Applicazione della trasformata di Laplace alle equazioni differenziali

(a) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 0, \; y(0) = 1, \; \frac{dy(0)}{dt} = -1$

(b) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 2, \; y(0) = 0, \; \frac{dy(0)}{dt} = 1$

(c) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \delta(t-1), \; y(0) = 1, \; \frac{dy(0)}{dt} = -1$

(d) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = f(t), \; y(0) = y_0, \; \frac{dy(0)}{dt} = y'(0)$

Applicando il Teorema della trasformata della derivata [Pag 13 g)]

$$L\{f^{(1)}(t)\} = sF(s) - f(0)$$

$$L\{f^{(2)}(t)\} = s^2 F(s) - s\,f(0) - f^{(1)}(0)$$

a) $s^2 Y(s) - s\,y(0) - y^{(1)}(0) + 2\,[sY(s) - y(0)] + 2\,Y(s) = 0$

$s^2 Y(s) + 2\,sY(s) + 2\,Y(s) = s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$[s^2 + 2\,s + 2]\,Y(s) = s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$$Y(s) = \frac{s\,y(0) + y^{(1)}(0) + 2\,y(0)}{s^2 + 2\,s + 2} = \frac{s\,1 + y^{(1)}(0) + 2\,y(0)}{s^2 + 2\,s + 2} = \frac{s \cdot 1 + (-1) + 2 \cdot 1}{s^2 + 2\,s + 2} = \frac{s - 1 + 2}{s^2 + 2\,s + 2} = \frac{s + 1}{s^2 + 2\,s + 2}$$

b) $s^2 Y(s) - s\,y(0) - y^{(1)}(0) + 2\,[sY(s) - y(0)] + 2\,Y(s) = \frac{2}{s}$

($\frac{2}{s}$ Pag 11 b) Proprietà di linearità)

$s^2 Y(s) + 2\,sY(s) + 2\,Y(s) = \frac{2}{s} + s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$[s^2 + 2\,s + 2]\,Y(s) = \frac{2}{s} + s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$$Y(s) = \frac{\frac{2}{s}}{s^2 + 2\,s + 2} + \frac{s\,y(0) + y^{(1)}(0) + 2\,y(0)}{s^2 + 2\,s + 2} = \frac{2}{s[s^2 + 2\,s + 2]} + \frac{s \cdot 0 + 1 + 2 \cdot 0}{s^2 + 2\,s + 2} = \frac{2}{s[s^2 + 2\,s + 2]} + \frac{1}{s^2 + 2\,s + 2}$$

c) $s^2 Y(s) - s\,y(0) - y^{(1)}(0) + 2\,[sY(s) - y(0)] + 2\,Y(s) = e^{-s}$

(e^{-s} Pag 12 c) Teorema della traslazione nel tempo)

$s^2 Y(s) + 2\,sY(s) + 2\,Y(s) = e^{-s} + s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$[s^2 + 2\,s + 2]\,Y(s) = e^{-s} + s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$$Y(s) = \frac{e^{-s}}{s^2 + 2\,s + 2} + \frac{s\,y(0) + y^{(1)}(0) + 2\,y(0)}{s^2 + 2\,s + 2} = \frac{e^{-s}}{s^2 + 2\,s + 2} + \frac{s \cdot 1 + (-1) + 2 \cdot 1}{s^2 + 2\,s + 2} = \frac{e^{-s}}{s^2 + 2\,s + 2} + \frac{s + 1}{s^2 + 2\,s + 2}$$

d) $s^2 Y(s) - s\,y(0) - y^{(1)}(0) + 2\,[sY(s) - y(0)] + 2\,Y(s) = F(s)$

$s^2 Y(s) + 2\,sY(s) + 2\,Y(s) = F(s) + s\,y(0) + y^{(1)}(0) + 2\,y(0)$

$[s^2 + 2\,s + 2]\,Y(s) = F(s) + s\,y_0 + y'(0) + 2\,y_0$

$$Y(s) = \frac{F(s)}{s^2 + 2\,s + 2} + \frac{s\,y0 + y'(0) + 2\,y0}{s^2 + 2\,s + 2}$$