

Usando la trasformata di Laplace risolvere i problemi di Cauchy:

1.  $3y'' - 6y' + 4y = 4H_2(t), \quad y(0) = 1, \quad y'(0) = 1.$

Sol.:  $y(t) = e^t \cos \frac{t}{\sqrt{3}} + H_2(t) \left[ 1 + e^{t-2} \left( \sqrt{3} \sin \frac{t-2}{\sqrt{3}} - \cos \frac{t-2}{\sqrt{3}} \right) \right]$

2.  $4y'' + 4y' + y = H_3(t)(t-3)^2 e^{-\frac{1}{2}(t-3)}, \quad y(0) = 1, \quad y'(0) = -1.$

Sol.:  $y(t) = e^{-t/2} \left( 1 - \frac{t}{2} \right) + \frac{1}{48} H_3(t)(t-3)^4 e^{-\frac{1}{2}(t-3)}$

3.  $2y'' - y' - y = 4\delta(t-3) - H_3(t), \quad y(0) = 1, \quad y'(0) = 1.$

Sol.:  $y(t) = e^t + H_3(t) \left[ t - 3 - 2e^{-\frac{1}{2}(t-3)} + e^{t-3} \right]$

4.  $9y'' - 6y' + y = 2H_4(t)e^{\frac{1}{3}(t-4)}, \quad y(0) = 1, \quad y'(0) = 1.$

Sol.:  $y(t) = \left( 1 + \frac{2}{3}t \right) e^{t/3} + \frac{1}{9} H_4(t)(t-4)^2 e^{\frac{1}{3}(t-4)}$

5.  $3y'' + 12y' + 13y = 3\delta(t-1) + 13H_1(t), \quad y(0) = 1, \quad y'(0) = -2.$

Sol.:  $y(t) = e^{-2t} \cos \frac{t}{\sqrt{3}} + H_1(t) \left[ 1 - \left( \cos \frac{t-1}{\sqrt{3}} - \sqrt{3} \sin \frac{t-1}{\sqrt{3}} \right) e^{-2(t-1)} \right]$

6.  $3y'' + 2y' - y = 5\delta(t-2) + H_2(t), \quad y(0) = 1, \quad y'(0) = -1.$

Sol.:  $y(t) = e^{-t} + H_2(t) \left[ 2e^{\frac{1}{3}(t-2)} - e^{-(t-2)} - 1 \right]$

7.  $9y'' + 12y' + 4y = 6H_5(t)(t-5)e^{-\frac{2}{3}(t-5)}, \quad y(0) = 1, \quad y'(0) = -1.$

Sol.:  $y(t) = \left( 1 - \frac{t}{3} \right) e^{-\frac{2}{3}t} + \frac{1}{9} H_5(t)(t-5)^3 e^{-\frac{2}{3}(t-5)}$

8.  $3y'' - 18y' + 28y = 28H_4(t) - 6\delta(t-4), \quad y(0) = 1, \quad y'(0) = -3.$

Sol.:  $y(t) = e^{3t} \cos \frac{t}{\sqrt{3}} + H_4(t) \left[ 1 - e^{3(t-4)} \left( \cos \frac{t-4}{\sqrt{3}} + \sqrt{3} \sin \frac{t-4}{\sqrt{3}} \right) \right]$

9.  $4y'' - 12y' + 9 = 20H_5(t)(t-5)^3 e^{\frac{3}{2}(t-5)}, \quad y(0) = 1, \quad y'(0) = 2.$

Sol.:  $y(t) = (2 + 2t) e^{\frac{3}{2}t} + \frac{1}{6} H_5(t)(t-5)^5 e^{\frac{3}{2}(t-5)}$

10.  $4y'' - 7y' - 2y = 10\delta(t-3) - 2H_3(t), \quad y(0) = 1, \quad y'(0) = 2.$

Sol.:  $y(t) = e^{2t} + H_3(t) \left[ 1 - 2e^{-\frac{1}{4}(t-3)} + e^{2(t-3)} \right]$

11.  $y'' + 4y = (1 - H_4(t)), \quad y(0) = 3, \quad y'(0) = -2.$

Sol.:  $y(t) = 3 \cos 2t - \sin 2t + \frac{1}{4}(1 - \cos 2t) + \frac{1}{4}(1 - \cos 2(t-4))H_4(t)$

12.  $y'' - 2y' + y = te^t, \quad y(0) = 0, \quad y'(0) = 0.$

Sol.:  $y(t) = \frac{1}{6} t^3 e^t$

13.  $y'' - 4y' + 4y = 3\delta(t-1) + \delta(t-2), \quad y(0) = 1, \quad y'(0) = 1.$

Sol.:  $y(t) = (1-t)e^{2t} + 3(t-1)H_1(t)e^{2(t-1)} + (t-2)H_2(t)e^{2(t-2)}$

14.  $y'' + 2y' + y = 2(t - 3)H_3(t)$ ,  $y(0) = 2$ ,  $y'(0) = 1$ .  
Sol.:  $y(t) = (2 + 3t)e^{-t} + 2H_3(t)[(t - 5) + (t - 1)e^{-(t-3)}]$
15.  $y'' + y' + y = 1 + e^{-t}$ ,  $y(0) = 3$ ,  $y'(0) = -5$ .  
Sol.:  $y(t) = 1 + e^{-t} + \left(\cos \frac{\sqrt{3}}{2}t - \frac{7}{\sqrt{3}} \sin \frac{\sqrt{3}}{2}t\right) e^{-t/2}$
16.  $y'' + 2y' + y = e^{-t} + 3\delta(t - 1)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .  
Sol.:  $y(t) = \frac{1}{2}t^2e^{-t} + 3(t - 1)e^{-(t-1)}$ ,  $t > 1$
17.  $y'' + y = (1 - H_{\pi/2}(t)) \cos t$ ,  $y(0) = 3$ ,  $y'(0) = -1$ .  
Sol.:  $y(t) = \left(t - \frac{\pi}{2}\right) \sin t - \cos t$
18.  $y'' + y = \sin t$ ,  $y(0) = 1$ ,  $y'(0) = 2$ . Sol.:  $y(t) = \cos t + \frac{5}{2} \sin t - \frac{1}{2}t \cos t$
19.  $y'' + y = \sin t + \delta(t - \pi)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .  
Sol.:  $y(t) = \frac{1}{2}(\sin t - t \cos t) - H_{\pi}(t) \sin t$
20.  $y'' - 2y' + y = [H_1(t) - H_2(t)]t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .  
Sol.:  $y(t) = te^t + H_1(t)[2 + t + (2t - 5)e^{t-1}] - H_2(t)[1 + t + (2t - 7)e^{t-2}]$

Risolvere i seguenti problemi di Cauchy:

1.  $y' = \frac{3x^2 + 4x + 2}{2y}$ ,  $y(0) = -1$  Sol:  $y(x) = -\sqrt{x^3 + 2x^2 + 2x + 1}$
2.  $y' + \frac{1}{2}y = \frac{3}{2}$ ,  $y(0) = 2$  Sol:  $y(x) = 3 - e^{-x/2}$
3.  $y' = \frac{3x^2 + 4x + 2}{7y}$ ,  $y(0) = -2$  Sol:  $y(x) = -\sqrt{\frac{2}{7}(x^3 + x^2 + 2x + 14)}$
4.  $y' - \frac{y}{2} = e^{-x}$ ,  $y(0) = -1$  Sol:  $y(x) = -\frac{2}{3}e^{-x} - \frac{1}{3}e^{-x/2}$
5.  $y'' + 5y' + 6y = 6$ ,  $y(0) = 1$ ,  $y'(0) = 3$  Sol:  $y = 9e^{-2x} - 7e^{-3x} + 1$
6.  $y' = \frac{4x^3 + 4x + 2}{5y}$ ,  $y(0) = 1$  Sol:  $y(x) = \sqrt{\frac{2}{5}(x^4 + 2x^2 + 2x + \frac{5}{2})}$
7.  $y' + \frac{1}{2}y = \frac{3}{2}$ ,  $y(0) = 4$  Sol:  $y(x) = 3 + e^{-x/2}$
8.  $y' = \frac{4x^3 + 4x + 2}{3y}$ ,  $y(0) = 2$  Sol:  $y(x) = \sqrt{\frac{2}{3}(x^3 + 2x^2 + 2x + 6)}$
9.  $y' - \frac{y}{2} = e^{-x}$ ,  $y(0) = 1$  Sol:  $y(x) = -\frac{2}{3}e^{-x} + \frac{5}{3}e^{x/2}$
10.  $y'' + 3y' + 2y = 2$ ,  $y(0) = -1$ ,  $y'(0) = -3$  Sol:  $y(x) = -7e^{-x} + 5e^{-2x} + 1$

N.B: LE SOLUZIONI NON SONO GARANTITE!!!