{R_4}{R_2 + R_4} = 20 \cdot \frac{2K}{4K} = 10V$

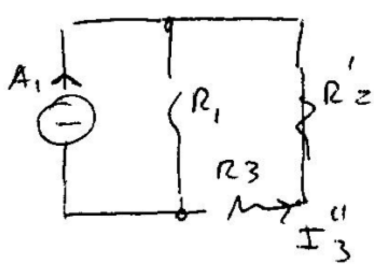


$$I_3' = -\frac{E_1 + E_2'}{R_1 + R' + R_3} = -\frac{20+10}{(2+1+2)K} = -6\,mA$$

✱ $E_1 = 0$ $E_2 = 0$ $A_1 \neq 0$



⇒

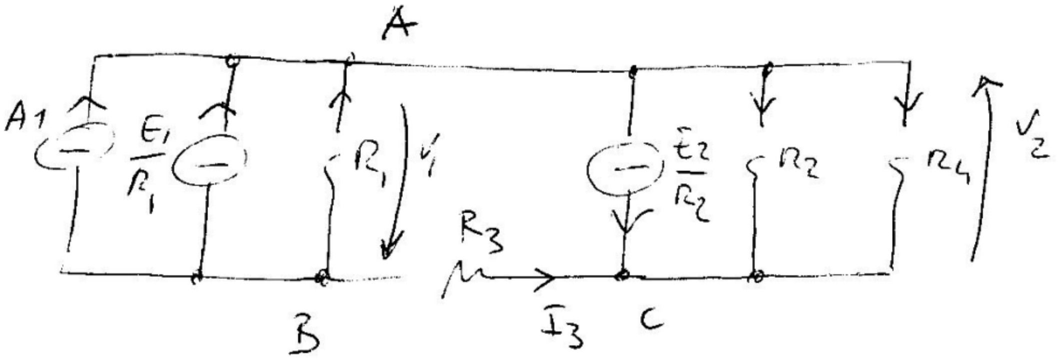


$R' = R_2 \parallel R_4 = 1K$

P.C. _ $$I_3'' = -A_1 \frac{R_1}{R_1 + R_3 + R'} = -2\,mA \cdot \frac{2K}{5K} = -0{,}8\,mA$$

$$I_3 = I_3' + I_3'' = -6\,mA - 0{,}8\,mA = -6{,}8\,mA.$$

per il punto h.



Fatto la trasformazione dei generatori E_1 ed E_2 con relative resistenze serie negli equivalenti generatori di corrente,

le eq. di Kirchhoff risolventi sono

- $$A_1 + \frac{E_1}{R_1} + \frac{V_1}{R_1} = \frac{E_2}{R_2} + \frac{V_2}{R_2} + \frac{V_2}{R_4}$$ (NODO A)

- $$A_1 + \frac{E_1}{R_1} + \frac{V_1}{R_1} = -I_3$$ (NODO B)

- $$V_1 + V_2 = R_3 I_3$$ (MAGLIA ABC)