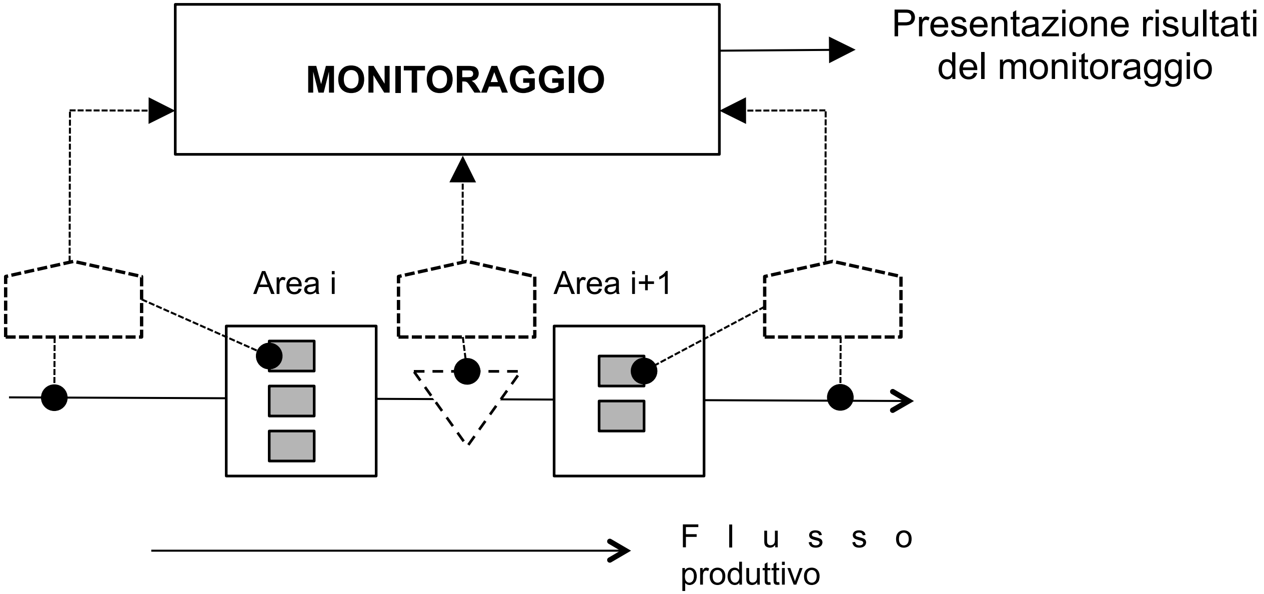
Analisi della prestazione dei sistemi produttivi – Legge di Little

Analisi delle prestazioni

- Per analisi (o anche monitoraggio) delle prestazioni di un sistema composto da risorse indipendenti, ma a consumo interdipendente (come ad es. un sistema produttivo che produce un insieme di pezzi diversi, in cui ciascuno richiede le risorse del sistema secondo un ciclo prestabilito), si intende lo studio dei legami fra le variabili di prestazione del sistema,avendo imposto determinate condizioni al contorno
- I metodi di analisi delle prestazioni sono di particolare importanza sia come strumenti di valutazione delle prestazioni prevedibili per un nuovo sistema, che per analizzare i legami fra i dati delle misure di prestazione a consuntivo di sistemi esistenti
- L'analisi delle prestazioni di un sistema complesso può essere effettuata con metodi diversi: alcuni molto semplici, ma approssimati; altri molto precisi e dettagliati (simulazione), ma onerosi da applicare
- Tra i più semplici e sintetici metodi di interpretazione del funzionamento di questi sistemi vi sono la legge di Little e l'analisi di carico (detta anche analisi di input/output o diagrammi di troughtput)

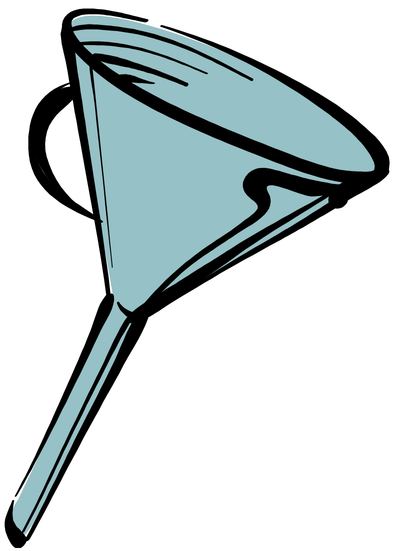
Analisi delle prestazioni

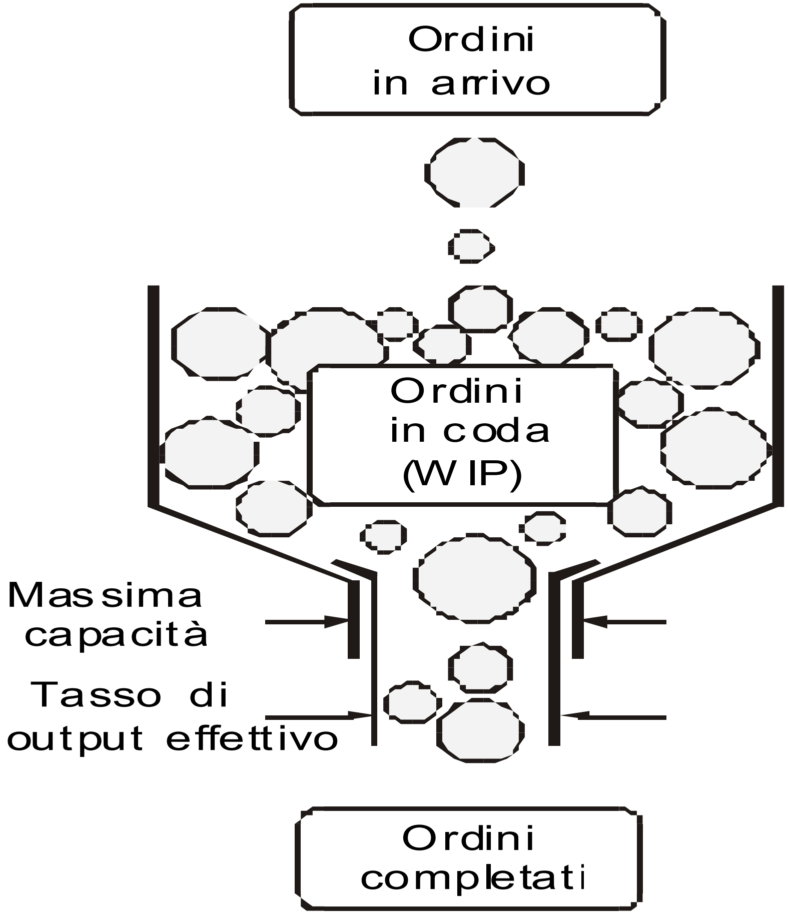
- Si misurano le prestazioni caratteristiche del sistema
- Con il monitoraggio si può
  - Ottenere informazioni in tempo reale sul sistema
  - Fare un'analisi dello storico di funzionamento
  - Calcolare e analizzare l'andamento di opportuni indici di prestazione



Analisi delle prestazioni

- Analogia del Funnel Model





Relazione di base

- **WIP critico (*WIP**):** livello di WIP per il quale un sistema produttivo in stato stazionario raggiunge il massimo throughput (THmax) con il minimo tempo di attraversamento (LTmin)

$$\boldsymbol{WIP^* = \text{THmax} \times \text{LTmin}}$$

The Penny Fab

- Caratteristiche:
  - 4 macchine identiche in serie
  - Tempo di processamento 2 h per pezzo (penny) in ogni macchina
  - Non esiste variabilità
  - I pezzi sono inseriti nel sistema in moda da mantenere il WIP costante (CONWIP)
    - Quando un penny è finito, allora può essere caricato un nuovo penny da lavorare nel sistema

- Parametri:

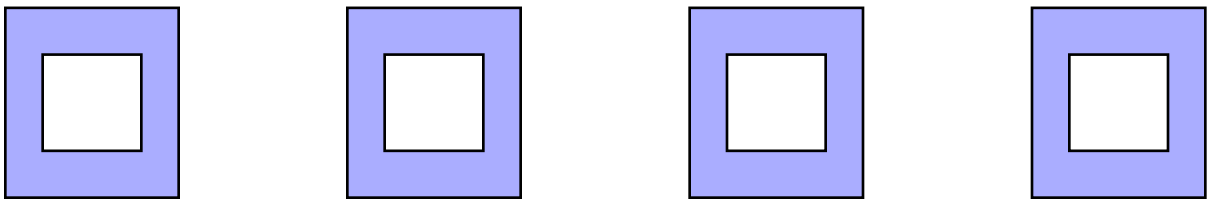
  THcb = 0,5 penny/h

  LTmin = 8 ore

  WIP critico WIP*= 0,5 x 8 ore = 4 penny

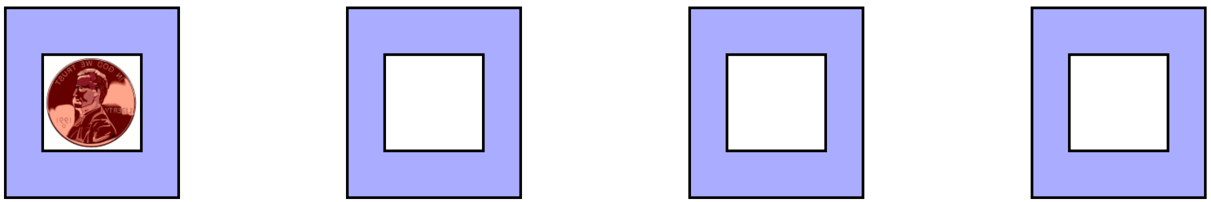
  Non esiste variabilità

The Penny Fab



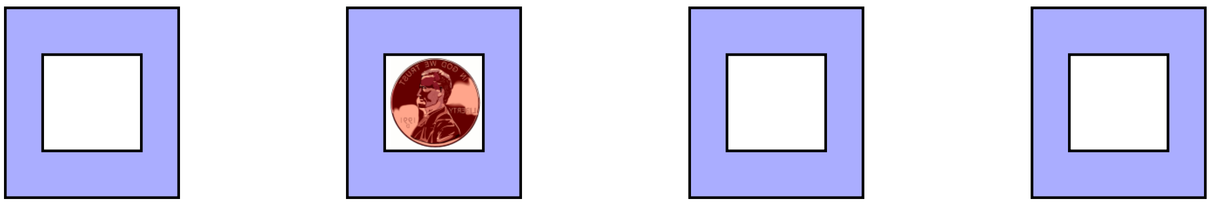
- Factory Physics, Wallace J. Hopp and Mark L. Spearman, Irwin/McGraw-Hill, 1996
  - Penny Fab represents a simple production line that makes **giant one-cent pieces** used extensively in Fourth of July parades
  - The line consists of four machines in sequence
    - The first machine is a punch press that cuts penny blanks, the second stamps Lincoln's face on one side and the Lincoln Memorial on the back, the third puts a rim on the penny, and the fourth cleans away any burrs
    - After each penny is processed, it is moved immediately to the next machine
  - The line runs 24 hours per day, with breaks and lunches covered by spare operators
  - The market for giant pennies is assumed to be unlimited, so that all product made is sold; thus, more throughput is unambiguously better for this system

The Penny Fab (WIP=1)



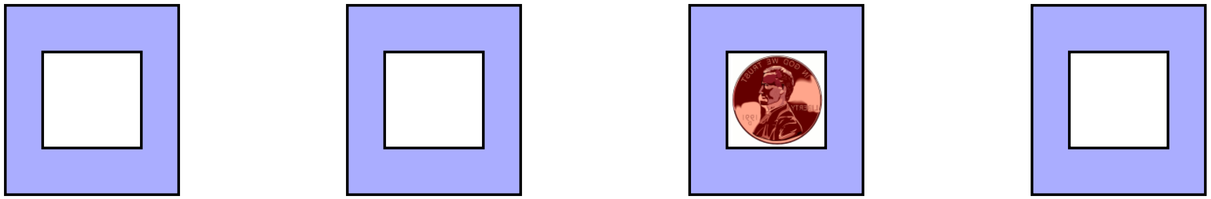
Time = 0 hours

The Penny Fab (WIP=1)



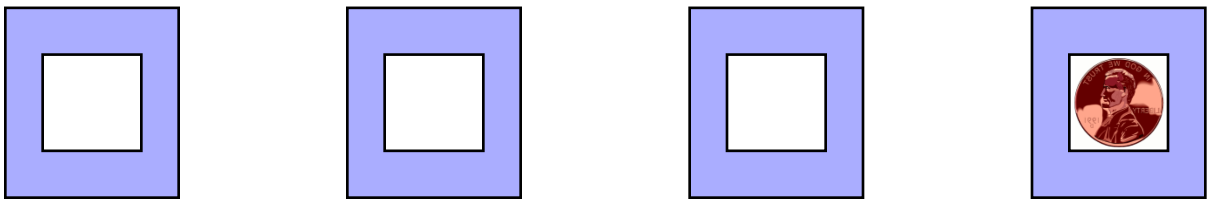
Time = 2 hours

The Penny Fab (WIP=1)



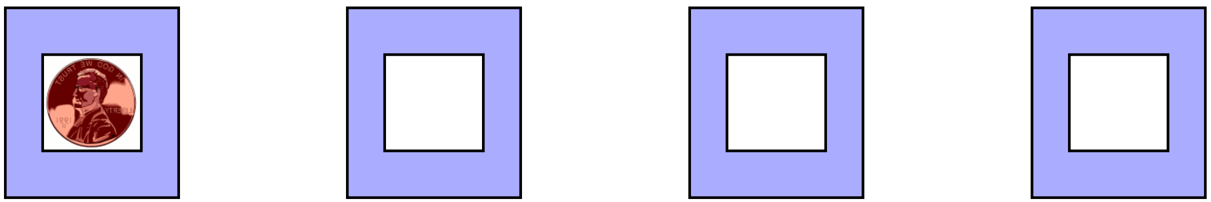
Time = 4 hours

The Penny Fab (WIP=1)



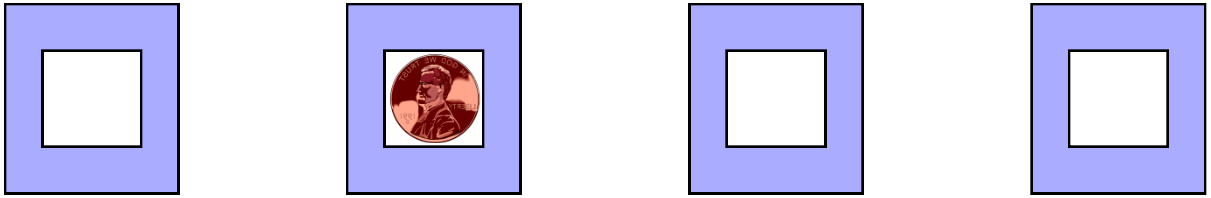
Time = 6 hours

The Penny Fab (WIP=1)



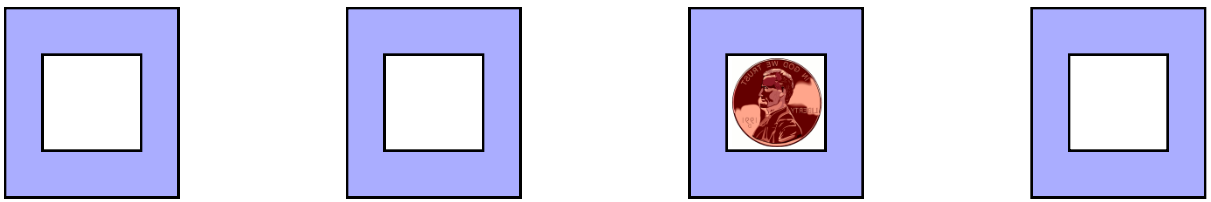
Time = 8 hours

The Penny Fab (WIP=1)



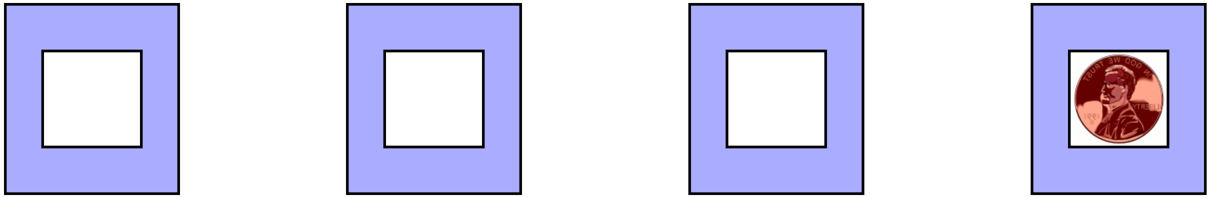
Time = 10 hours

The Penny Fab (WIP=1)

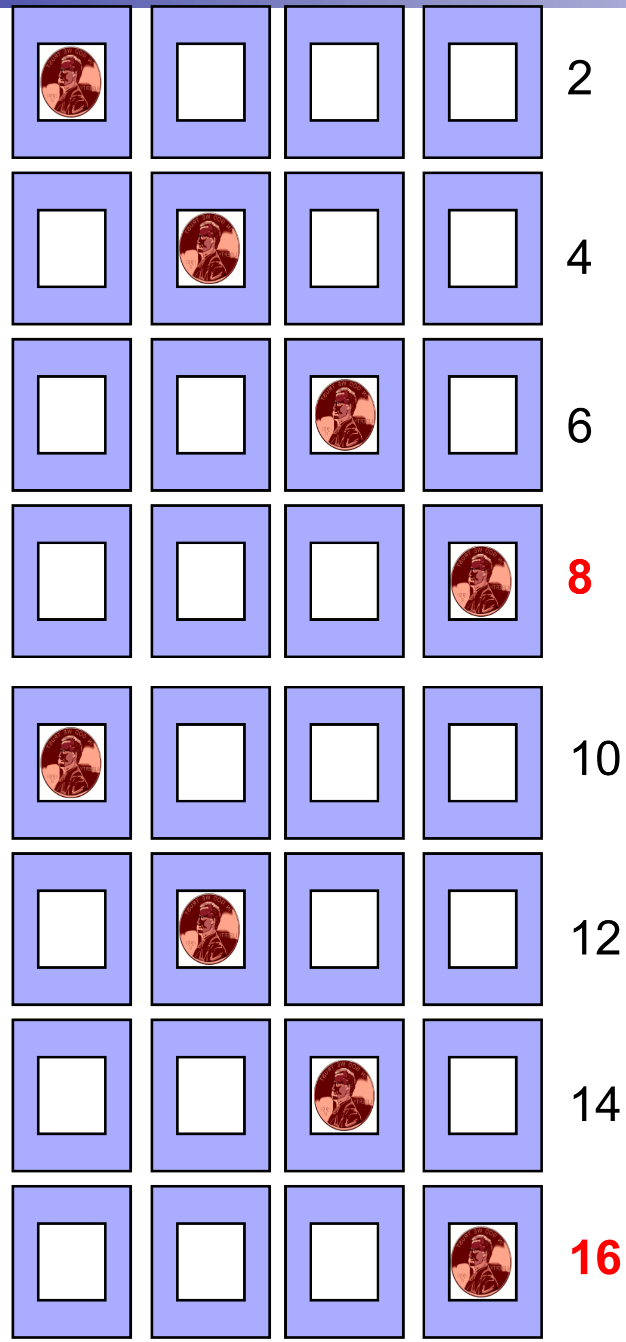


Time = 12 hours

The Penny Fab (WIP=1)



Time = 14 hours

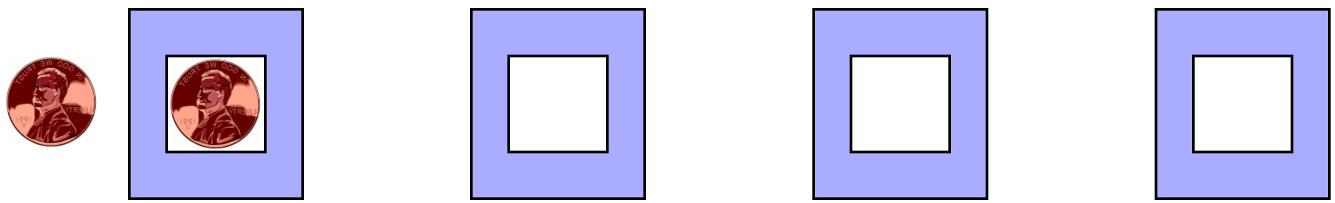


2 MONETE IN 16 ORE =
2/16 = 0,125

Performance con WIP = 1

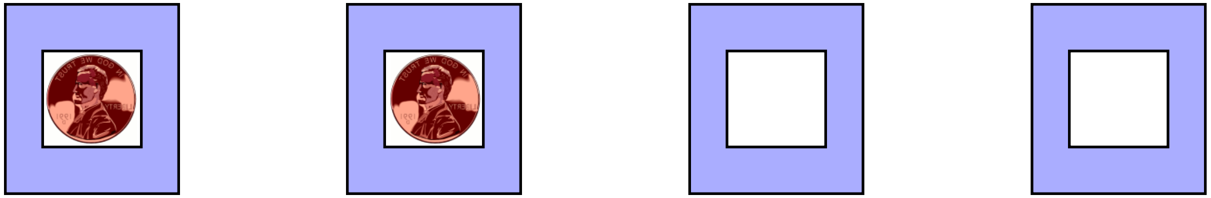
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2			
3			
4			
5			
6			

The Penny Fab (WIP=2)



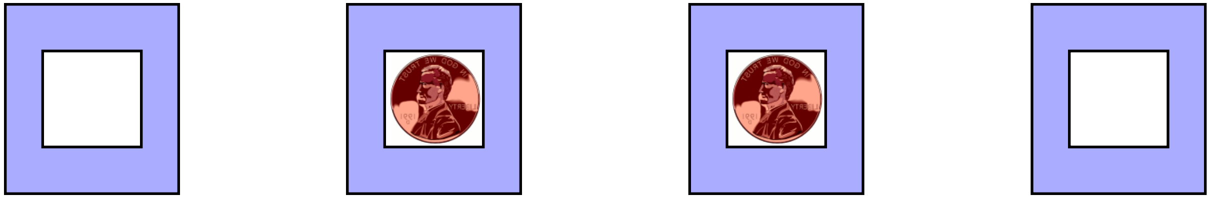
Time = 0 hours

The Penny Fab (WIP=2)



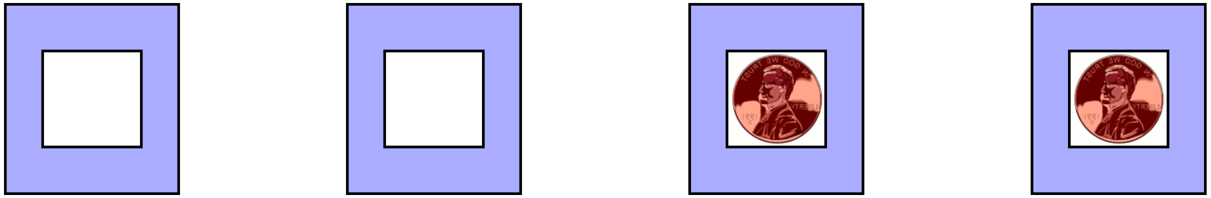
Time = 2 hours

The Penny Fab (WIP=2)



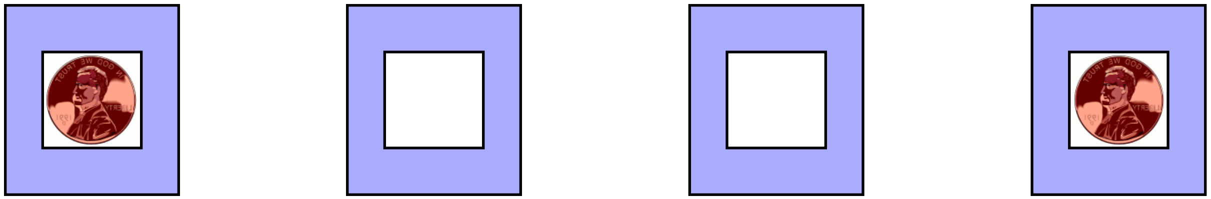
Time = 4 hours

The Penny Fab (WIP=2)



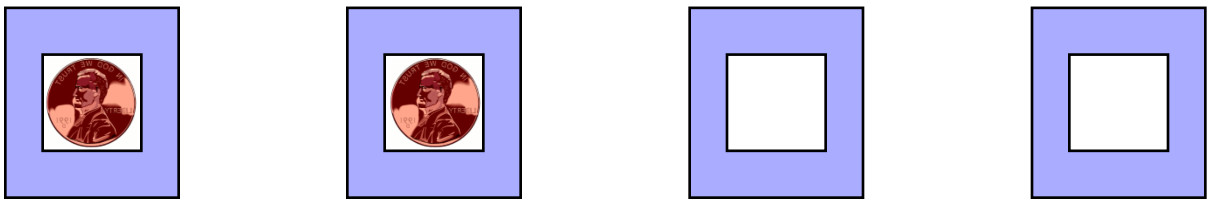
Time = 6 hours

The Penny Fab (WIP=2)



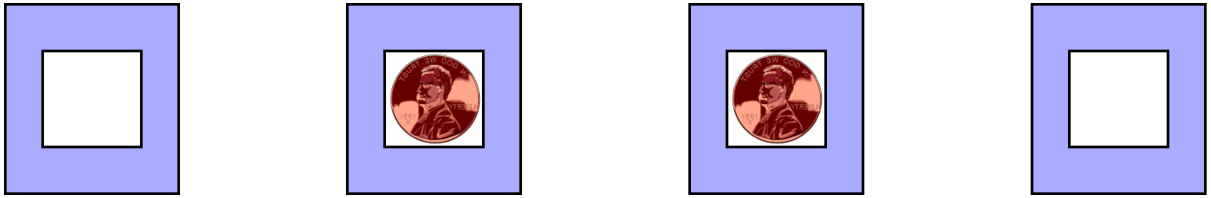
Time = 8 hours

The Penny Fab (WIP=2)



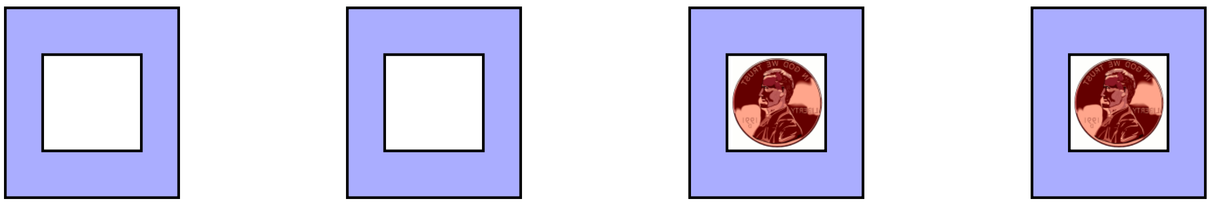
Time = 10 hours

The Penny Fab (WIP=2)

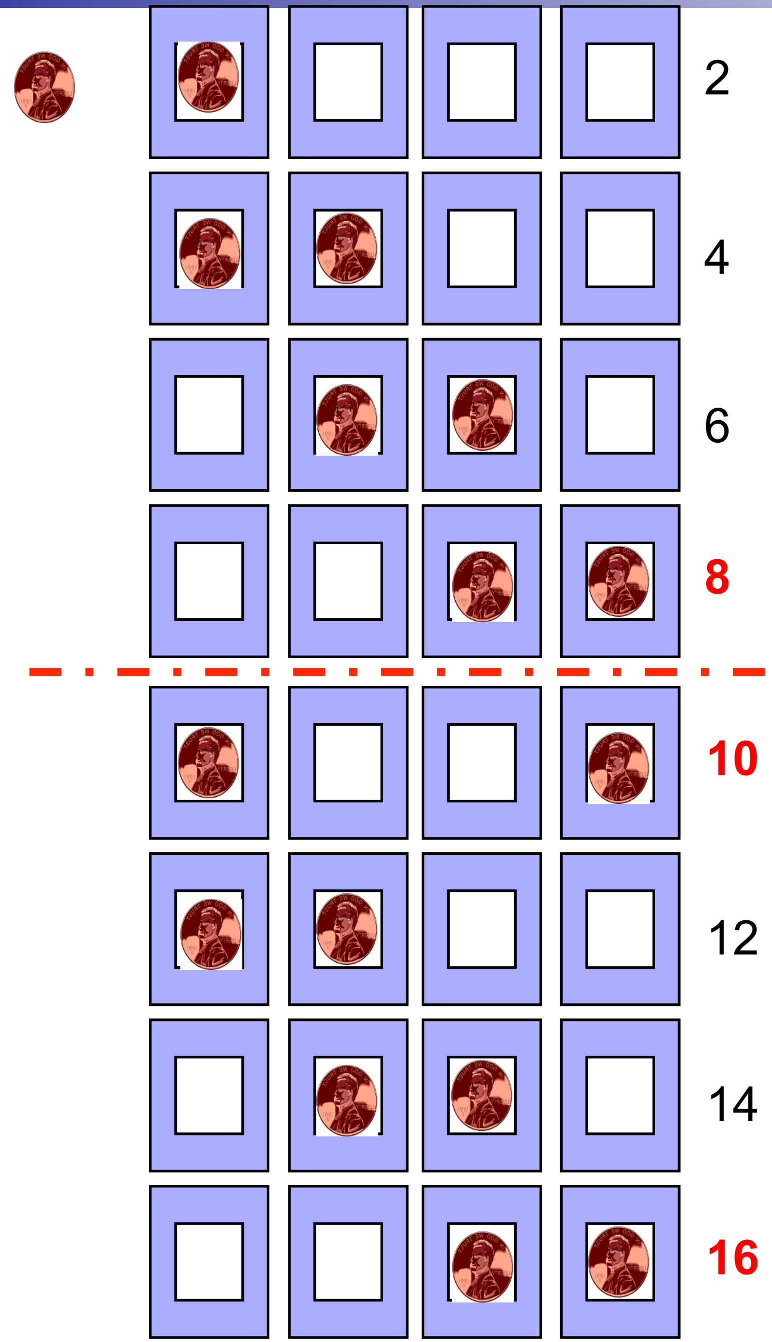


Time = 12 hours

The Penny Fab (WIP=2)



Time = 14 hours

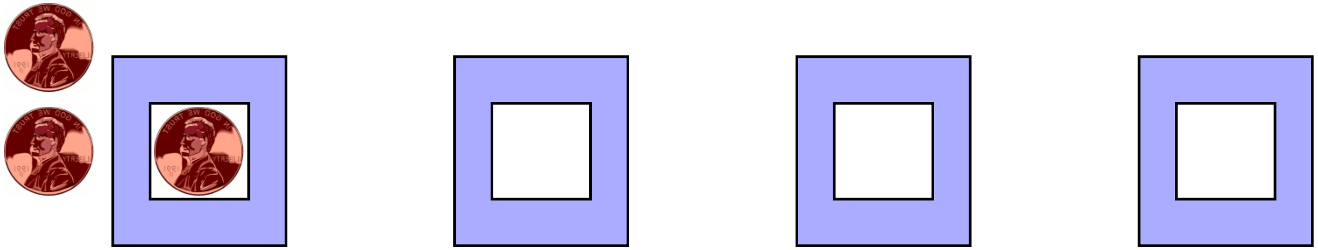


Dopo la fase di transitorio (riempimento del sistema) 2 MONETE IN 8 ORE = 2/8 = 0,250

Performance con WIP = 2

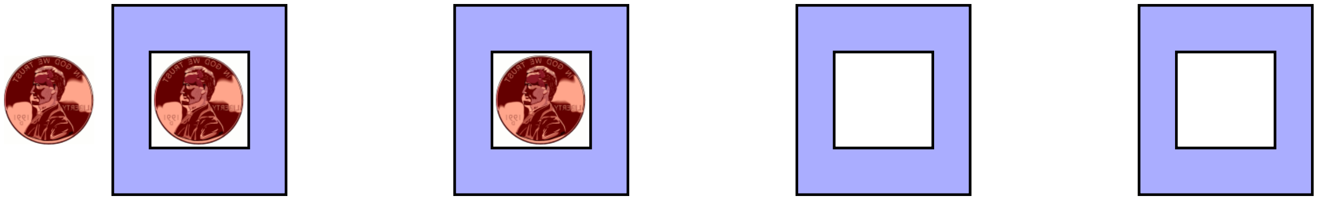
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3			
4			
5			
6			

The Penny Fab (WIP=3)



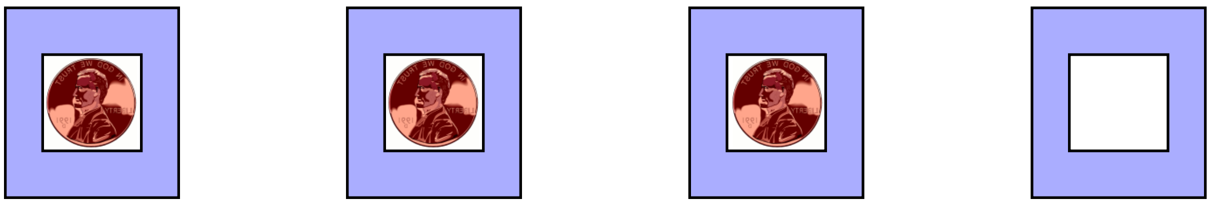
Time = 0 hours

The Penny Fab (WIP=3)



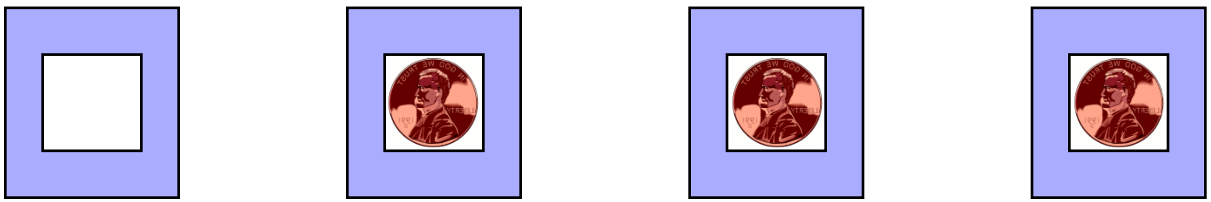
Time = 2 hours

The Penny Fab (WIP=3)



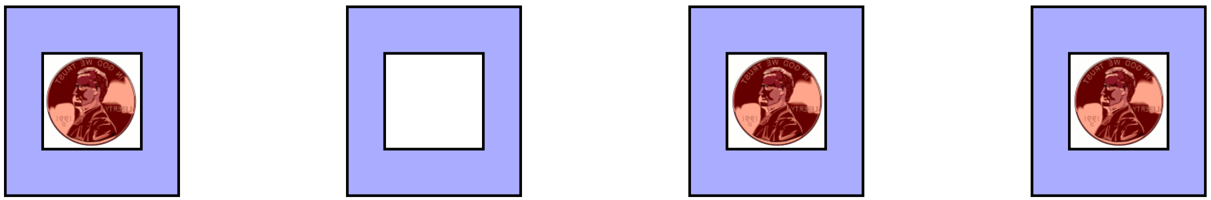
Time = 4 hours

The Penny Fab (WIP=3)



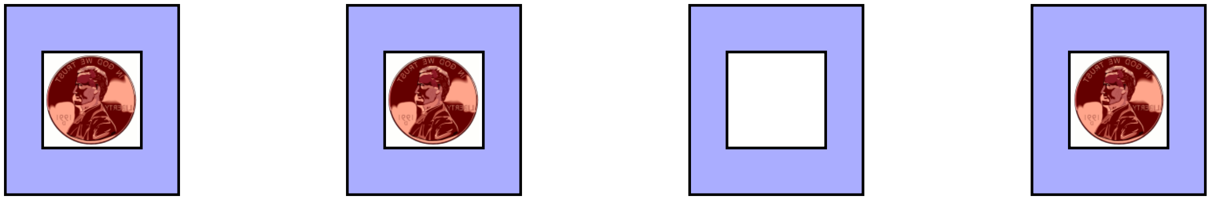
Time = 6 hours

The Penny Fab (WIP=3)



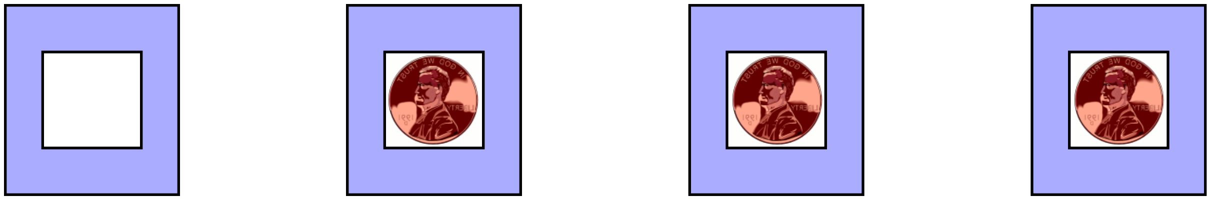
Time = 8 hours

The Penny Fab (WIP=3)

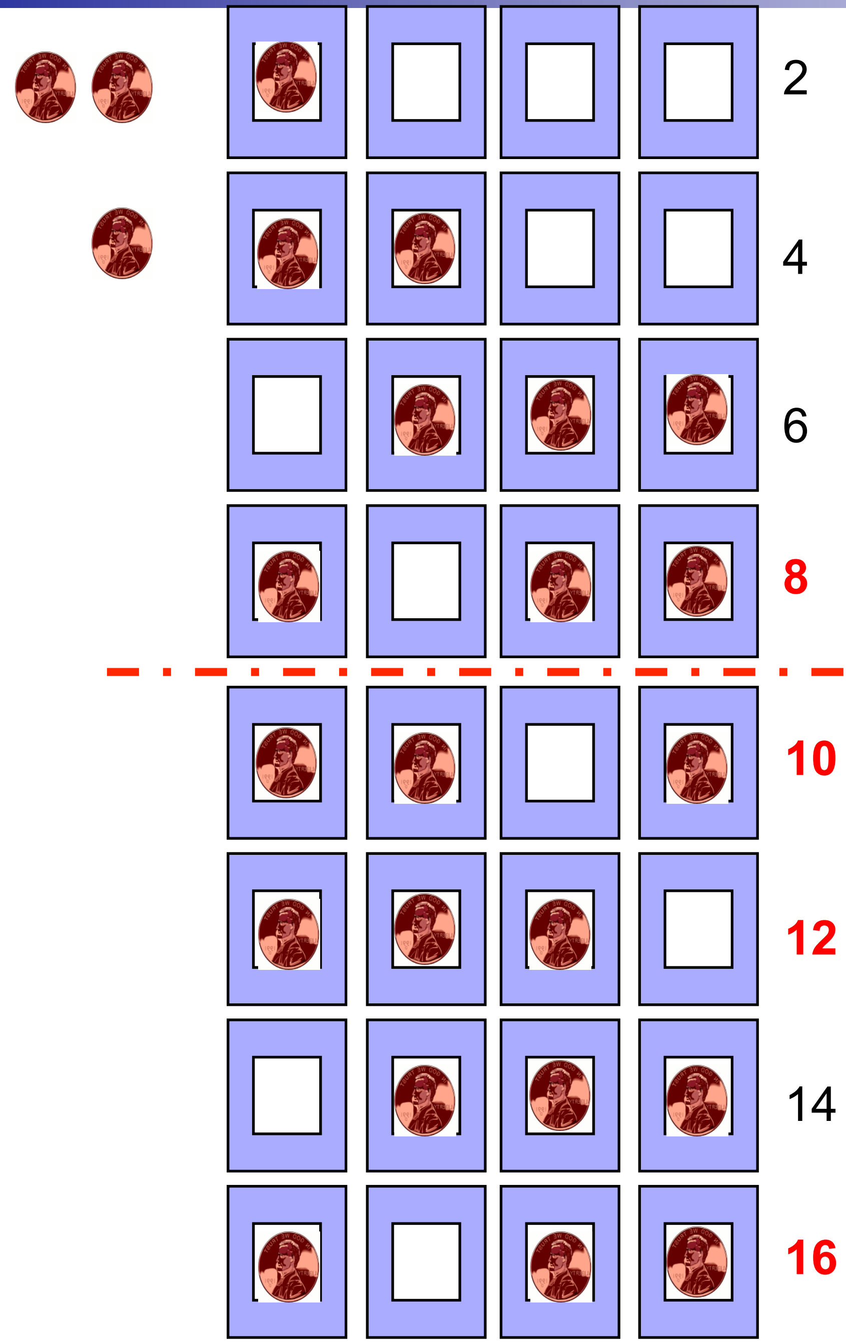


Time = 10 hours

The Penny Fab (WIP=3)



Time = 12 hours

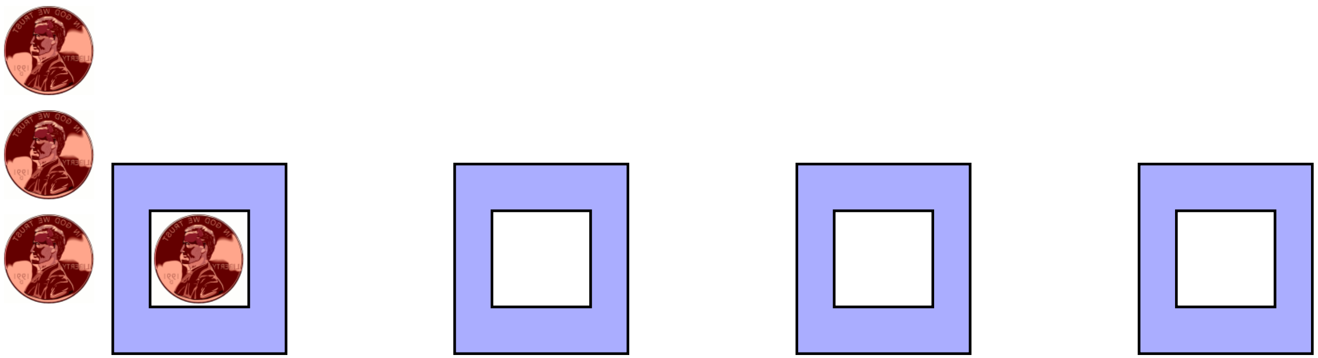


Dopo la fase di transitorio
(riempimento del sistema)
3 MONETE IN 8 ORE =
3/8 = 0,375

Performance con WIP = 3

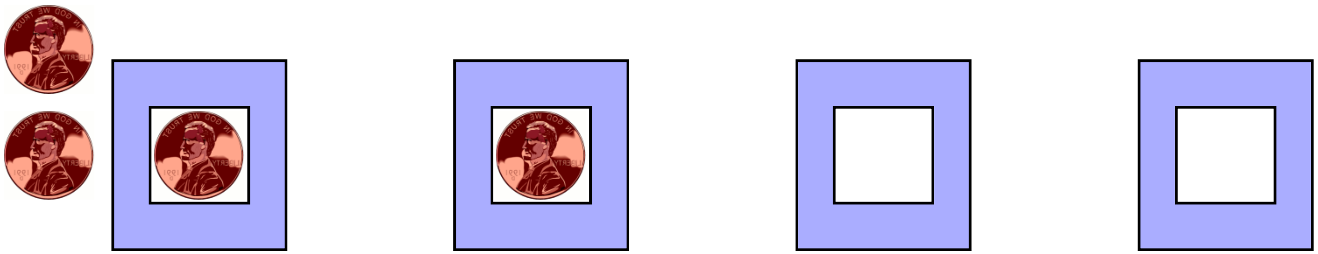
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4			
5			
6			

The Penny Fab (WIP=4)



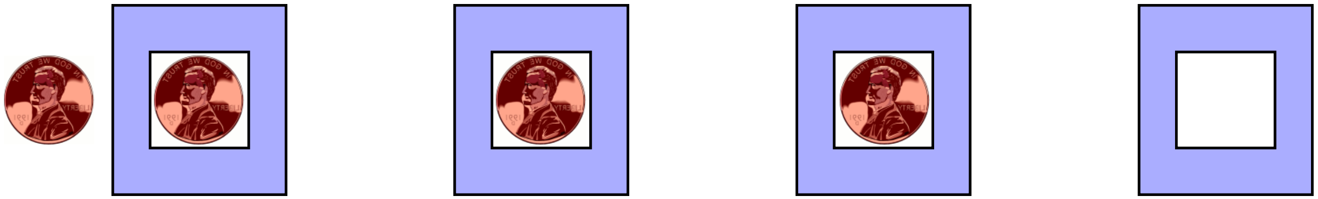
Time = 0 hours

The Penny Fab (WIP=4)



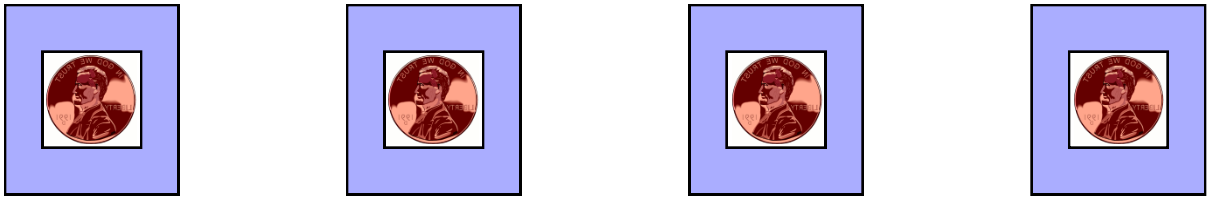
Time = 2 hours

The Penny Fab (WIP=4)



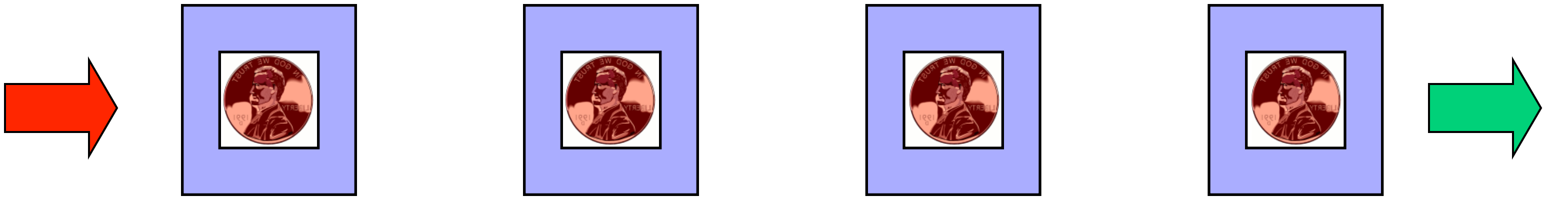
Time = 4 hours

The Penny Fab (WIP=4)



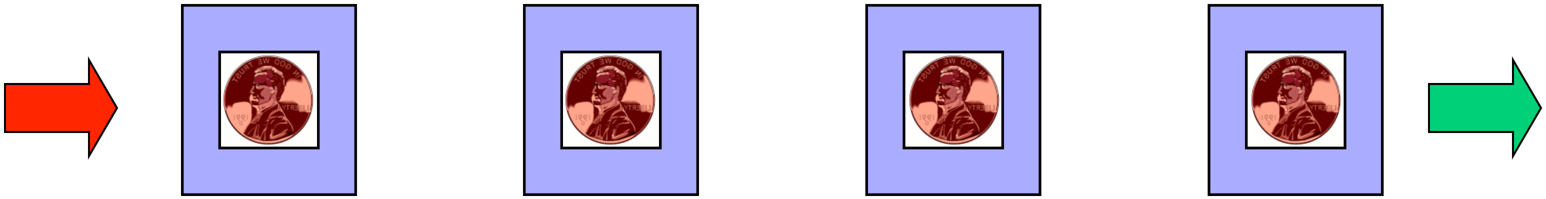
Time = 6 hours

The Penny Fab (WIP=4)



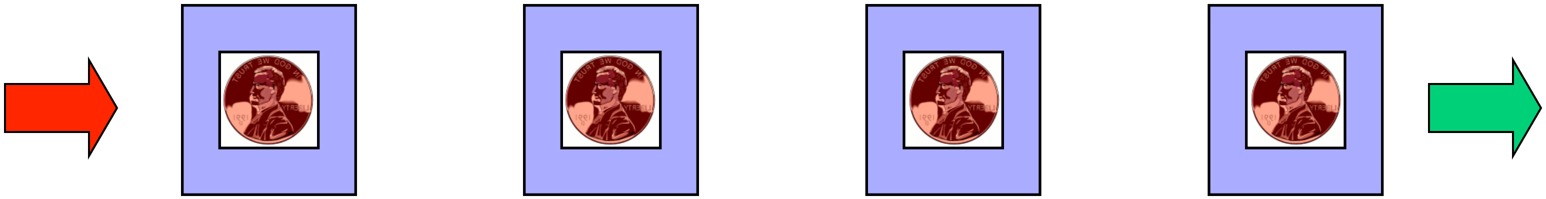
Time = 8 hours

The Penny Fab (WIP=4)



Time = 10 hours

The Penny Fab (WIP=4)

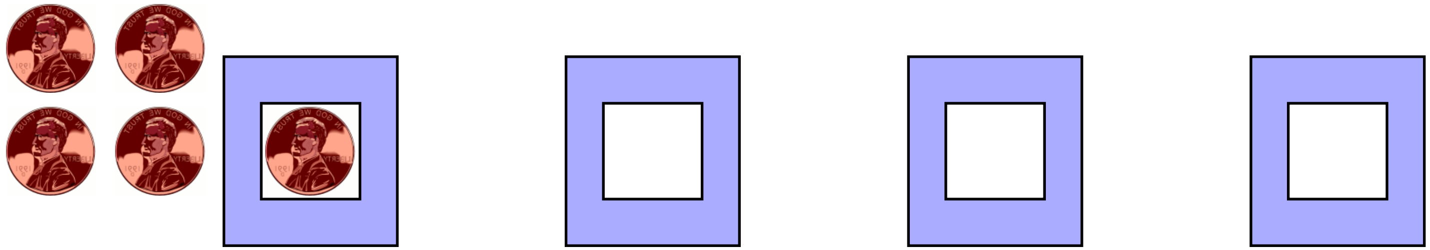


Time = 12 hours

Performance con WIP = 4

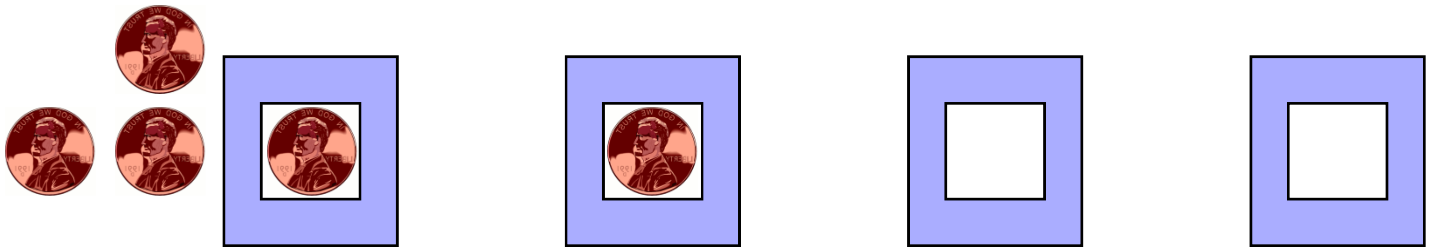
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5			
6			

The Penny Fab (WIP=5)



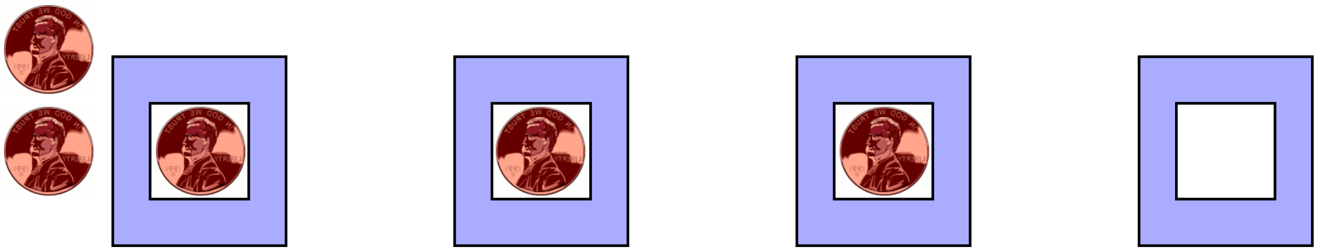
Time = 0 hours

The Penny Fab (WIP=5)



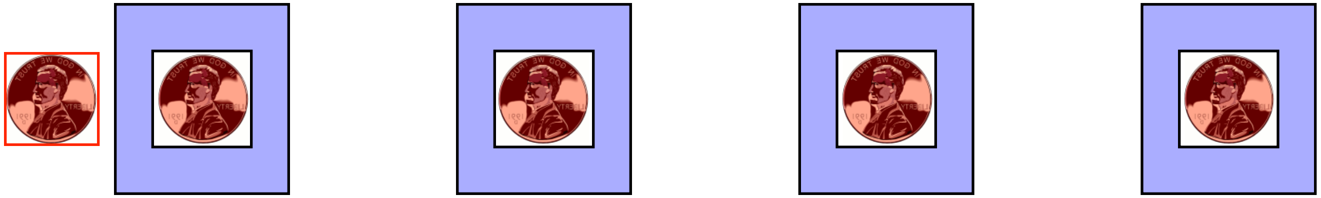
Time = 2 hours

The Penny Fab (WIP=5)



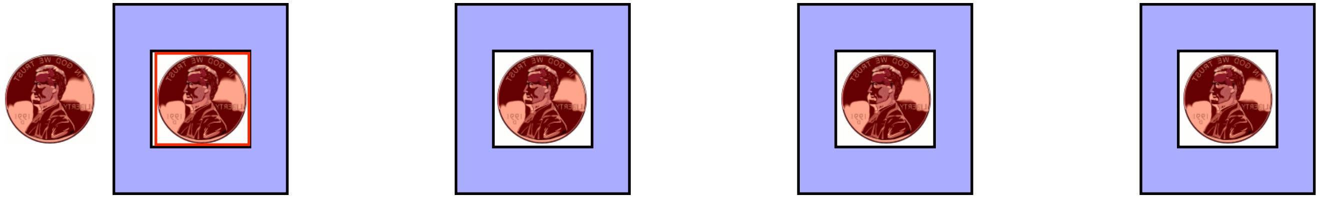
Time = 4 hours

The Penny Fab (WIP=5)



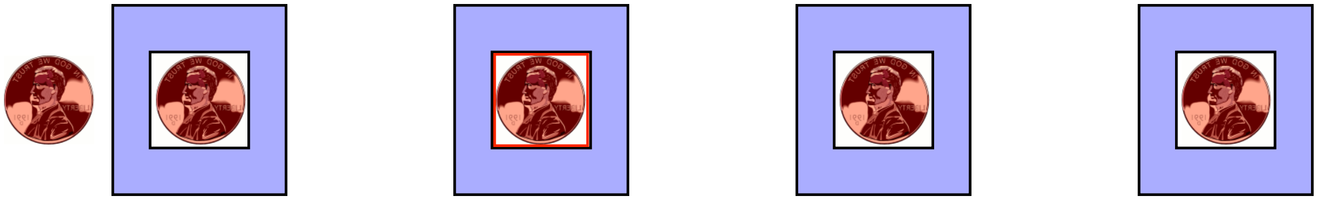
Time = 6 hours

The Penny Fab (WIP=5)



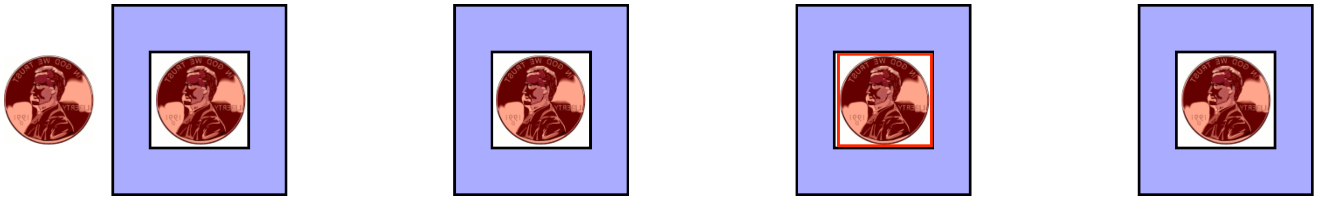
Time = 8 hours

The Penny Fab (WIP=5)



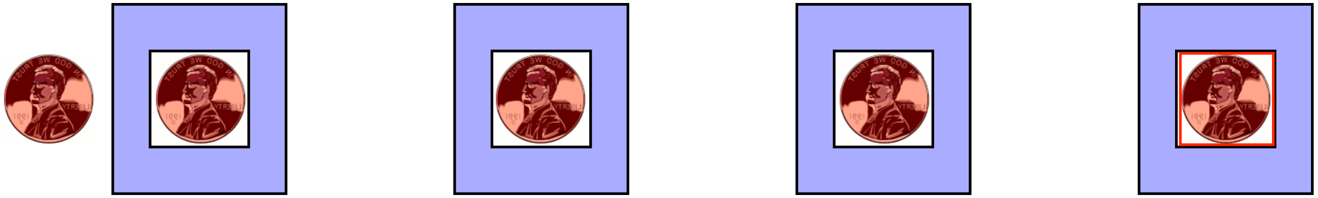
Time = 10 hours

The Penny Fab (WIP=5)



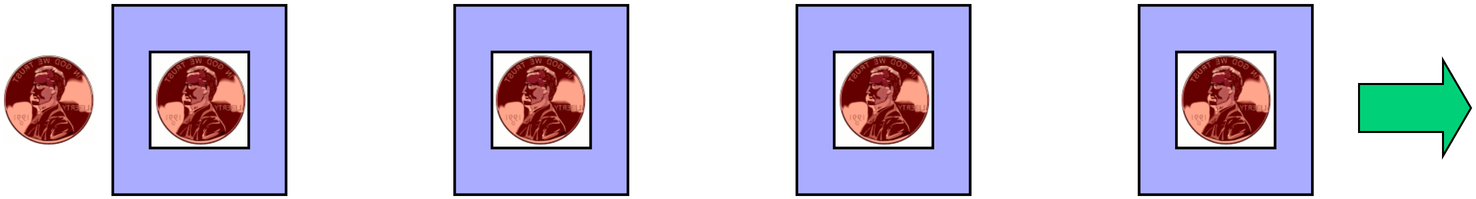
Time = 12 hours

The Penny Fab (WIP=5)



Time = 14 hours

The Penny Fab (WIP=5)

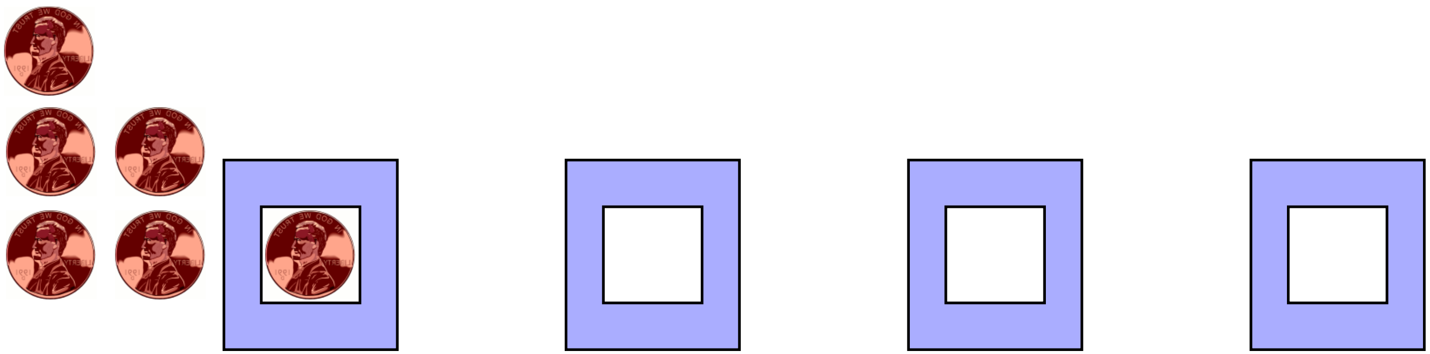


Time = 16 hours

Performance con WIP = 5

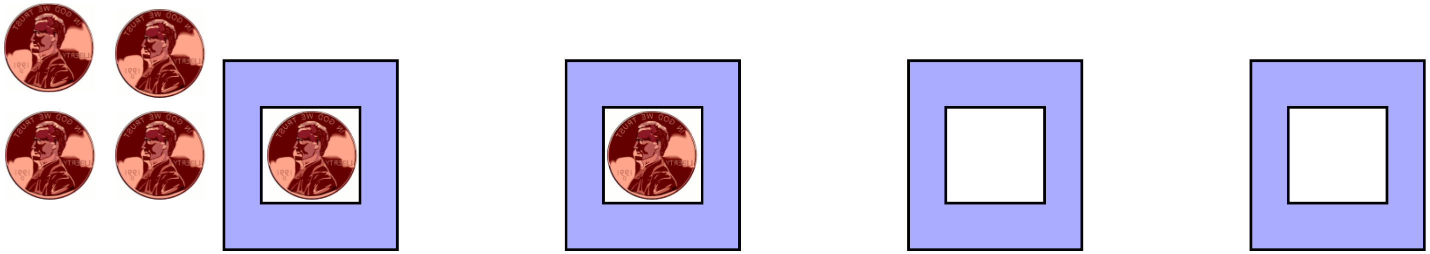
WIP [pz]	**TH** [pz/h]	**LT** [h]	**THxLT** [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5	0,500	10	5
6			

The Penny Fab (WIP=6)



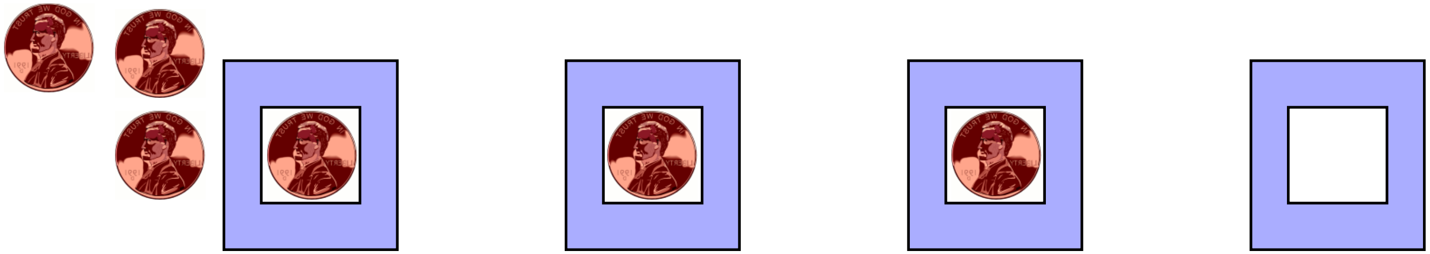
Time = 0 hours

The Penny Fab (WIP=6)



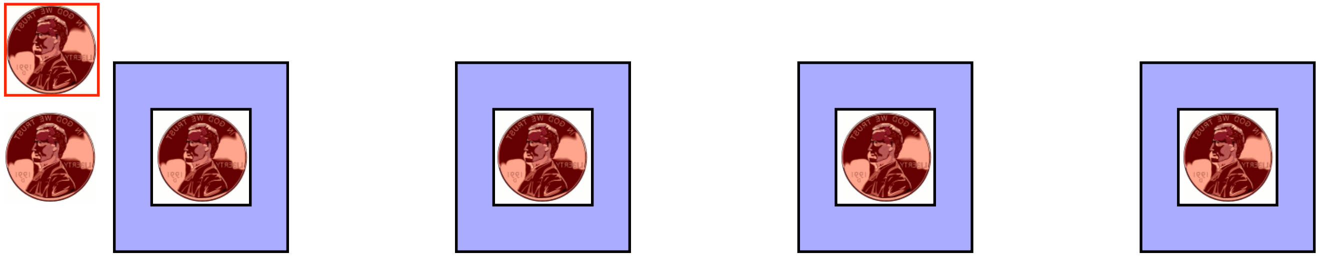
Time = 2 hours

The Penny Fab (WIP=6)



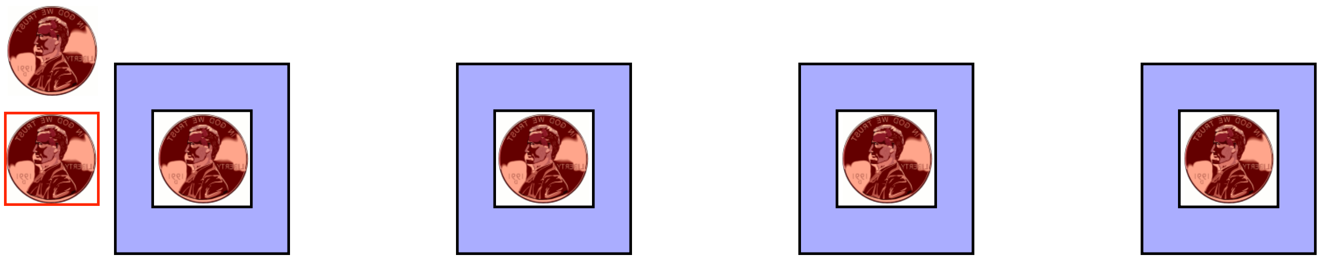
Time = 4 hours

The Penny Fab (WIP=6)



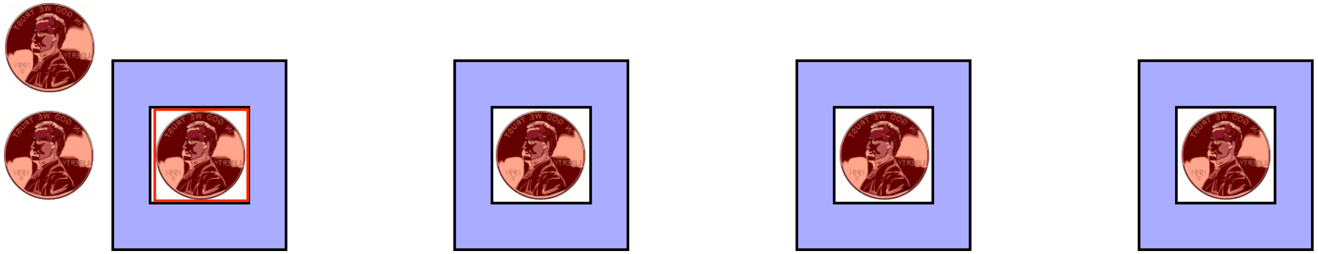
Time = 6 hours

The Penny Fab (WIP=6)



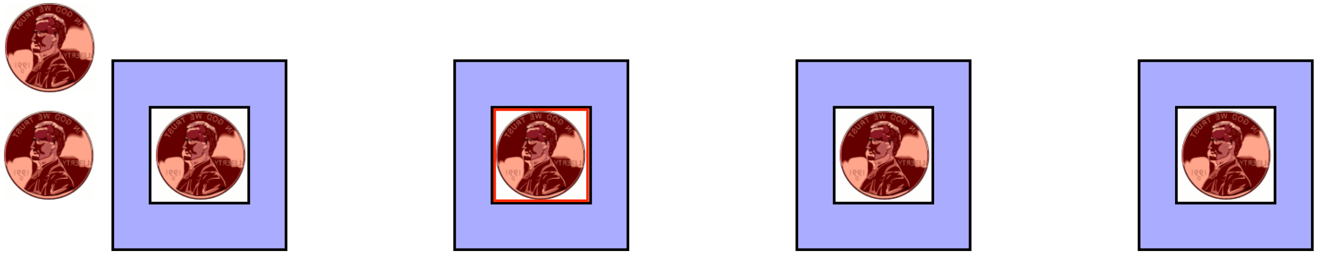
Time = 8 hours

The Penny Fab (WIP=6)



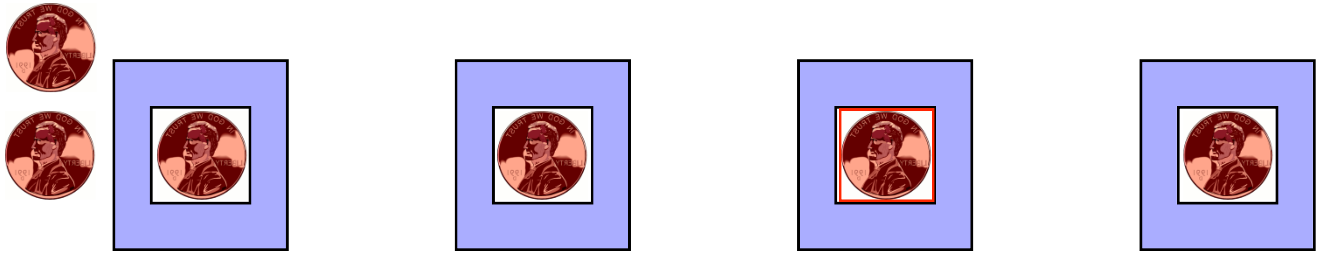
Time = 10 hours

The Penny Fab (WIP=6)



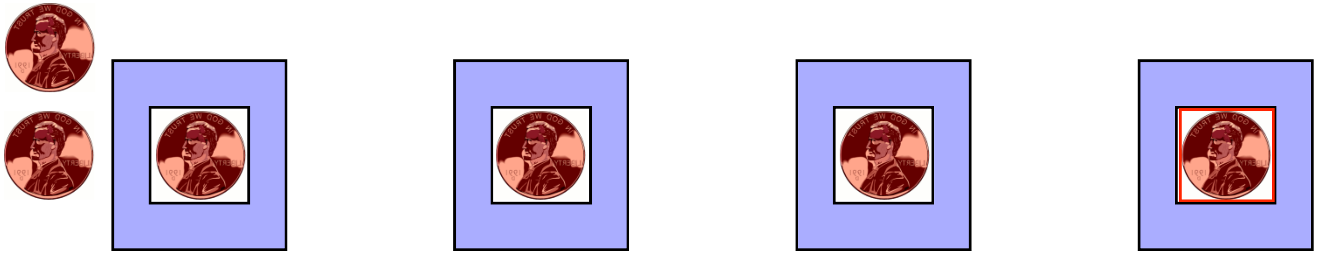
Time = 12 hours

The Penny Fab (WIP=6)



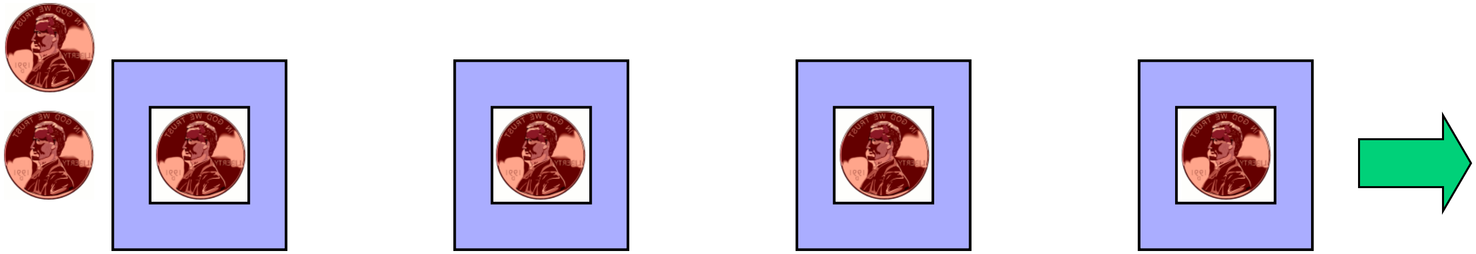
Time = 14 hours

The Penny Fab (WIP=6)



Time = 16 hours

The Penny Fab (WIP=6)

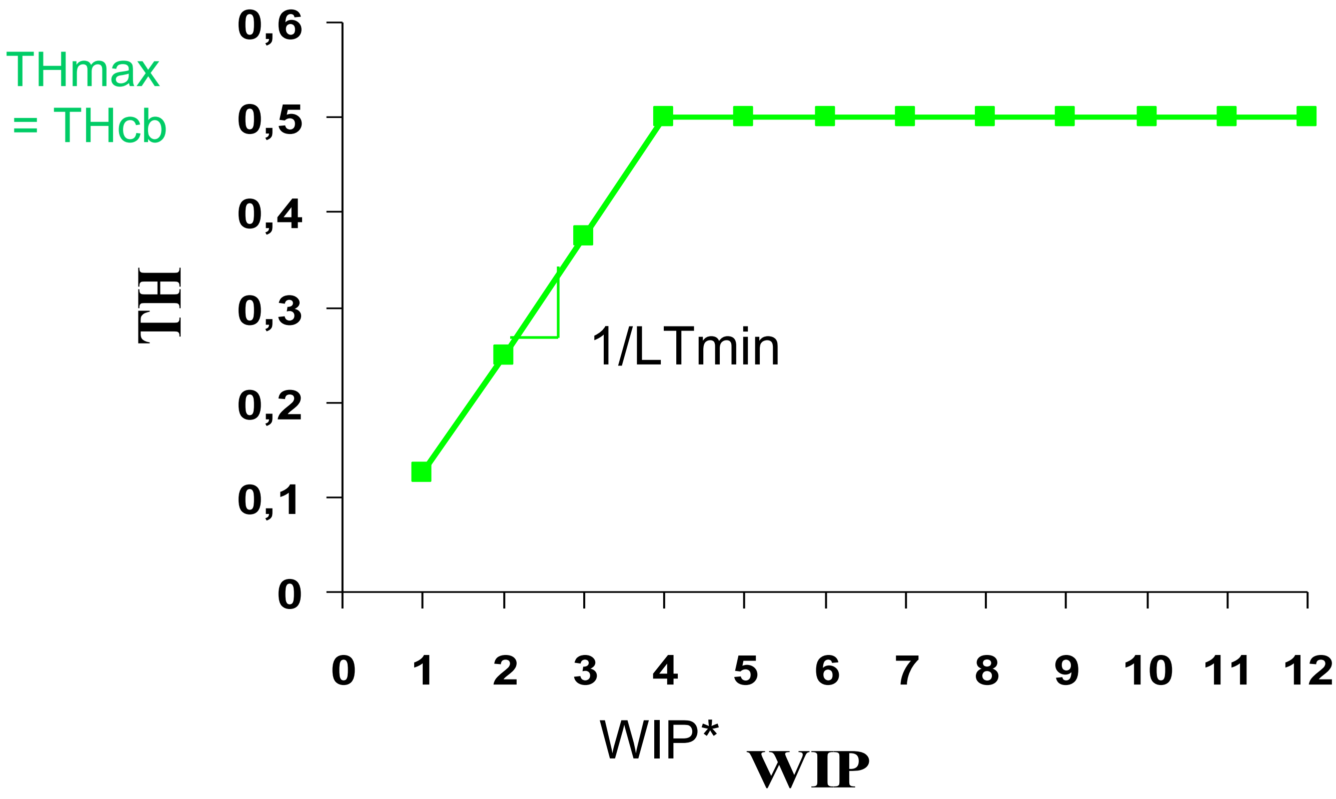


Time = 18 hours

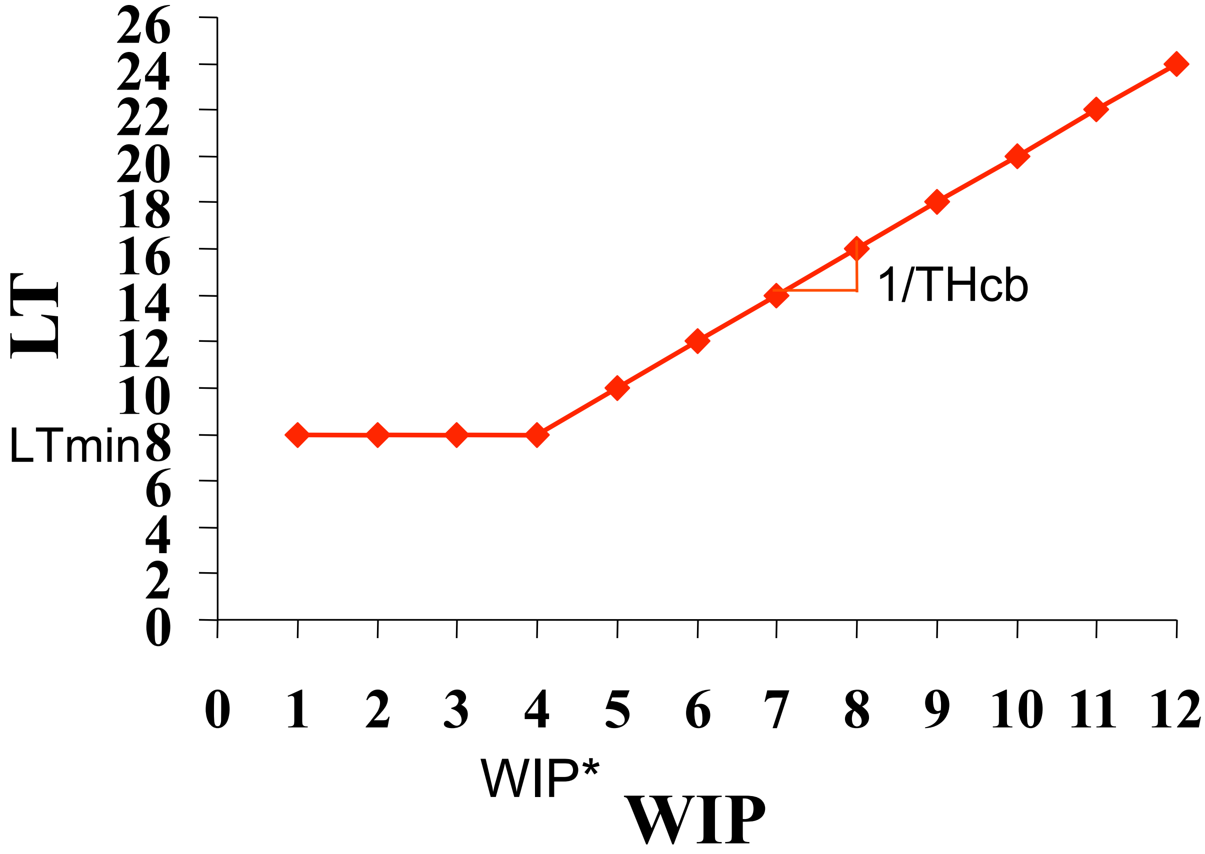
Performance con WIP = 6

WIP [pz]	TH [pz/h]	LT [h]	THxLT [pz]
1	0,125	8	1
2	0,250	8	2
3	0,375	8	3
4	0,500	8	4
5	0,500	10	5
6	0,500	12	6

TH vs. WIP


THmax
= THcb
0,6
0,5
0,4
0,3
0,2
0,1
0
TH
1/LTmin
0 1 2 3 4 5 6 7 8 9 10 11 12
WIP*
WIP

LT vs. WIP


LT
26
24
22
20
18
16
14
12
10
LTmin8
6
4
2
0
1/THcb
0 1 2 3 4 5 6 7 8 9 10 11 12
WIP*
WIP

THmax e LTmin

Il LTmin è naturalmente la somma dei minimi tempi di processamento necessari a realizzare il primo penny!

- Il minimo LT (LTmin) per un dato livello di WIP è

$$LT_{min} = \begin{cases} \sum Tprocesso, & \text{if } wip \leq WIP^* \\ wip / TH_{max}, & \text{altrimenti} \end{cases}$$

- Il massimo throughput (THmax) per un dato livello di WIP è

$$TH_{max} = \begin{cases} wip / LT_{min}, & \text{if } wip \leq WIP^* \\ TH_{cb}, & \text{altrimenti} \end{cases}$$

Il THmax possibile è dato dal TH (ritmo) del collo di bottiglia del sistema, cioè dalla risorsa più lenta

THmax e LTmin

- Nel Penny Fab, THcb = 0.5 e LTmin = 8
  - WIP* = 0.5 × 8 = 4,

$$LT_{\min} = \begin{cases} 8, & \text{if } wip \leq 4 \\ 2wip, & \text{altrimenti} \end{cases}$$

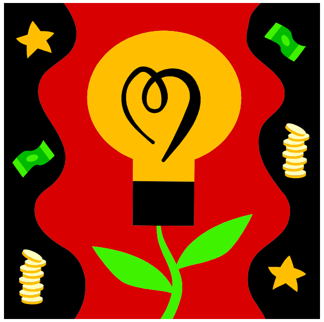
$$TH_{\max} = \begin{cases} wip/8, & \text{if } wip \leq 4 \\ 0.5, & \text{altrimenti} \end{cases}$$

Legge di Little

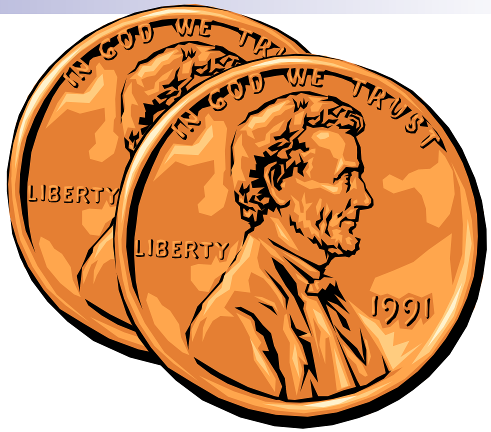
- La relazione fondamentale tra WIP, LT e Th nel lungo termine è

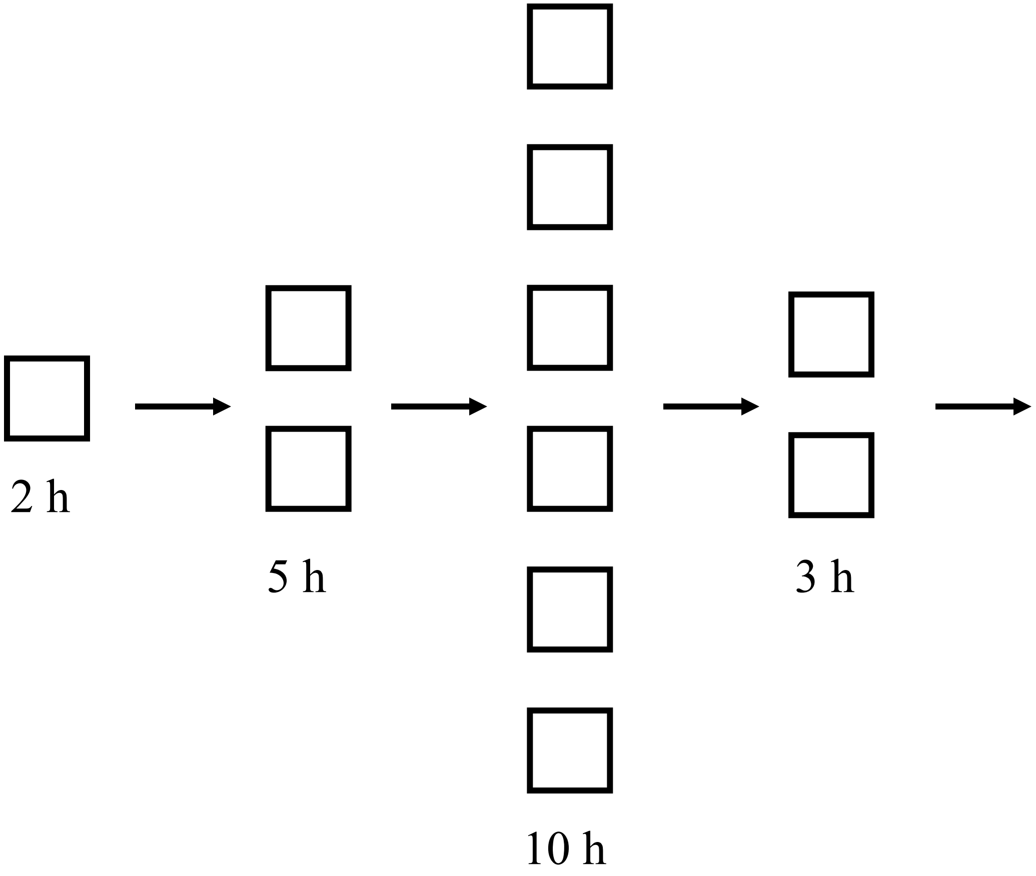
$$WIP = TH \times LT \qquad pz = \frac{pz}{h} \times h$$

- LT = WIP/TH



Penny Fab Two



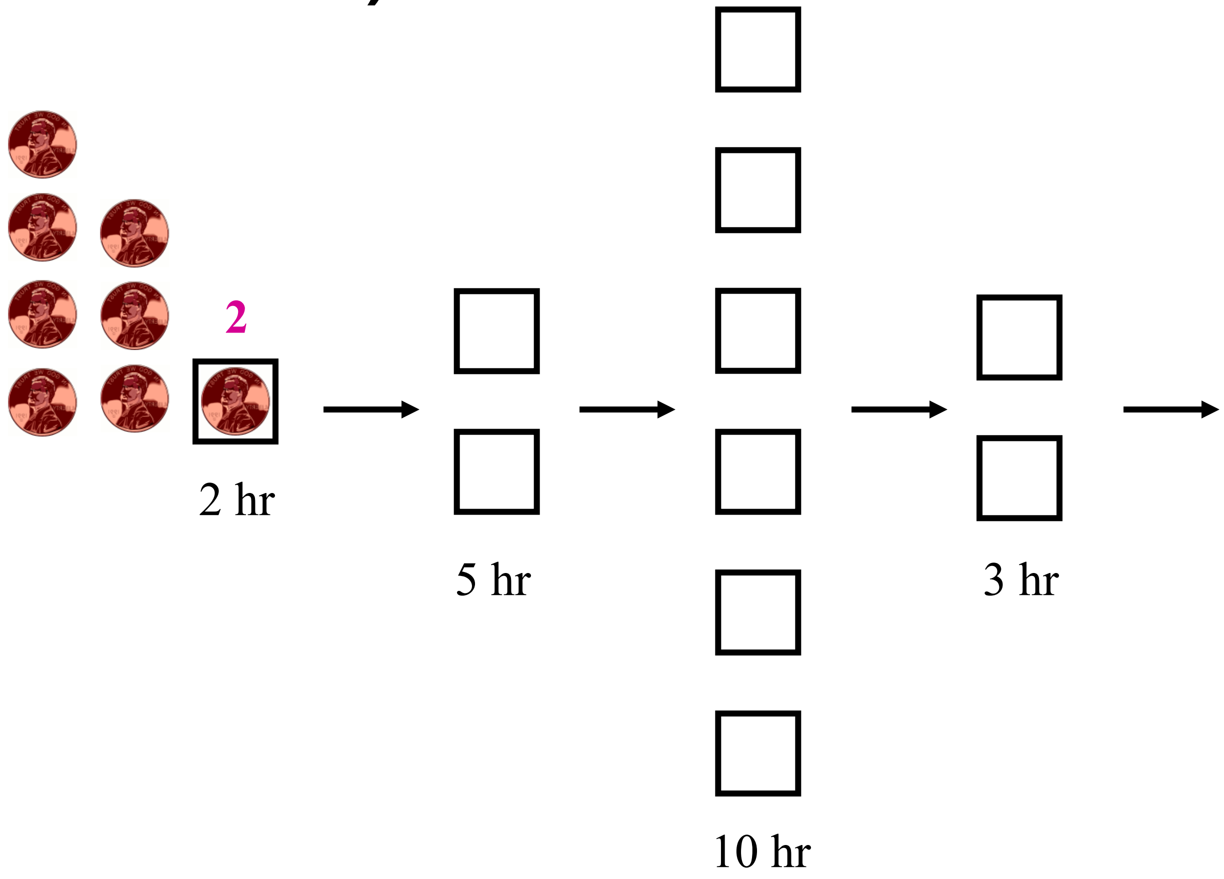


Penny Fab Two

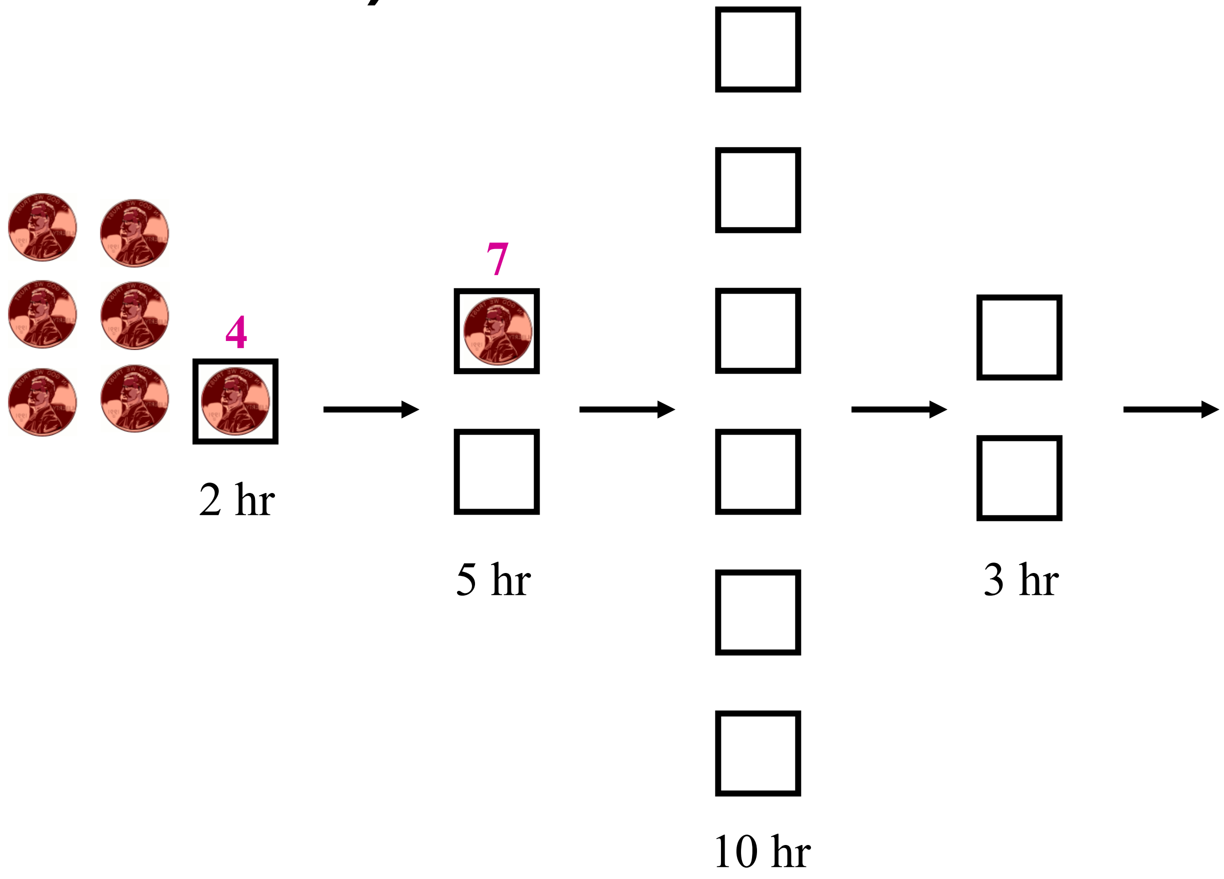
Reparto	**Numero di macchine**	**Tempo di processamento** [h]	**TH di reparto** [job/h]
1	1	2	0,5
2	2	5	0,4
3	6	10	0,6
4	2	3	0,67

$THcb =$ **0.4 p/h** $LTmin =$ **20 h** $WIP^* =$ **8 penny**

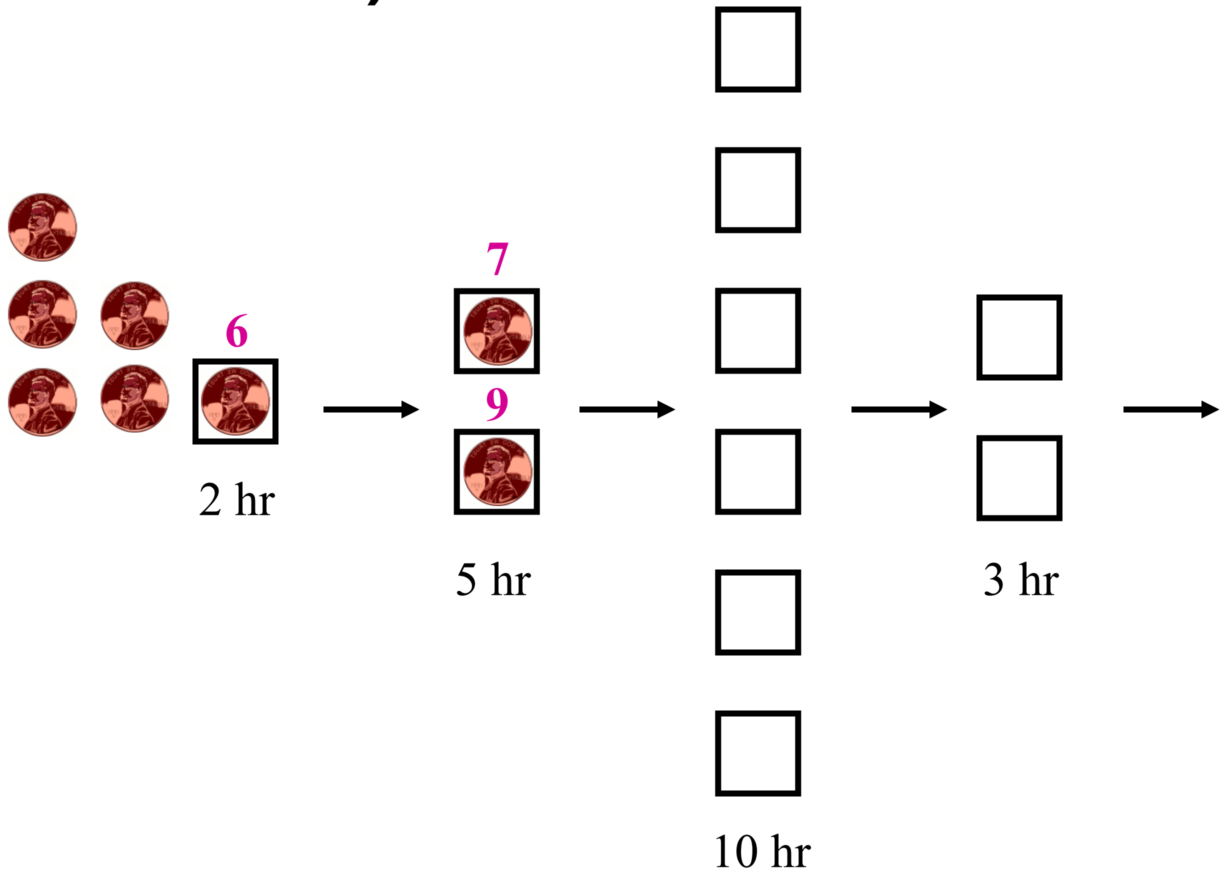
Penny Fab Two Simulation (Time=0)



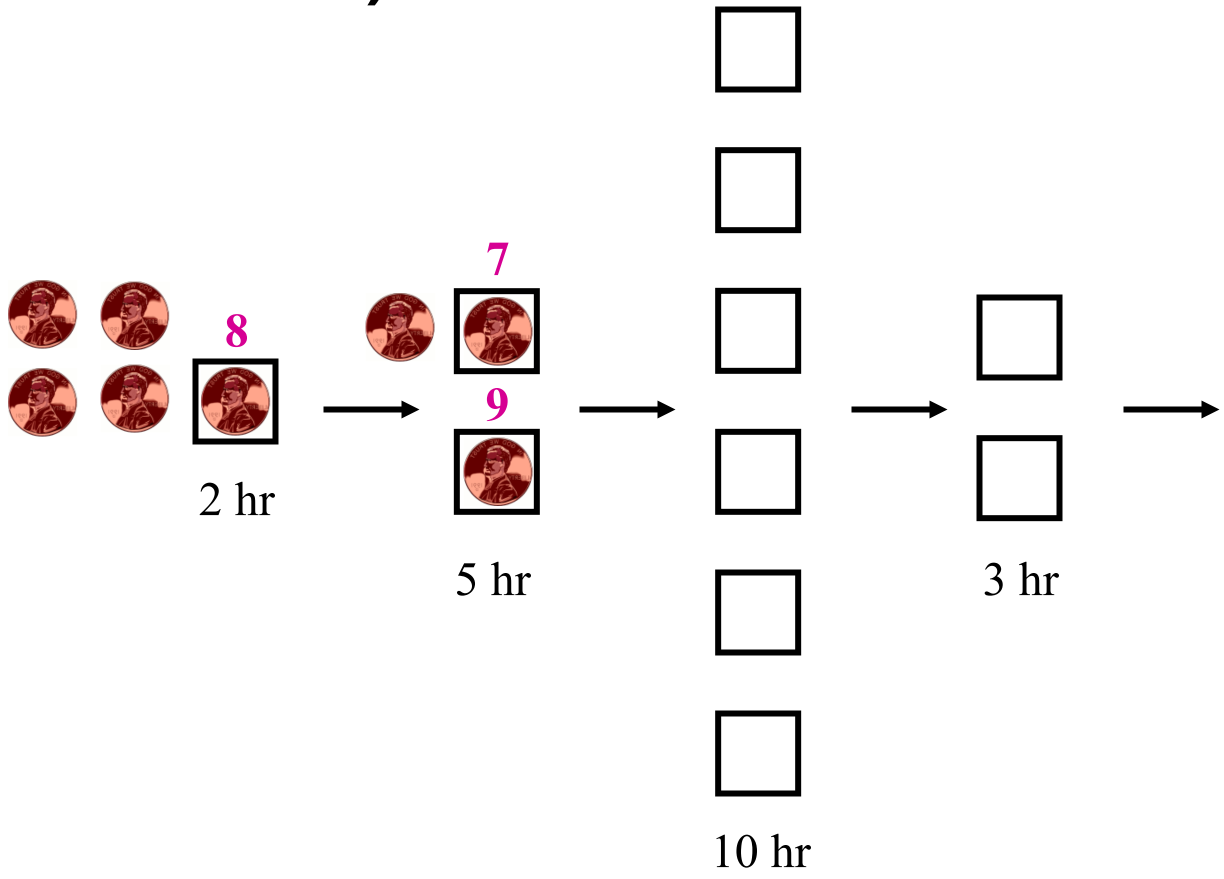
Penny Fab Two Simulation (Time=2)



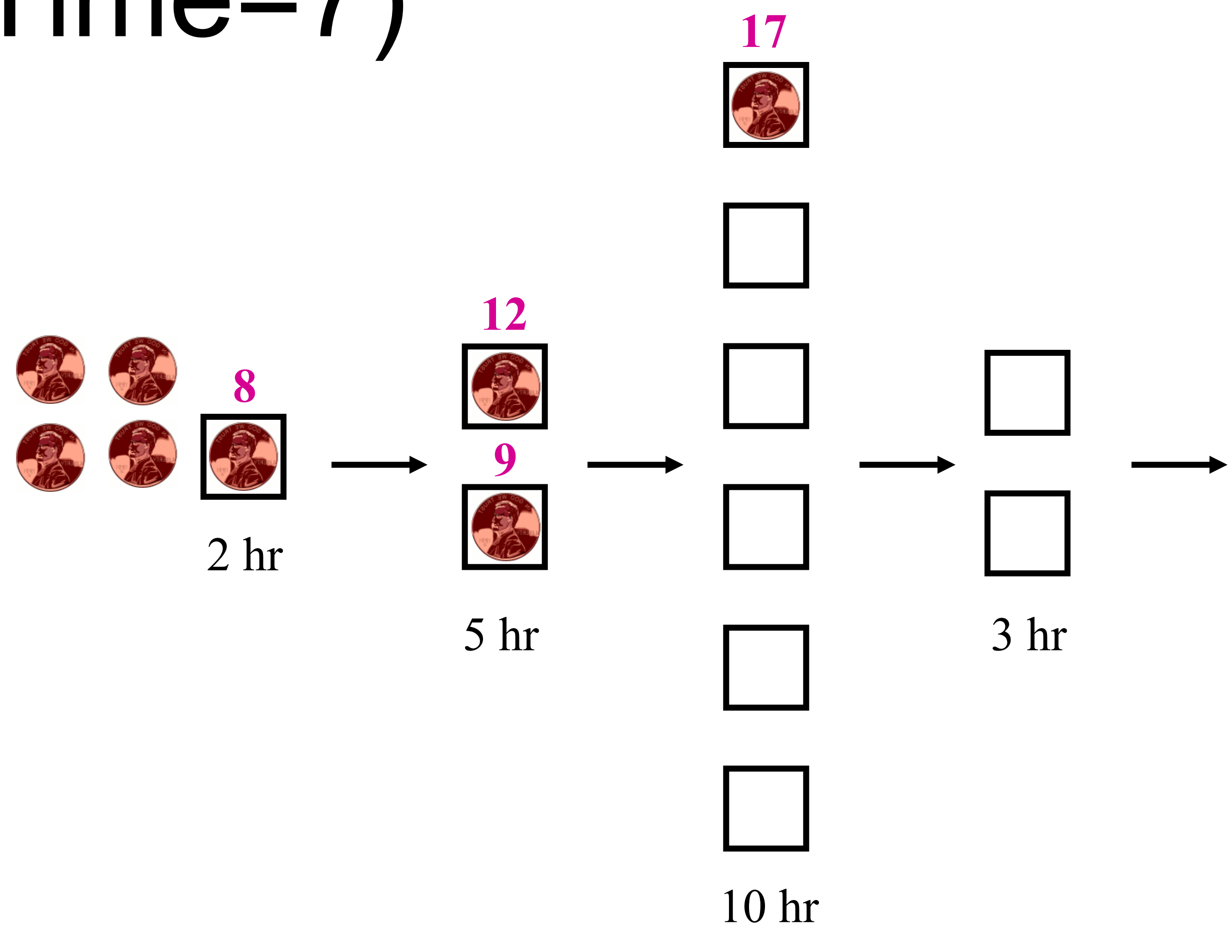
Penny Fab Two Simulation (Time=4)



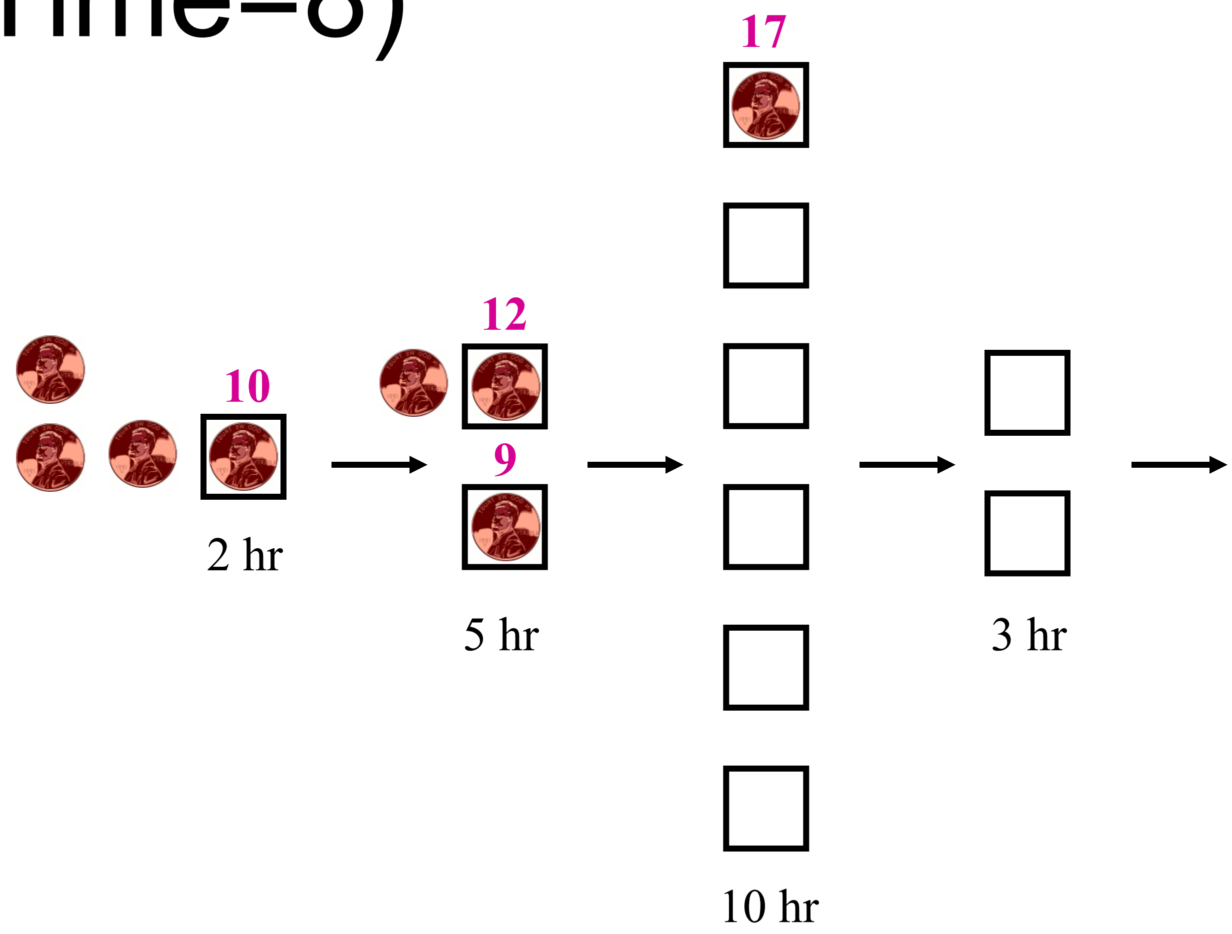
Penny Fab Two Simulation (Time=6)



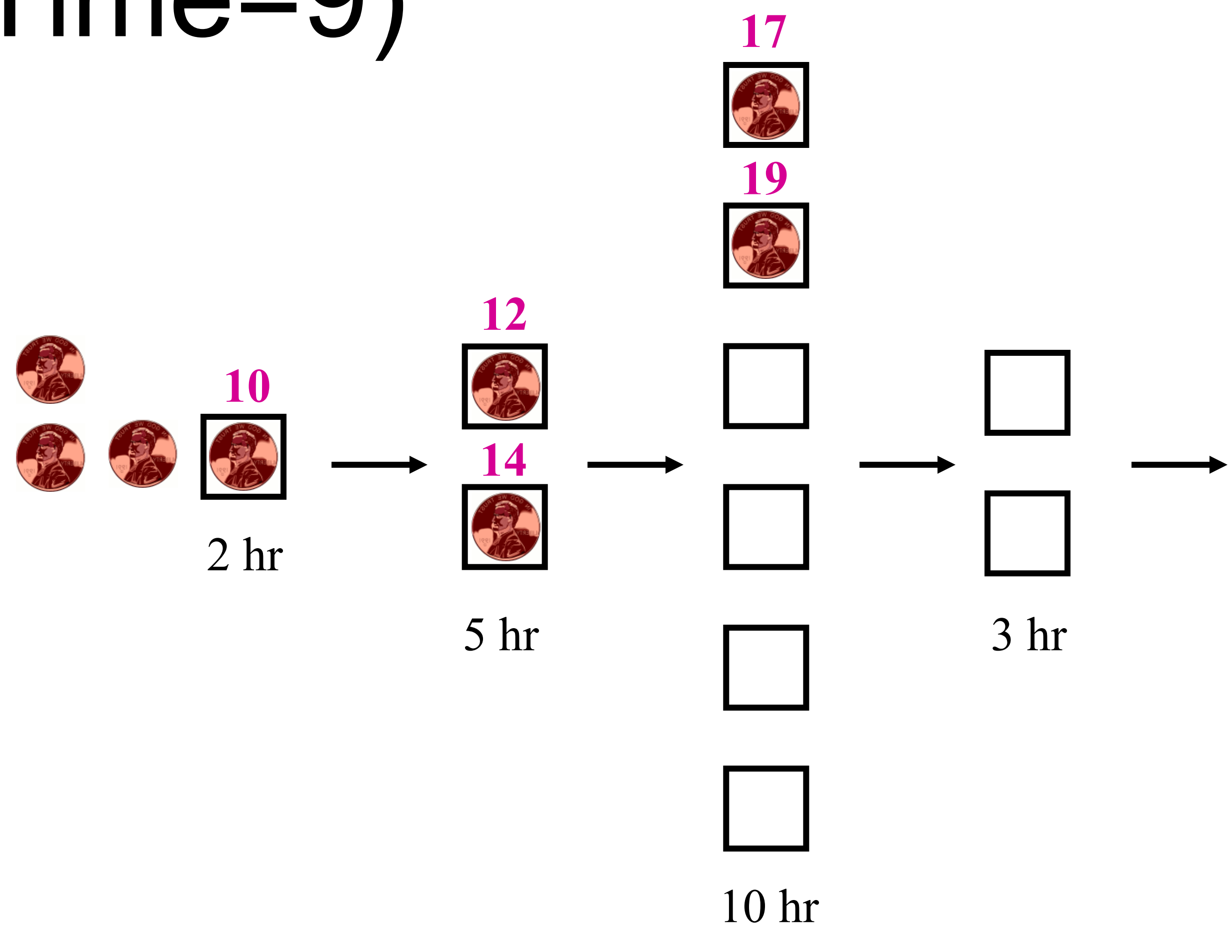
Penny Fab Two Simulation (Time=7)



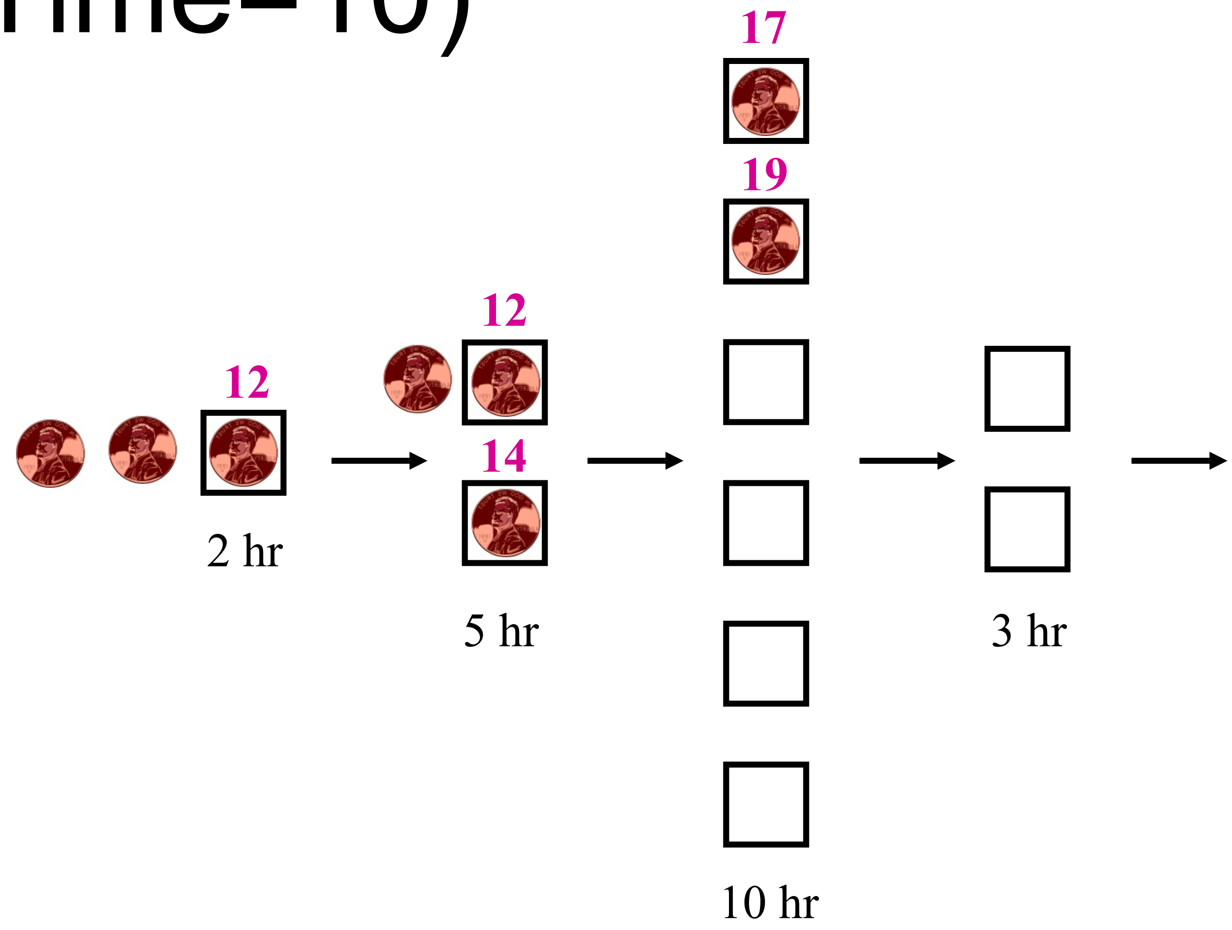
Penny Fab Two Simulation (Time=8)



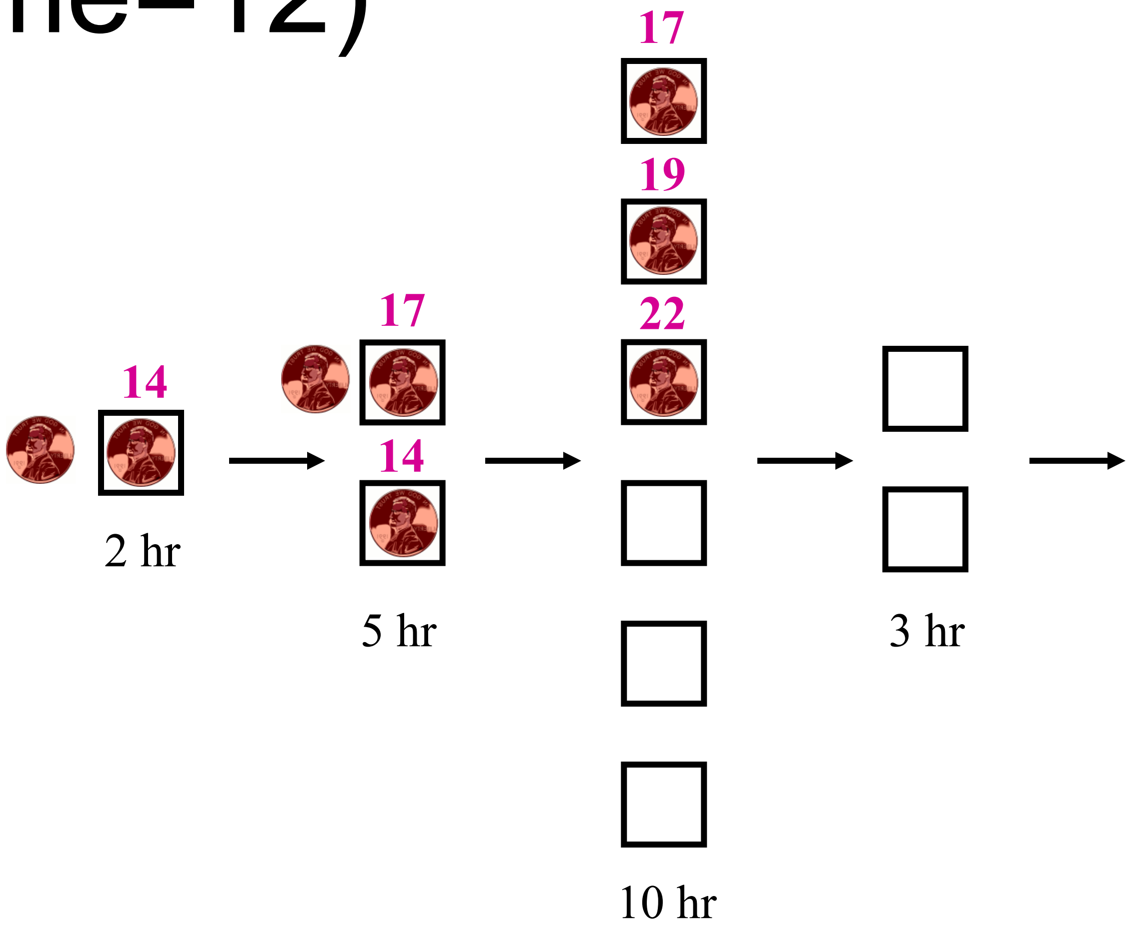
Penny Fab Two Simulation (Time=9)



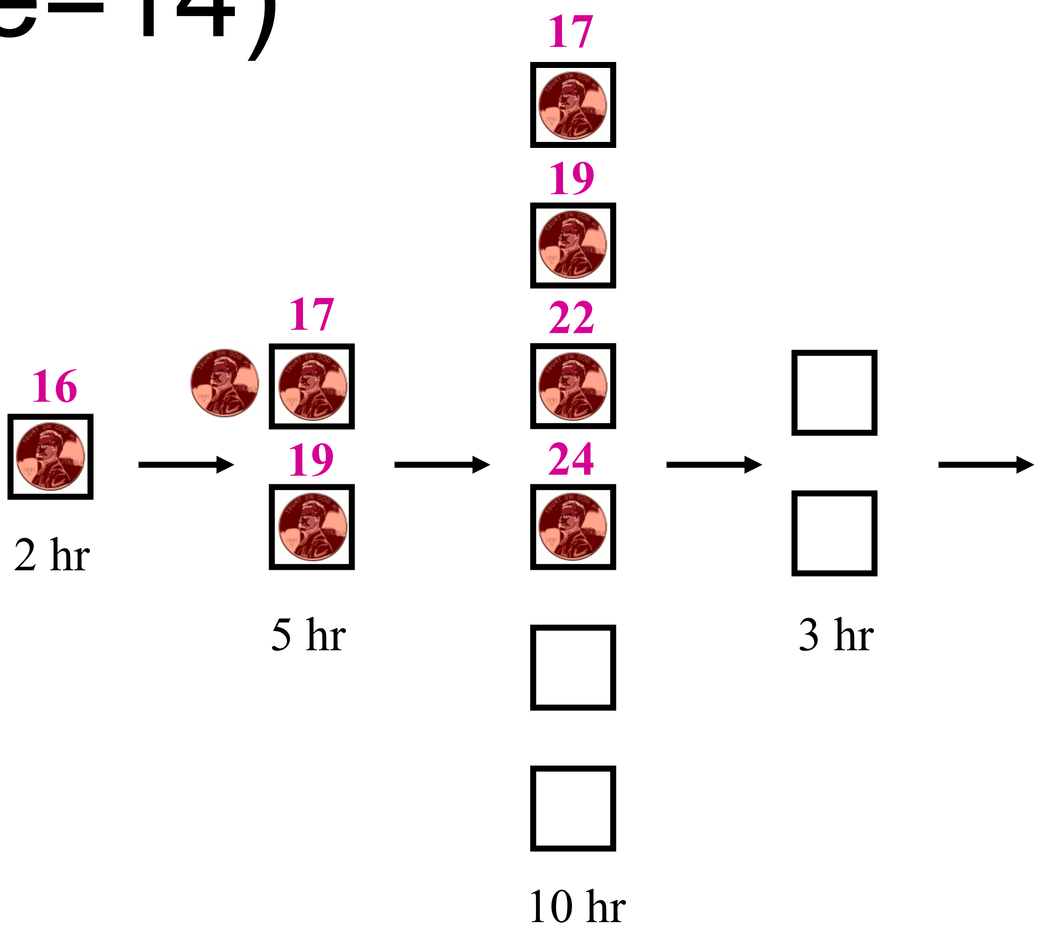
Penny Fab Two Simulation (Time=10)



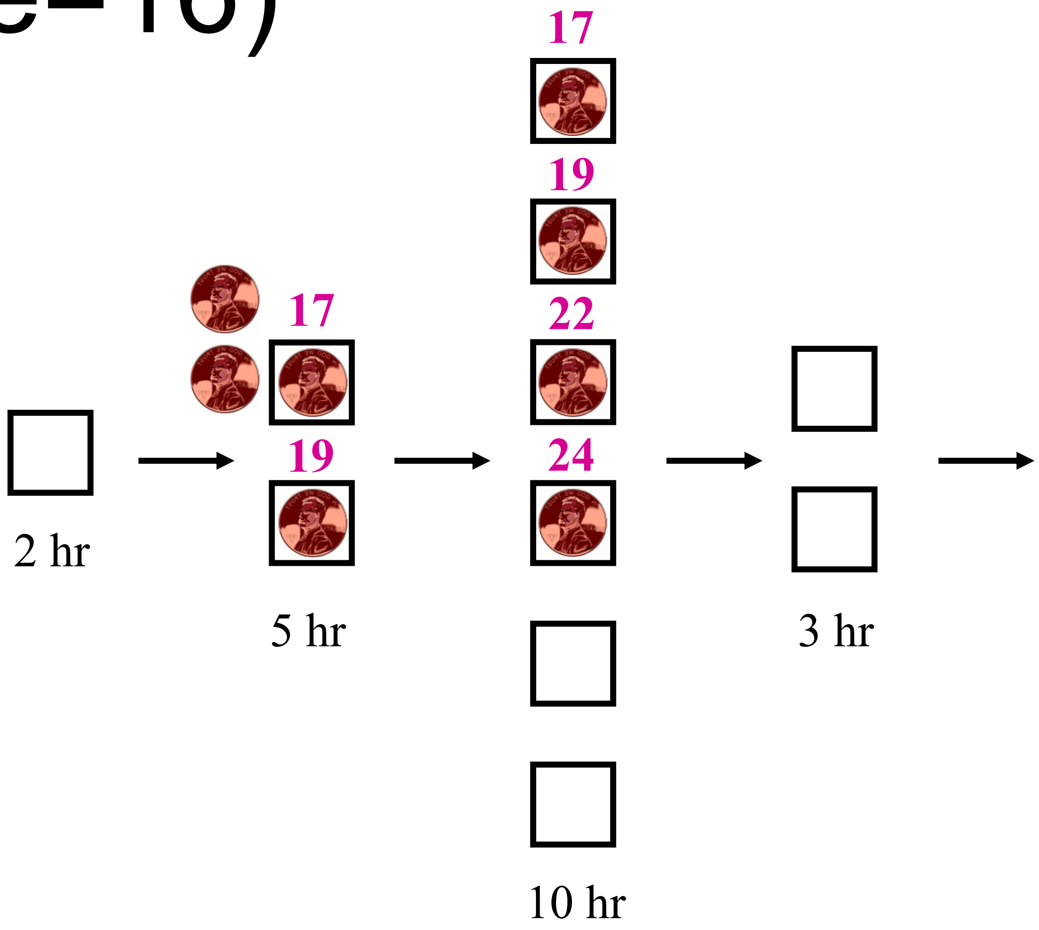
Penny Fab Two Simulation (Time=12)



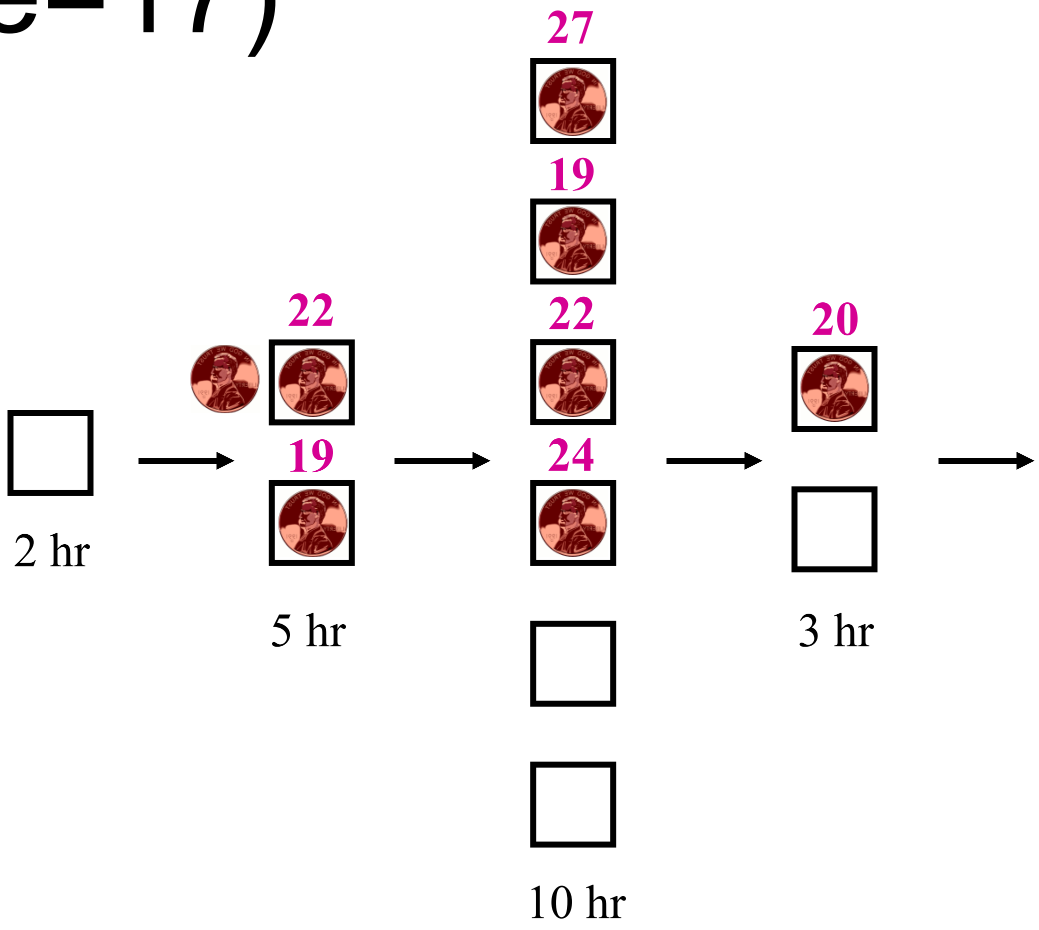
Penny Fab Two Simulation (Time=14)



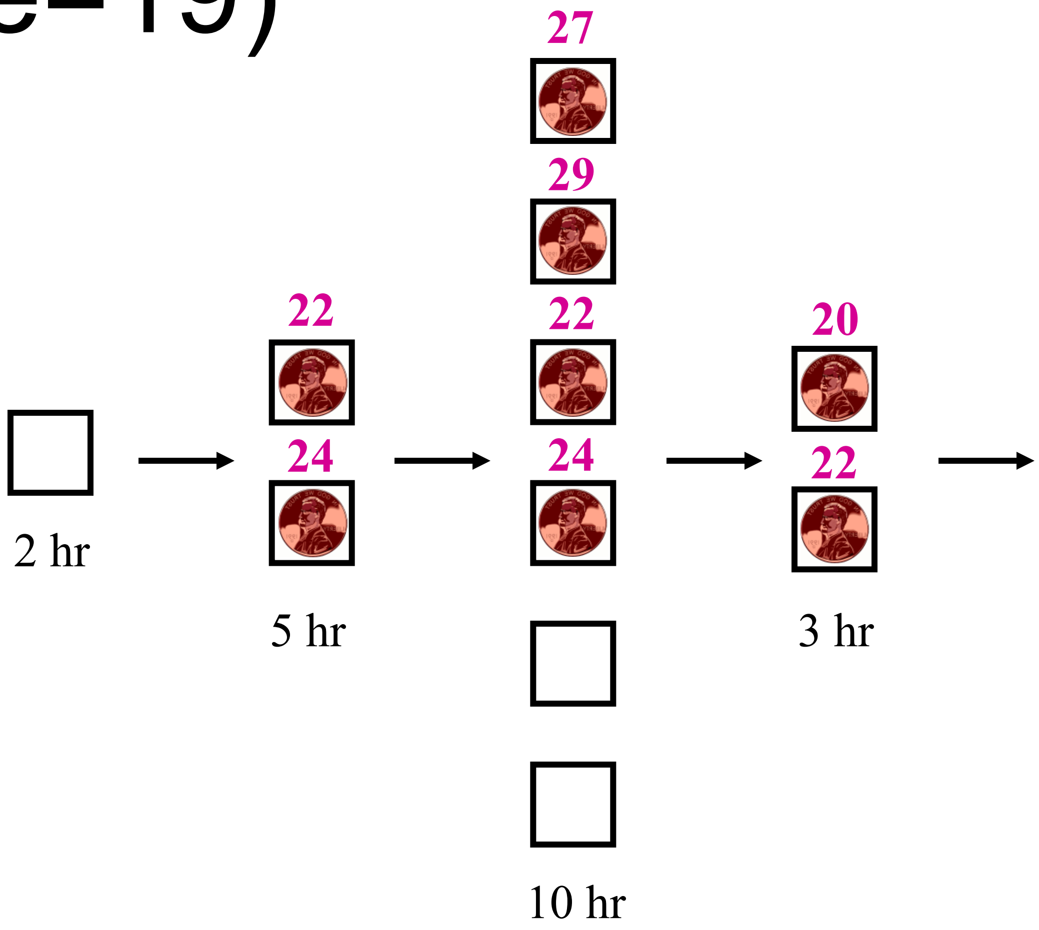
Penny Fab Two Simulation (Time=16)



Penny Fab Two Simulation (Time=17)

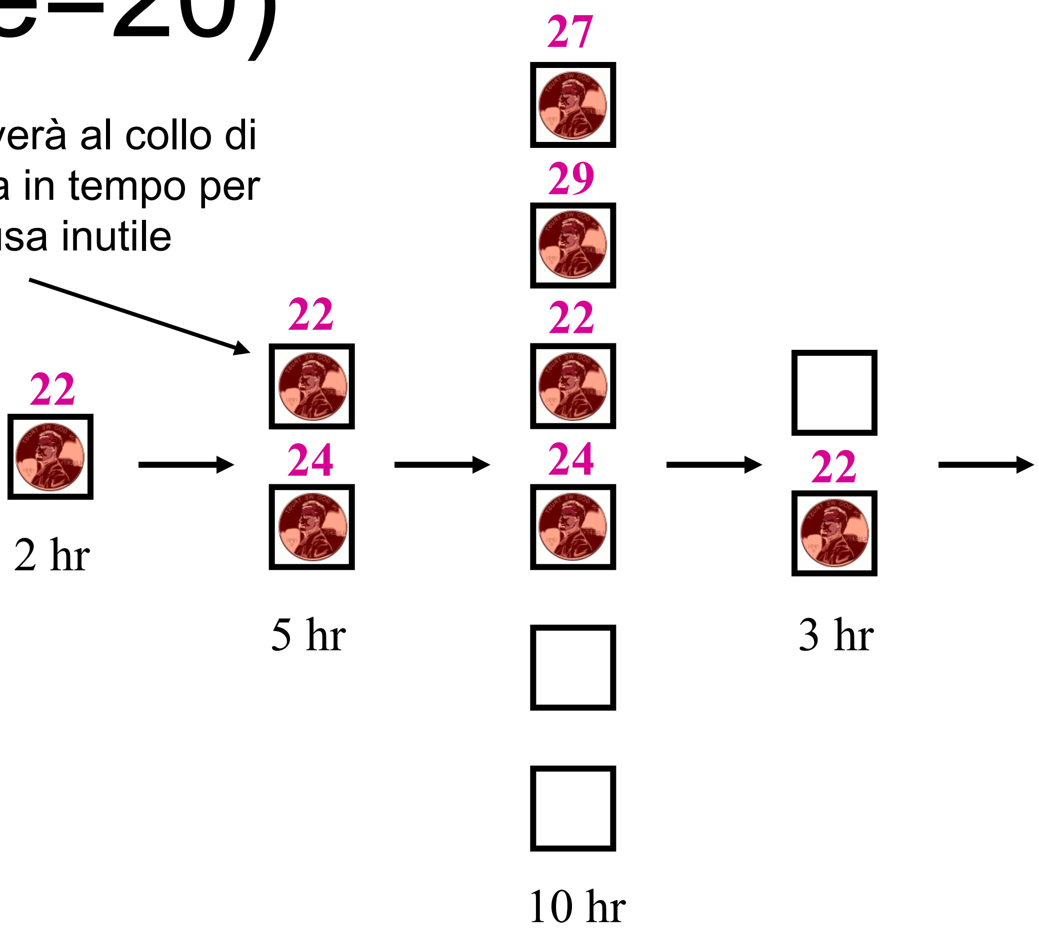


Penny Fab Two Simulation (Time=19)

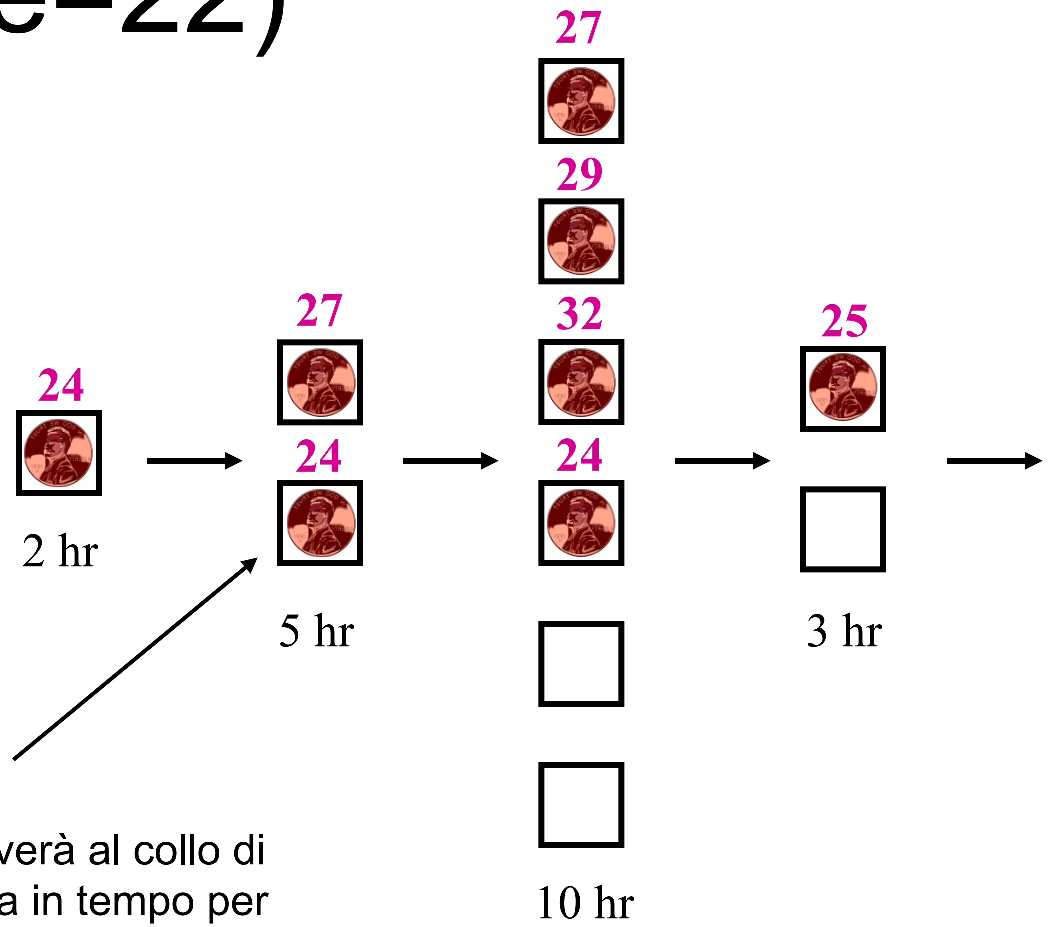


Penny Fab Two Simulation (Time=20)

Nota: il job arriverà al collo di bottiglia appena in tempo per evitare una pausa inutile (starvation)

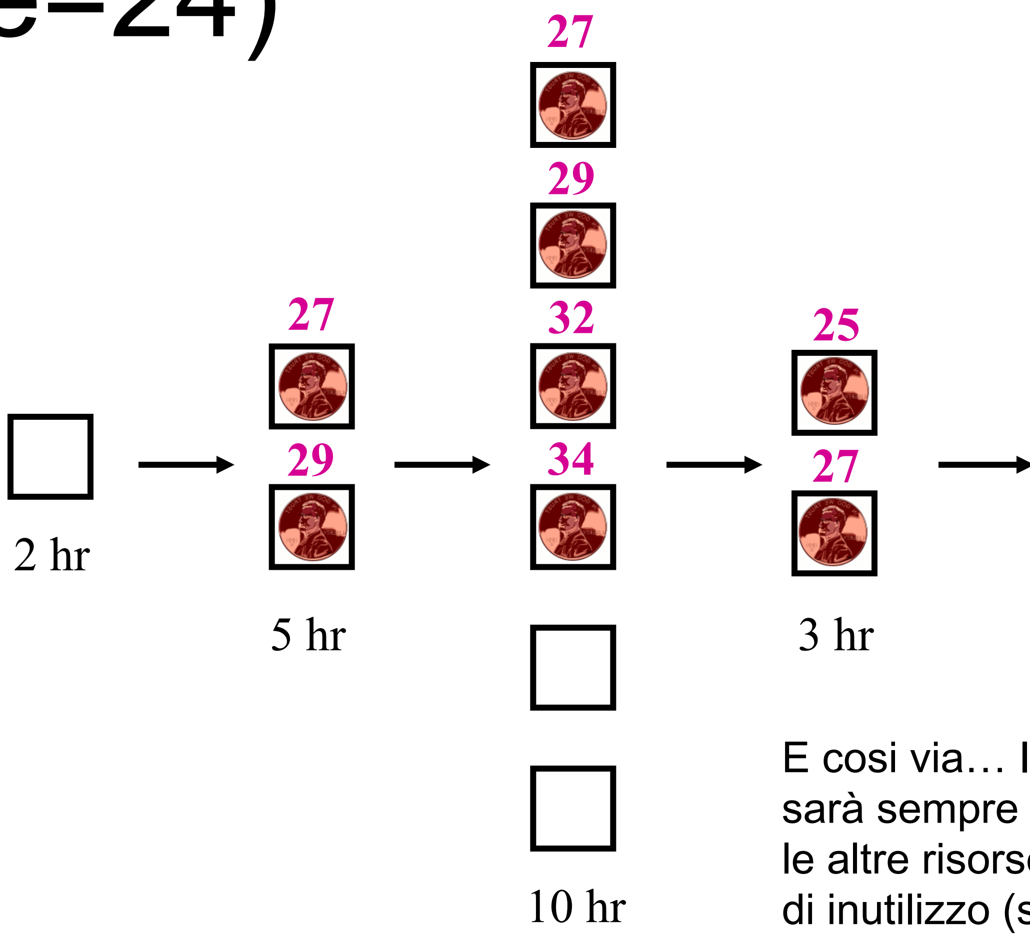


Penny Fab Two Simulation (Time=22)



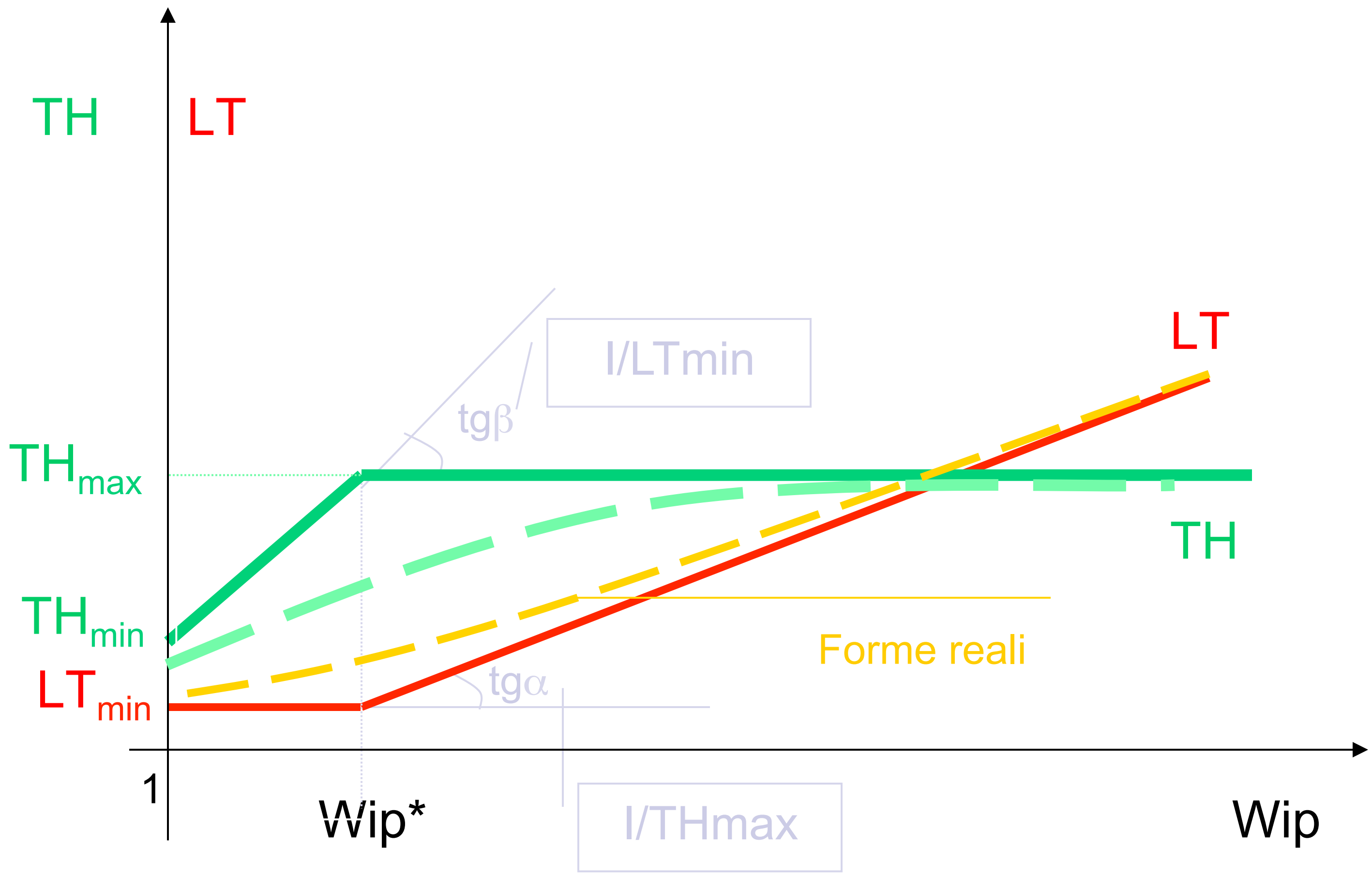
Nota: il job arriverà al collo di bottiglia appena in tempo per evitare una pausa inutile (starvation)

Penny Fab Two Simulation (Time=24)



E cosi via… Il collo di bottiglia sarà sempre occupato mentre le altre risorse avranno periodi di inutilizzo (starvation)

Legge di Little



Conclusioni

- Nella realtà, tranne che per casi particolari, l'ipotesi di non interferenza fra i pezzi caricati in un sistema, anche per bassi livelli di Wip, non si realizza, per cui il reale andamento delle curve di LT e TH in funzione di Wip si discosta dall'andamento teorico
- Un caso particolare è costituito da una linea di produzione con tempi perfettamente bilanciati e deterministici, come quello della Penny Fab
  - In tal caso il Wip* è pari al numero di stazioni e il TH cresce fino al valore max a parità di LT = LTmin; da questo valore in poi LT cresce linearmente, ma senza alcun beneficio
  - Per linee non perfettamente bilanciate il valore limite Wip* risulta invece inferiore al numero di macchine presenti (come è il caso di Penny Fab Two).
- L considerazioni fatte sono valide nel caso i pezzi nel sistema produttivo non interferiscano tra di loro
  - Se, per esempio, i tempi di lavorazione sono aleatori, anche in una linea perfettamente bilanciata, un pezzo potrebbe rimanere in attesa perché il pezzo sulla macchina a valle ha richiesto un tempo di lavorazione superiore alla media
- Quindi, in tutti i casi diversi dalla situazione teorica definita, l'andamento delle leggi è diverso da sistema a sistema e deve essere ricavato sperimentalmente (ad esempio mediante simulazione
- Concludendo, in base alla legge di Little, la riduzione del LT di un sistema richiede necessariamente che venga ridotto il valore del Wip, se si vuole che il TH rimanga costante
  - Di conseguenza, la presenza di grandi code in un sistema è un indicatore della possibilità di riduzione del LT (attraverso opportuni interventi da identificare), così come del WIP