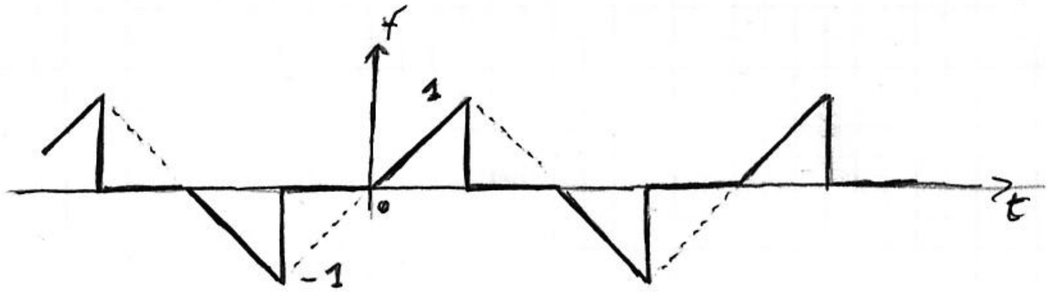
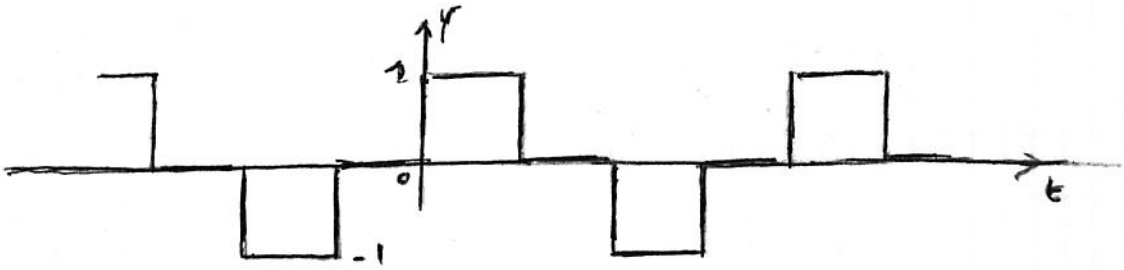
(1)



1A SCRIVERE LE RELAZIONI CHE CONSENTONO IL CALCOLO DIRETTO DEI COEFFICIENTI DELLO SVILUPPO IN SERIE DI FOURIER DELLA f.d O. (IMPOSTARE, MA NON SVILUPPARE I CALCOLI)

1B OSSERVANDO LA f d O, COSA SI PUÒ DIRE, A PRIORI, RELATIVAMENTE AI COEFFICIENTI DELLO SVILUPPO IN SERIE ?

(2)



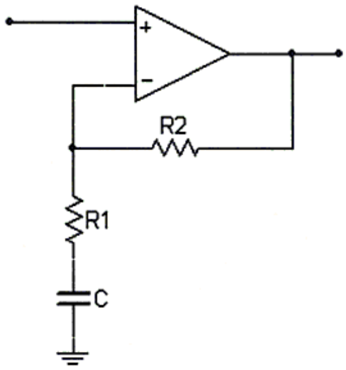
2A SCOMPORRE LA fdO NELLA SOMMA DI UNA FUNZIONE PARI E UNA FUNZIONE DISPARI E DETERMINARNE LO SVILUPPO IN SERIE DI FOURIER (SONO NOTI GLI SVILUPPI IN SERIE ALLEGATI)

2B DETERMINARE IL VALORE EFFICACE DELLA f d O —

3) Determinare la risposta $v_o(t)$ fornita da un sistema descritto dalla funzione di trasferimento F(s) sollecitato dal segnale $v_i(t)$

$$F(s) = \frac{1+5s}{1+s} \qquad\qquad v_i(t) = u(t) \quad \begin{Bmatrix} 0\, V\ per\ t < 0 \\ 1\, V\ per\ t \geq 0 \end{Bmatrix}$$

4) Con riferimento al circuito in fig.



$R_1 = 1\Omega$

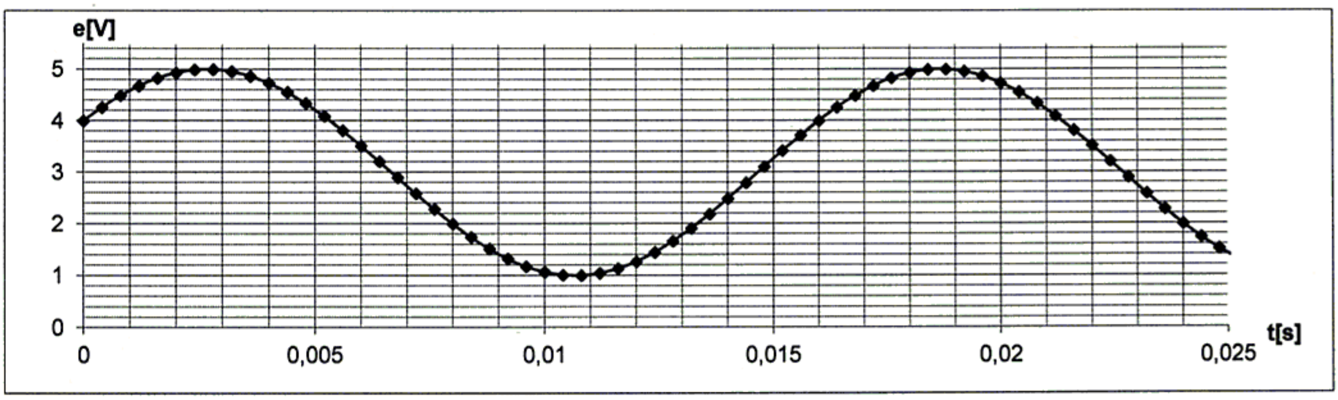
C = 1F

$R_2 = 4\Omega$

a) determinare la funzione di trasferimento
b) determinare il guadagno a frequenze molto basse (f → 0) e molto alte (f → ∞)
c) la risposta $v_0(t)$ (a transitorio esaurito) al segnale sinusoidale

$$v_i(t) = 2\ sen(0{,}8t)$$

5) a) determinare l'espressione analitica, lo spettro delle ampiezze e il valore efficace del segnale e(t) in figura

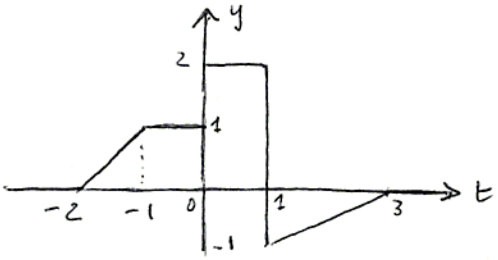


b) determinare la risposta $v_o(t)$ al segnale e(t) - di cui sopra - applicato ad un filtro passa-basso passivo ideale con guadagno unitario nella banda passante di 10 Hz

6) DETERMINARE LA TRASFORMATA DI LAPLACE DELLE SEG. FUNZIONI

a) $y(t) = \frac{3}{2} + 2e^{-t}\left(-\frac{3}{4}\cos t + \frac{7}{4}\sin t\right)$ per $t \geq 0$

b)



SCRIVERE ANCHE L'ESPRESSIONE ANALITICA $y(t)$ DELLA FUNZIONE IN FIGURA

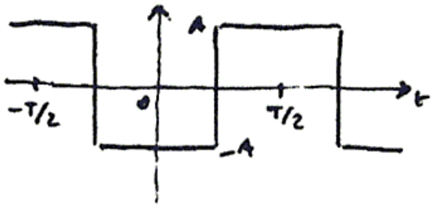
DETERMINARE LE FUNZIONI CORRISPONDENTI ALLE SEG. TRASFORMATE DI LAPLACE

$$F_1(s) = \frac{7}{s} + \frac{2}{3\left(s+\frac{1}{2}\right)} - \frac{3}{s^2}$$

$$F_2(s) = \frac{-s+1}{(s^2+1)(s+1)}$$

ALLEGATO ALL'ES. 2

7)



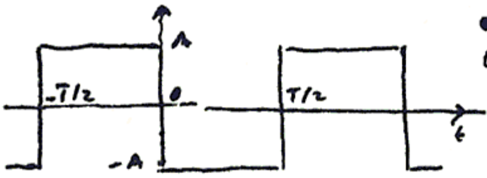
ONDA QUADRA PARI

LA FUNZIONE È OPPOSTA AD f_1

$f_7(t) = -f_1(t)$

$$f_7(t) = -\frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

8)



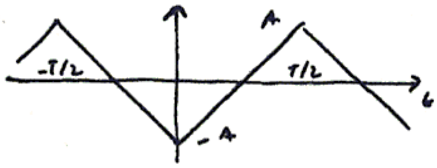
ONDA QUADRA DISPARI

LA FUNZIONE È OPPOSTA AD f_2

$f_8(t) = -f_2(t)$

$$f_8(t) = -\frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

9)



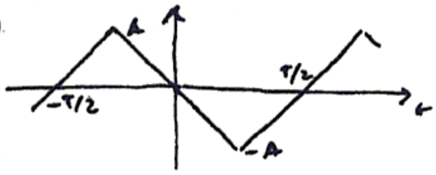
ONDA TRIANGOLARE PARI

LA FUNZIONE È OPPOSTA AD f_3

$f_9(t) = -f_3(t)$

$$f_9(t) = -\frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

10)



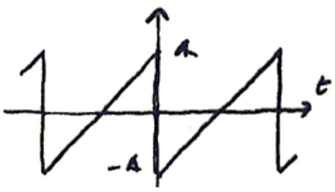
ONDA TRIANGOLARE DISPARI

LA FUNZIONE È OPPOSTA AD f_4

$f_{10}(t) = -f_4(t)$

$$f_{10}(t) = -\frac{8A}{\pi^2}\left(\sin\omega t - \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t - \dots\right)$$

11)



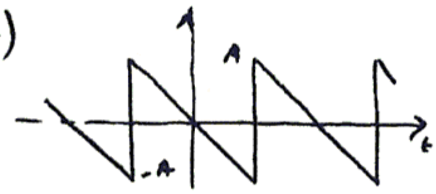
ONDA A DENTE DI SEGA

LA FUNZIONE È OPPOSTA AD f_5

$f_{11}(t) = -f_5(t)$

$$f_{11}(t) = -\frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

12)



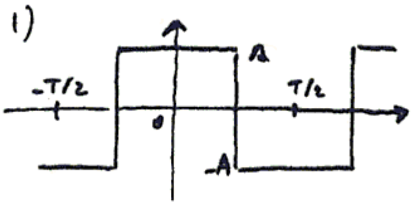
ONDA A DENTE DI SEGA

LA FUNZIONE È OPPOSTA AD f_6

$f_{12}(t) = -f_6(t)$

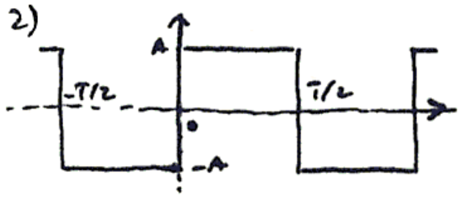
$$f_{12}(t) = -\frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \dots\right)$$

1)



$$f_1(t) = \frac{4A}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right]$$

2)

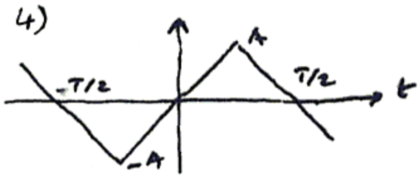


$$f_2(t) = \frac{4A}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \dots\right]$$

3)

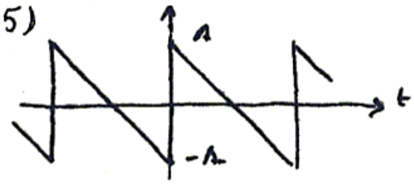
$$f_3(t) = \frac{8A}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right]$$

4)



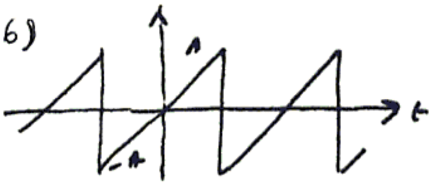
$$f_4(t) = \frac{8A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t - \dots\right]$$

5)



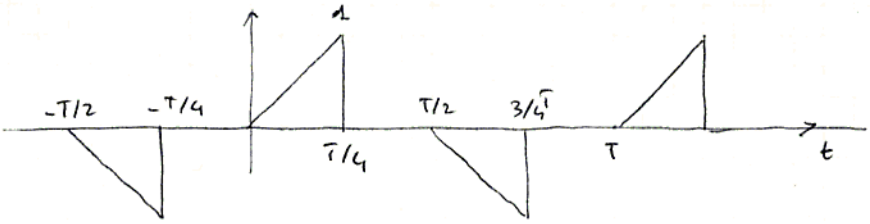
$$f_5(t) = \frac{2A}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \dots\right)$$

6)



$$f_6(t) = \frac{2A}{\pi}\left(\sin\omega t - \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t - \dots\right)$$

①



1A $\quad f(t) = A_0 + \sum_{1}^{\infty} a_n \cos n\omega t + \sum_{1}^{\infty} b_n \sin n\omega t \quad$ con $\omega = \frac{2\pi}{T} \quad n = 1, 2, 3 \dots$

$$f(t) = \begin{cases} f_1 = \frac{1}{T/4} t = \frac{4}{T} t & \text{per } 0 < t < T/4 \\ f_2 = -\frac{4}{T}(t - T/2) & \text{per } T/2 < t < \frac{3}{4}T \\ 0 & \text{per } T/4 < t < T/2 \\ 0 & \text{per } \frac{3}{4}T < t < T \end{cases} \quad \text{oppure} \quad \begin{cases} f_1 = \frac{4}{T} t & \text{per } 0 < t < T/4 \\ f_3 = -\frac{4}{T}\left(t + \frac{T}{2}\right) & \text{per } -T/2 < t < -T/4 \\ 0 & \text{per } -T/4 < t < 0 \\ 0 & \text{per } \frac{T}{4} < t < \frac{T}{2} \end{cases}$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\left[\int_0^{T/4} f_1(t)\,dt + \int_{T/2}^{3/4 T} f_2(t)\,dt\right] = 0 \quad \text{oppure} \quad A_0 = \frac{1}{T}\left[\int_0^{T/4} f_1(t)\,dt + \int_{-T/2}^{-T/4} f_3(t)\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\cos n\omega t\,dt + \int_{T/2}^{\frac{3}{4}T} f_2(t)\cos n\omega t\,dt\right]$$

oppure

$$a_n = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\cos n\omega t\,dt + \int_{-T/2}^{-T/4} f_3(t)\cos n\omega t\,dt\right]$$

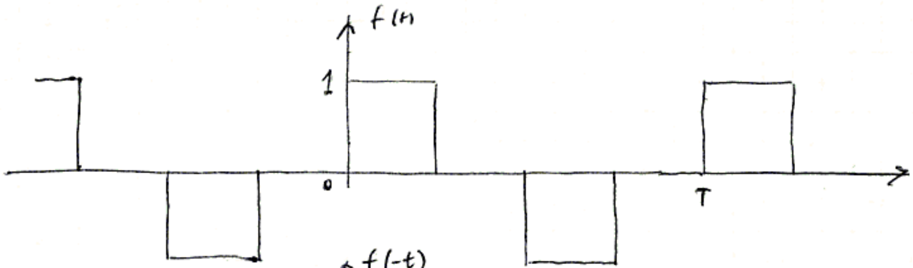
$$b_n = \frac{2}{T}\int_T f(t)\sin n\omega t\,dt = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\sin n\omega t\,dt + \int_{T/2}^{\frac{3}{4}T} f_2(t)\cos n\omega t\,dt\right]$$

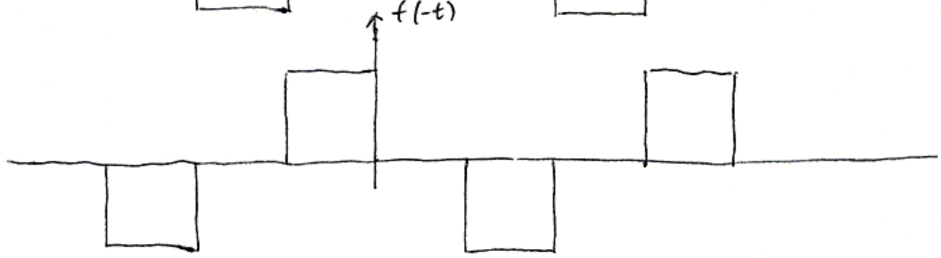
oppure

$$b_n = \frac{2}{T}\left[\int_0^{T/4} f_1(t)\sin n\omega t\,dt + \int_{-T/2}^{-T/4} f_3(t)\sin n\omega t\,dt\right]$$

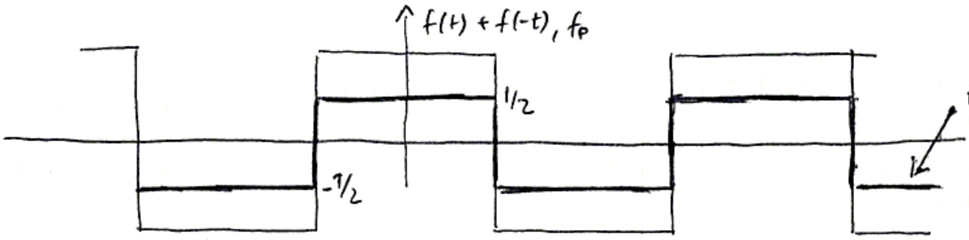
1B
- $f(t)$ NON È NÉ PARI NÉ DISPARI $\rightarrow$ $f(t) \neq f(-t)$ E $f(t) \neq -f(-t)$
  $\exists$ SIA COEFF. a_n SIA COEFF. b_n $[a_n \neq 0,\ b_n \neq 0]$
- $f(t)$ È EMISIMMETRICA (SIMMETRIA DI SEMIONDA) $\rightarrow f(t) = -f(t + T/2)$
  $\exists$ SOLO ARMONICHE DISPARI $[a_{2n} = 0$ E $b_{2n} = 0]$

2A)





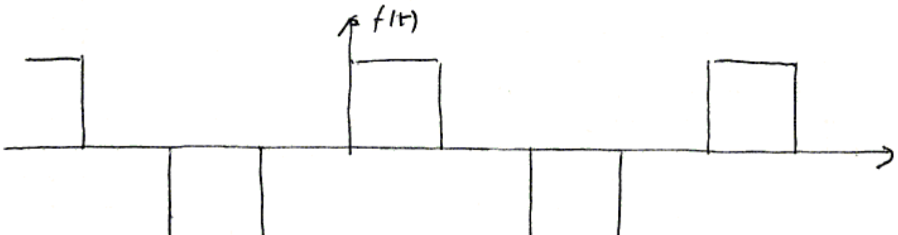
f. SPECCHIO
RISPETTO ALL'ASSE DELLE ORDINATE

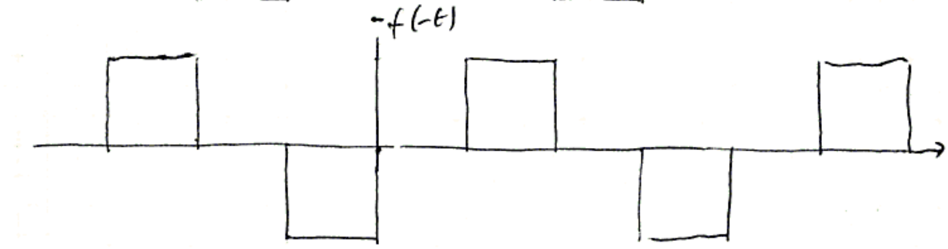


$$f_P = \frac{f(t)+f(-t)}{2}$$

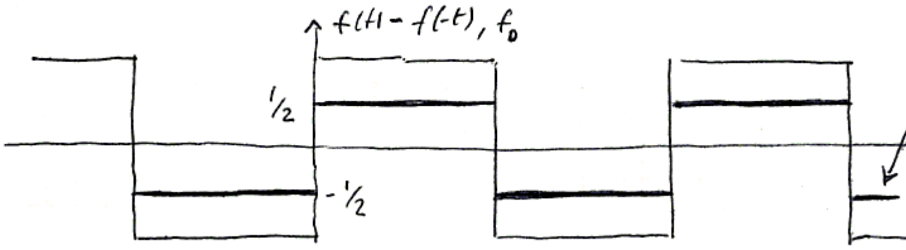
$$f_P = \frac{2}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t \ldots\right]$$

(ALLEGATO CON A=1/2)





f. OPPOSTA
DELLA f. SPECCHIO



$$f_D = \frac{f(t)-f(-t)}{2}$$

$$f_D = \frac{2}{\pi}\left[\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t \ldots\right]$$

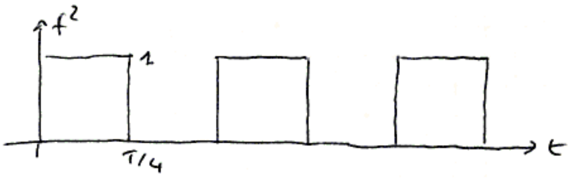
$$f(t) = f_P + f_D = \frac{2}{\pi}\left[\cos\omega t + \sin\omega t - \frac{1}{3}(\cos 3\omega t - \sin 3\omega t) + \frac{1}{5}(\cos 5\omega t + \sin 5\omega t) - \cdots\right]$$

2B

$$f_{RMS} = \sqrt{\frac{1}{T}\int_T f^2 dt} =$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T/4} dt + \int_{T/2}^{3/4 T} dt\right]} =$$

$$= \sqrt{\frac{1}{T}\, 2\, \frac{T}{4}} = \sqrt{\frac{1}{2}}$$



$f^2(t) = 1$ per $0 < t < T/4$
$f^2(t) = 1$ per $\frac{T}{2} < t < \frac{3}{4}T$

3) 4) 5) a) v. soluzione al file 2009 1a simulazione 5AI riportato di seguito

3) $V_i(s) = \frac{1}{s}$

$$V_o(s) = F(s) \cdot V_i(s) = \frac{1+5s}{1+s} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{1+s}$$

$$A = \lim_{s\to 0} s \cdot V_o(s) = \left.\frac{1+5s}{1+s}\right|_{s=0} = 1$$

$$B = \lim_{s\to -1} (s+1) \cdot V_o(s) = \left.\frac{1+5s}{s}\right|_{s=-1} = 4$$

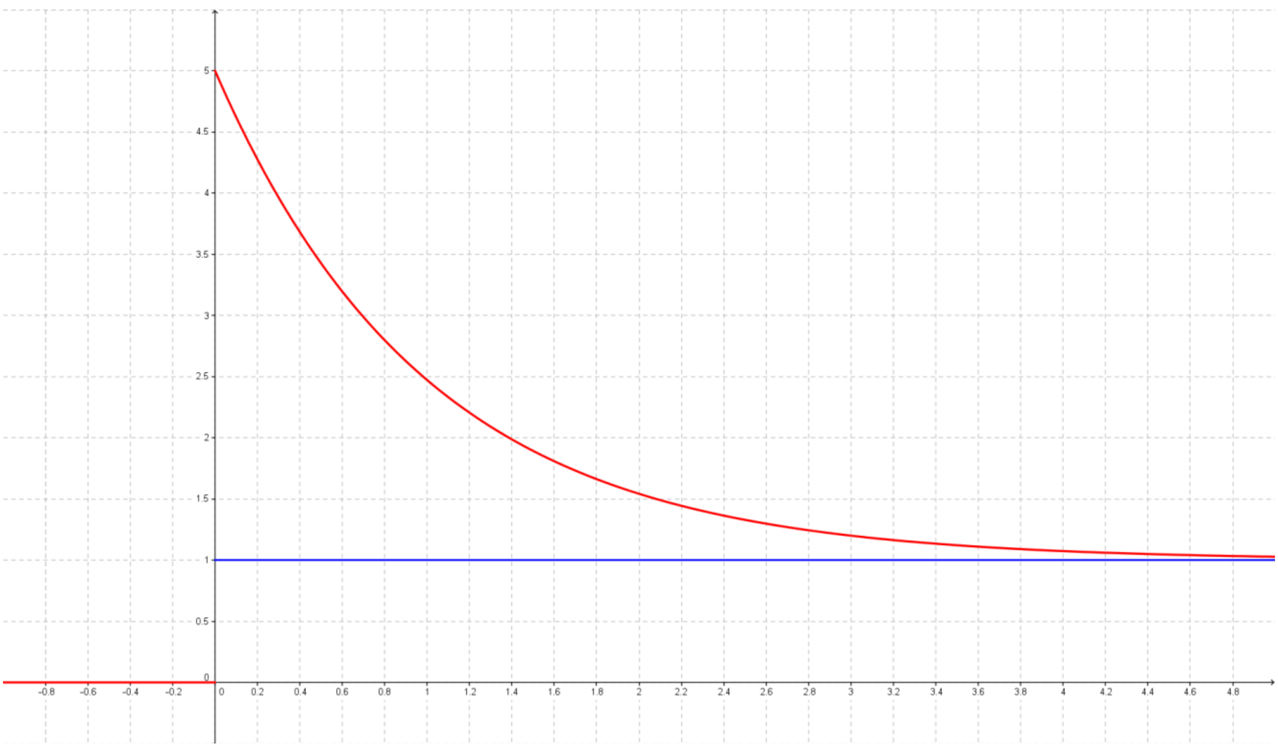
$$V_o(s) = \frac{1}{s} + \frac{4}{1+s}$$

$$v_o(t) = \mathcal{L}^{-1}[V_o(s)] = (1 + 4 \cdot e^{-t}) \cdot u(t)$$

In altro modo

$$\frac{1+5s}{1+s} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{1+s} = \frac{A + As + Bs}{s(1+s)}$$

$$\begin{cases} A = 1 \\ A + B = 5 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 4 \end{cases}$$



4a) $F(s) = \frac{V_o(s)}{V_i(s)} = \frac{z_1 + z_2}{z_1}$ con $\begin{cases} Z_1 = R_1 + \frac{1}{sC} = \frac{1+sR_1C}{sC} \\ Z_2 = R_2 \end{cases}$

$$F(s) = \frac{\frac{1+sR_1C}{sC} + R_2}{\frac{1+sR_1C}{sC}} = \frac{1+s(R_1+R_2)C}{1+sR_1C}$$

sostituendo

$$F(s) = \frac{1+5s}{1+s}$$

poli e zeri

poli: D(s) = 0 → 1 + s = 0 → s = -1 (p_1)

zeri: N(s) = 0 → 1 + 5s = 0 → s = $-\frac{1}{5}$ (z_1)

s $\rightarrow j\omega$ $\quad \bar{F}(j\omega) = \frac{1+j5\omega}{1+j\omega}$

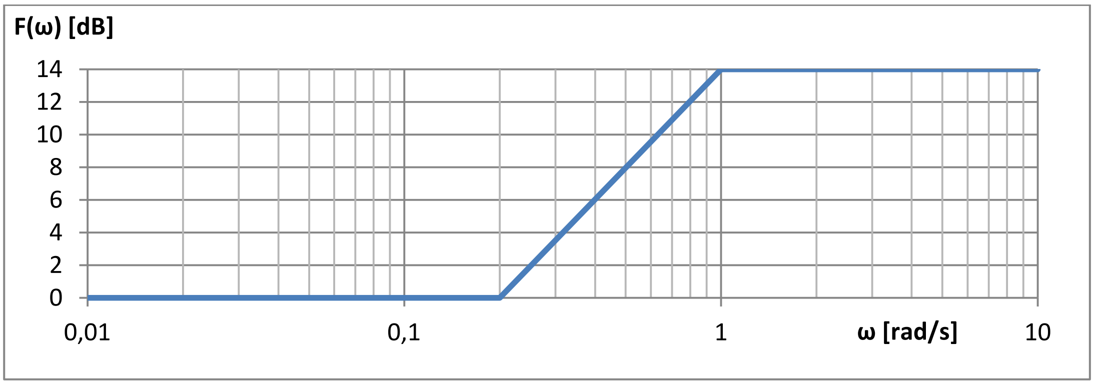
espressione del modulo e della fase della FdT

$$F(\omega) = \frac{\sqrt{1+(5\omega)^2}}{\sqrt{1+\omega^2}} \qquad \varphi_F(\omega) = \text{atg}(5\omega) - \text{atg}(\omega)$$

4b) alle freq. molto basse il cond. è un c.to aperto → $F(0) = 1$ $\;(0\,dB)$

alle freq. molto alte il cond. è un c.to c.to → $F(\infty) = 5$ $\;(14\,dB)$

diagramma di Bode del modulo della FdT



4c) $v_o(t) = V_o \cdot \sin(\bar{\omega}t + \psi)$ con $\begin{cases} \bar{\omega} = 0{,}8\,\frac{rad}{s} \\ V_o = F(\bar{\omega}) \cdot V_i \quad \begin{cases} V_i = 2\,V \\ \varphi_i = 0\,rad \end{cases} \\ \psi = \varphi_i + \varphi_F(\bar{\omega}) \end{cases}$

$$F(\bar{\omega}) = \frac{\sqrt{1+(5\cdot 0{,}8)^2}}{\sqrt{1+0{,}8^2}} = 3{,}22$$

$$\varphi_F(\omega) = \text{atg}(5\cdot 0{,}8) - \text{atg}(0{,}8) = 37{,}3°$$

$$v_o(t) = 6{,}44 \cdot \sin(0{,}8\,t + 37{,}3°)$$

5a) $e(t) = E_o + E_1 \cdot \sin(\omega_1 t + \varphi_1)$

$\omega_1 = \frac{2\pi}{T}\ rad/s$ con T = 0,016 s (f = 62,5 Hz) $\rightarrow$ $\omega_1 = 125\,\pi\ rad/s$

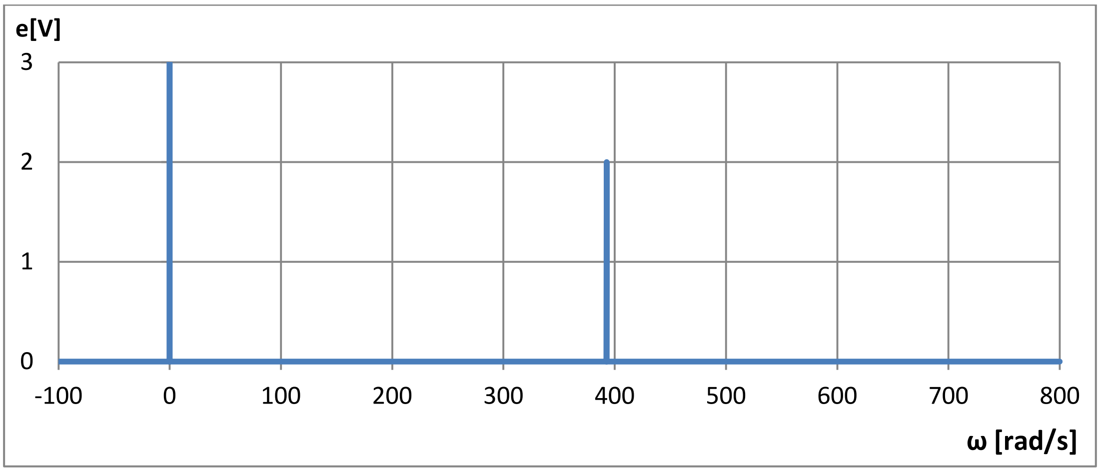
$E_o = \frac{e_{max} + e_{min}}{2} = \frac{5+1}{2} = 3\ V$

$E_1 = \frac{e_{max} - e_{min}}{2} = \frac{5-1}{2} = 2\ V$

$e(0) = E_o + E_1 \cdot \sin(\varphi_1)$ $\rightarrow \sin(\varphi_1) = \frac{e(0) - E_o}{E_1} = \frac{4-3}{2} = \frac{1}{2}$ $\rightarrow$ $\varphi_1 = 30°$

$e(t) = 3 + 2 \cdot \sin(\omega_1 t + 30°)$ con $\omega_1 = 125\,\pi\ rad/s$

Spettro delle ampiezze



$e_{RMS} = \sqrt{(E_o)^2 + \frac{1}{2}\ (E_1)^2}$ = 3,317 V

5b) Lo spettro delle ampiezze del segnale è riportato al punto precedente ed è costituito da due righe corrispondenti alla componente continua e alla sinusoide a freq. 62,5 Hz (pulsazione $125\,\pi\ rad/s$).

Il filtro passa basso ideale, con frequenza di taglio 10 Hz, blocca le componenti del segnale a frequenze superiori a questa e lascia passare inalterate le componenti a frequenze inferiori; in questo caso passa solo la componente continua con guadagno G=1, quindi

$v_o(t) = G \cdot E_o = 1 \cdot 3 = 3\ V$

6) a) IL PRIMO TERMINE RAPPRESENTA UN GRADINO DI AMPIEZZA $\frac{3}{2}$.

IL SECONDO RAPPRESENTA UNA FUNZIONE MOLTIPLICATA PER UN ESPONENZIALE, ALLA QUALE CORRISPONDE UNA TRASLAZIONE NEL DOMINIO DI LAPLACE, CIOÈ:

$$\text{SE } \mathcal{L}[f(t)] = F(s) \rightarrow \mathcal{L}[e^{-\alpha t} f(t)] = F(s+\alpha)$$

$$Y(s) = \frac{3}{2}\cdot\frac{1}{s} - \frac{3}{2}\frac{s+1}{(s+1)^2+1} + \frac{7}{2}\cdot\frac{1}{(s+1)^2+1} =$$

$$= \frac{3[(s+1)^2+1] - 3s(s+1) + 7s}{2s(s^2+2s+2)} = \frac{3s^2+6s+6-3s^2-3s+7s}{2s(s^2+2s+2)} = \frac{10s+6}{2s(s^2+2s+2)} =$$

$$= \frac{5s+3}{s(s^2+2s+2)}$$

b) $$y(t) = r(t+2) - r(t+1) + u(t) - 3u(t-1) + \frac{1}{2}r(t-1) - \frac{1}{2}r(t-3)$$

$$Y(s) = \frac{1}{s} - \frac{3}{s}e^{-s} + \frac{1}{2}\frac{1}{s^2}e^{-s} - \frac{1}{2}\frac{1}{s^2}e^{-3s}$$

c) $$f_1(t) = \left[7 + \frac{2}{3}e^{-\frac{1}{2}t} - 3t\right]u(t)$$

d) $$F_2(s) = \frac{-s+1}{(s^2+1)(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$C = \left.\frac{-s+1}{s^2+1}\right|_{s=-1} = \frac{2}{2} = 1$$

$$(As+B)(s+1) + C(s^2+1) = -s+1 \rightarrow As^2 + (A+B)s + B + Cs^2 + C = -s+1$$

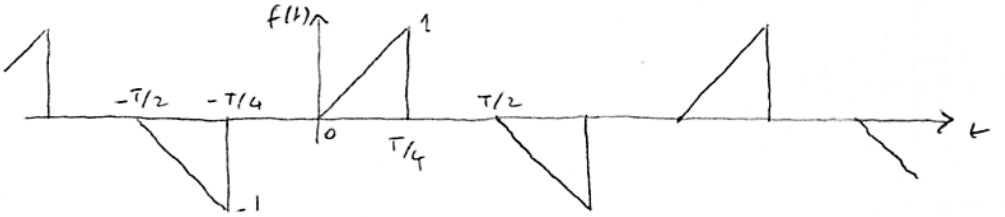
$$(A+C)s^2 + (A+B)s + B + C = -s+1$$

$$\begin{cases} A+C=0 \\ A+B=-1 \\ B+C=1 \\ C=1 \end{cases} \Rightarrow \begin{cases} C=1 \\ B=0 \\ A=-1 \end{cases}$$

$$F_2(s) = \frac{-s}{s^2+1} + \frac{1}{s+1}$$

$$f_2(t) = \left[-\cos t + e^{-t}\right]u(t)$$

SVILUPPO IN SERIE DI FOURIER DELLA SEG. FORMA D'ONDA



a) SOLUZIONE MEDIANTE CALCOLO DIRETTO DEI COEFFICIENTI

$$f(t) = \begin{cases} -\frac{4}{T}\left(t+\frac{T}{2}\right) = -\frac{4}{T}t - 2 & \text{per } -\frac{T}{2}<t<-\frac{T}{4} \\ \frac{4}{T}t & \text{per } 0<t<T/4 \\ 0 & \text{per } -\frac{T}{4}<t<0 \quad \text{e per } \frac{T}{4}<t<\frac{T}{2} \end{cases}$$

$$f(t) = A_0 + \sum_1^{\infty} a_n \cos n\omega t + \sum_1^{\infty} b_n \sin n\omega t \qquad \text{con } \omega = \frac{2\pi}{T} \quad n=1,2,\dots$$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt = \frac{1}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)dt + \int_0^{T/4}\frac{4}{T}t\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega t\,dt = \frac{2}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)\cos n\omega t\,dt + \int_0^{T/4}\frac{4}{T}t\cos n\omega t\,dt\right] =$$

$$= -\frac{8}{T^2}\int_{-T/2}^{-T/4} t\cos n\omega t\,dt - \frac{4}{T}\int_{-T/2}^{-T/4}\cos n\omega t\,dt + \frac{8}{T^2}\int_0^{T/4} t\cos n\omega t\,dt =$$

PER PARTI $\int t\cos n\omega t\,dt = \int t\,d\left(\frac{\sin n\omega t}{n\omega}\right) = t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}$

$$= -\frac{8}{T^2}\left[t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{(n\omega)^2}\right]_{-T/2}^{-T/4} - \frac{4}{T}\frac{\sin n\omega t}{n\omega}\Bigg|_{-T/2}^{-T/4} + \frac{8}{T^2}\left[t\frac{\sin n\omega t}{n\omega} + \frac{\cos n\omega t}{n^2\omega^2}\right]_0^{T/4} =$$

$$= -\frac{8}{T^2}\left[-\frac{T}{4}\frac{\sin n\omega(-T/4)}{n\omega} + \frac{\cos n\omega(-T/4)}{n^2\omega^2} + \frac{T}{2}\frac{\sin n\omega(-T/2)}{n\omega} - \frac{\cos n\omega(-T/2)}{n^2\omega^2}\right] - \frac{4}{T}\frac{\sin n\omega(-T/4)}{n\omega} +$$

$$+ \frac{4}{T}\frac{\sin n\omega(-T/2)}{n\omega} + \frac{8}{T^2}\left[\frac{T}{4}\frac{\sin n\omega T/4}{n\omega} + \frac{\cos n\omega T/4}{n^2\omega^2} - \frac{1}{n^2\omega^2}\right] =$$

$$= \cancel{-\frac{2}{T}\frac{\sin n\omega T/4}{n\omega}} \cancel{- \frac{8\cos n\omega T/4}{n^2\omega^2T^2}} \cancel{+ \frac{4}{T}\frac{\sin n\omega T/2}{n\omega}} + \frac{8}{T^2}\frac{\cos n\omega T/2}{n^2\omega^2} + \frac{4}{T}\frac{\sin n\omega T/4}{n\omega} +$$

$$\cancel{-\frac{4}{T}\frac{\sin n\omega T/2}{n\omega}} \cancel{+ \frac{2}{T}\frac{\sin n\omega T/4}{n\omega}} \cancel{+ \frac{8\cos n\omega T/4}{n^2\omega^2T^2}} - \frac{8}{n^2\omega^2T^2} =$$

$$\begin{cases} \omega\frac{T}{4} = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2} \\ \omega T/2 = \pi \\ \omega T = 2\pi \end{cases}$$

$$= \frac{8}{n^2 4\pi^2}\cos n\pi + \frac{4}{2n\pi}\sin n\pi/2 - \frac{8}{4\pi^2 n^2} =$$

$$= \frac{2}{n\pi}\sin n\frac{\pi}{2} - \frac{2}{n^2\pi^2}(1-\cos n\pi)$$

$n=1 \qquad a_1 = \frac{2}{\pi} - \frac{4}{\pi^2}$

$n=2 \qquad a_2 = 0$

$n=3 \qquad a_3 = -\frac{2}{3\pi} - \frac{4}{3^2\pi^2}$

$n=5 \qquad a_5 = \frac{2}{5\pi} - \frac{4}{5^2\pi^2}$

$$b_n = \frac{2}{T}\int_T f(t)\sin n\omega t\,dt = \frac{2}{T}\left[\int_{-T/2}^{-T/4}\left(-\frac{4}{T}t-2\right)\sin n\omega t + \int_0^{T/4}\frac{4}{T}t\sin n\omega t\,dt\right] =$$

$$= -\frac{8}{T^2}\int_{-T/2}^{-T/4} t\sin n\omega t\,dt - \frac{4}{T}\int_{-T/2}^{-T/4}\sin n\omega t\,dt + \frac{8}{T^2}\int_0^{T/4} t\sin n\omega t\,dt =$$

PER PARTI $\int t\sin n\omega t\,dt = \int t\, d\left(-\frac{\cos n\omega t}{n\omega}\right) = -\frac{t\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}$

$$= -\frac{8}{T^2}\left[-t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_{-T/2}^{-T/4} + \frac{4}{T}\frac{\cos n\omega t}{n\omega}\Bigg|_{-T/2}^{-T/4} + \frac{8}{T^2}\left[-t\frac{\cos n\omega t}{n\omega} + \frac{\sin n\omega t}{(n\omega)^2}\right]_0^{T/4} =$$

$$= -\frac{8}{T^2}\left[\frac{T}{4}\frac{\cos n\omega(-T/4)}{n\omega} + \frac{\sin n\omega(-T/4)}{n^2\omega^2} - \frac{T}{2}\frac{\cos n\omega(-T/2)}{n\omega} - \frac{\sin n\omega(-T/2)}{n^2\omega^2}\right] + \frac{4}{T}\frac{\cos n\omega(-T/4)}{n\omega} +$$

$$-\frac{4}{T}\frac{\cos n\omega(-T/2)}{n\omega} + \frac{8}{T^2}\left[-\frac{T}{4}\frac{\cos n\omega T/4}{n\omega} + \frac{\sin n\omega T/4}{n^2\omega^2}\right] =$$

$$= \cancel{-\frac{2}{T}\frac{\cos n\omega T/4}{n\omega}} + \frac{8\sin n\omega T/4}{n^2\omega^2T^2} + \cancel{\frac{4}{T}\frac{\cos n\omega T/2}{n\omega}} - \frac{8}{T^2}\frac{\sin n\omega T/2}{n^2\omega^2} + \cancel{\frac{4}{T}\frac{\cos n\omega T/4}{n\omega}} +$$

$$\cancel{-\frac{4}{T}\frac{\cos n\omega T/2}{n\omega}} \cancel{-\frac{2}{T}\frac{\cos n\omega T/4}{n\omega}} + \frac{8\sin n\omega T/4}{n^2\omega^2T^2} =$$

$$= 16\frac{\sin n\omega T/4}{n^2\omega^2T^2} - \frac{8\sin n\omega T/2}{n^2\omega^2T^2}$$

$\omega\frac{T}{4} = \frac{2\pi}{T}\cdot\frac{T}{4} = \frac{\pi}{2}$

$\omega\frac{T}{2} = \frac{2\pi}{T}\cdot\frac{T}{2} = \pi$

$n\omega T = 2\pi$

$$= \frac{16\sin n\pi/2}{4n^2\pi^2} - \frac{8}{4n^2\pi^2}\cdot\sin n\pi$$

$$= \frac{4}{n^2\pi^2}\sin n\pi/2$$

$n=1 \quad a_1 = \frac{4}{\pi^2}$

$n=2 \quad a_2 = 0$

$n=3 \quad a_3 = -\frac{4}{3^2\pi^2}$

$n=5 \quad a_5 = \frac{4}{5^2\pi^2}$

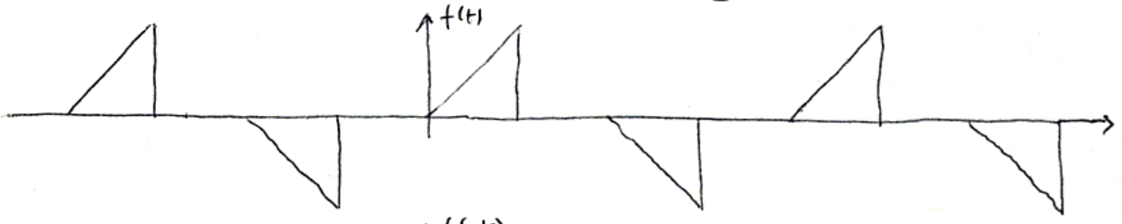
$n=7 \quad a_7 = -\frac{4}{7^2\pi^2}$

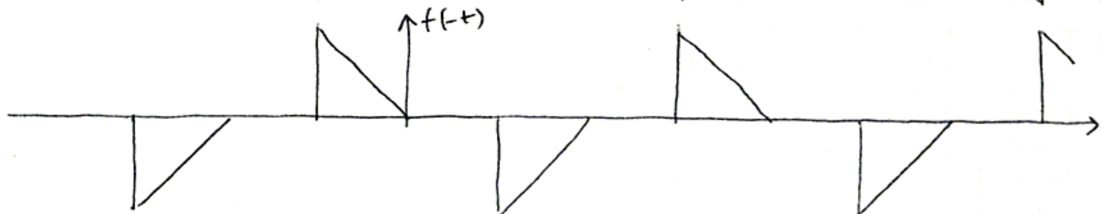
$$f(t) = \sum_{1}^{\infty}{}_n\left[\frac{2}{n\pi}\sin n\frac{\pi}{2} - \frac{2}{n^2\pi^2}(1-\cos n\pi)\right]\cos n\omega t + \sum_1 {}_n \frac{4}{n^2\pi^2}\sin n\frac{\pi}{2}\sin n\omega t =$$

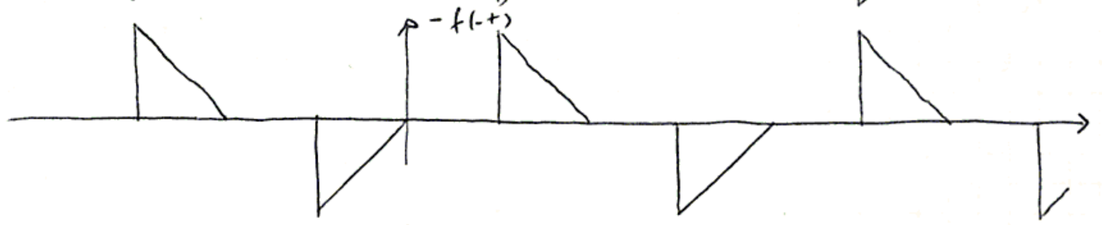
$$= \left(\frac{2}{\pi}-\frac{4}{\pi^2}\right)\cos\omega t - \left(\frac{2}{3\pi}-\frac{4}{3^2\pi^2}\right)\cos 3\omega t + \left(\frac{2}{5\pi}-\frac{4}{5^2\pi^2}\right)\cos 5\omega t - \dots + \frac{4}{\pi^2}\sin\omega t - \frac{4}{3^2\pi^2}\sin 3\omega t + \frac{4}{5^2\pi^2}\sin 5\omega t - \dots$$

b) SOLUZIONE MEDIANTE SCOMPOSIZIONE DELLA f d'O in SOMMA DI f d'O PARI E DISPARI

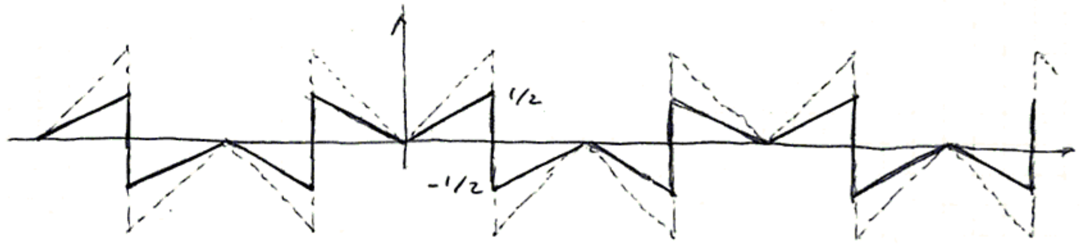
$$f_P = \frac{f(t) + f(-t)}{2} \qquad f_D = \frac{f(t) - f(-t)}{2}$$







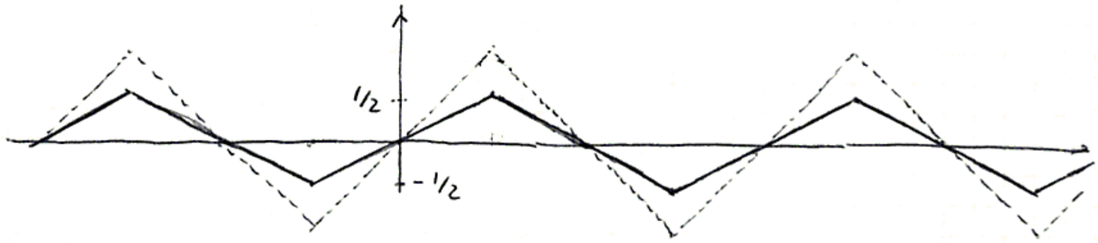
P



IN TRATTEGGIO $f(t) + f(-t)$

IN TRATTO CONTINUO f_P

D

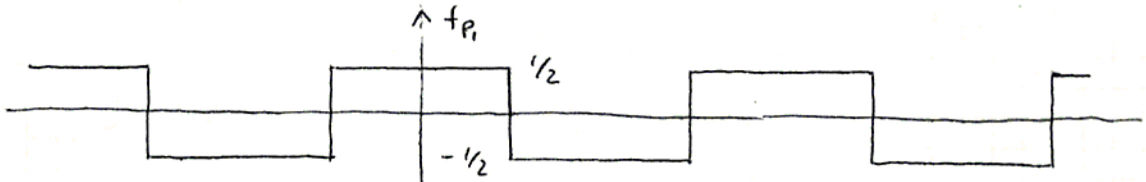


IN TRATTEGGIO $f(t) - f(-t)$

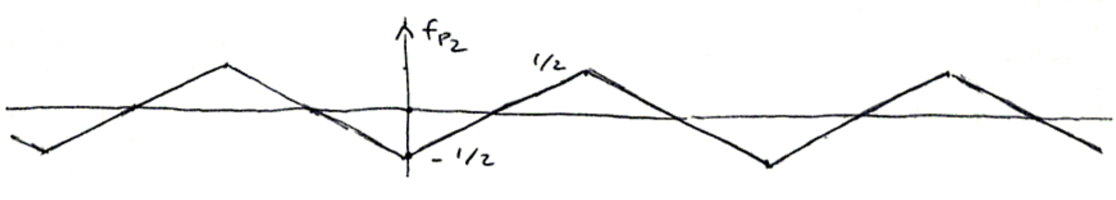
IN TRATTO CONTINUO f_D

LA FORMA D'ONDA PARI PUÒ ESSERE SCOMPOSTA NELLA SOMMA DI ALTRE 2 f.d'O PARI DI CUI È NOTO LO SVILUPPO IN SERIE.

P1



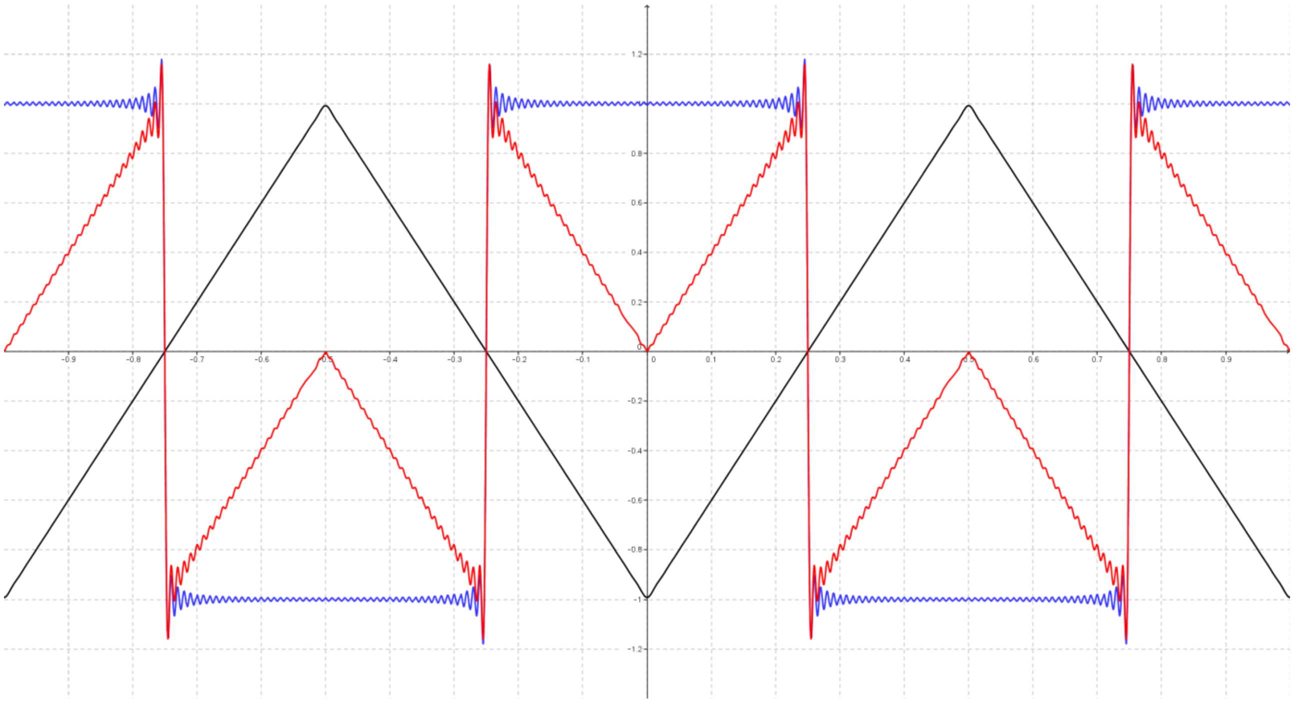
P2



$$f(t) = f_P + f_D = f_{P_1} + f_{P_2} + f_D$$

(FORME D'ONDA 1), 9) E 4) IN ALLEGATO 2 CON $A = \frac{1}{2}$)

CON
$$\begin{cases} f_{P_1} = \frac{2}{\pi}\left[\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t - \dots\right] \\ f_{P_2} = -\frac{4}{\pi^2}\left[\cos\omega t + \frac{1}{3^2}\cos 3\omega t + \frac{1}{5^2}\cos 5\omega t + \dots\right] \\ f_D = \frac{4A}{\pi^2}\left[\sin\omega t - \frac{1}{3^2}\sin 3\omega t + \frac{1}{5^2}\sin 5\omega t + \dots\right] \end{cases}$$

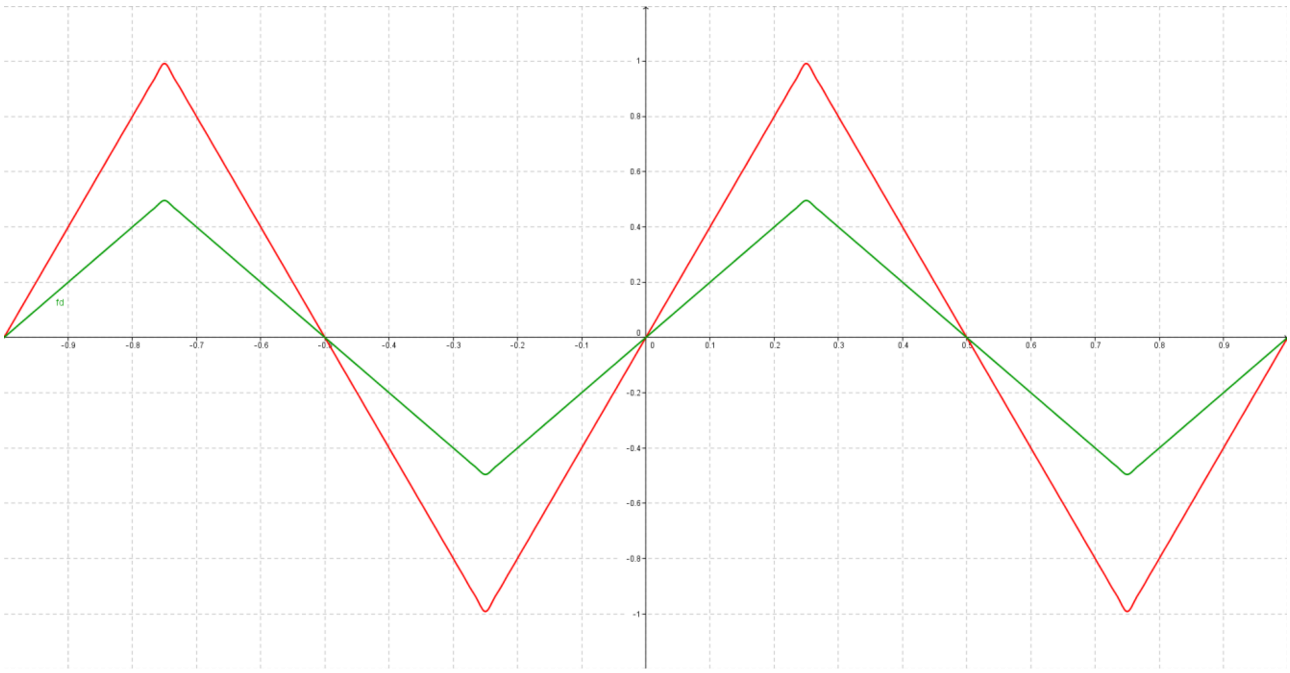


$$\text{QuadraPari}(x) = 4 / \pi\, (\cos(2\pi x) - 1 / 3 \cos(2\,(3)\,\pi x) + 1 / 5 \cos(2\,(5)\,\pi x) - 1 / 7 \cos(2\,(7)\,\pi x) + 1 / 9 \cos(2\,(9)\,\pi x) - 1 / 11 \cos(2\,(11)\,\pi x) + 1 / 13 \cos(2\,(13)\,\pi x) - 1 / 15 \cos(2\,(15)\,\pi x) + 1 / 17 \cos(2\,(17)\,\pi x) - 1 / 19 \cos(2\,(19)\,\pi x) + 1 / 21 \cos(2\,(21)\,\pi x) - 1 / 23 \cos(2\,(23)\,\pi x) + 1 / 25 \cos(2\,(25)\,\pi x) - 1 / 27 \cos(2\,(27)\,\pi x) + 1 / 29 \cos(2\,(29)\,\pi x) - 1 / 31 \cos(2\,(31)\,\pi x) + 1 / 33 \cos(2\,(33)\,\pi x) - 1 / 35 \cos(2\,(35)\,\pi x) + 1 / 37 \cos(2\,(37)\,\pi x) - 1 / 39 \cos(2\,(39)\,\pi x) + 1 / 41 \cos(2\,(41)\,\pi x) - 1 / 43 \cos(2\,(43)\,\pi x) + 1 / 45 \cos(2\,(45)\,\pi x) - 1 / 47 \cos(2\,(47)\,\pi x) + 1 / 49 \cos(2\,(49)\,\pi x) - 1 / 51 \cos(2\,(51)\,\pi x) + 1 / 53 \cos(2\,(53)\,\pi x) - 1 / 55 \cos(2\,(55)\,\pi x) + 1 / 57 \cos(2\,(57)\,\pi x) - 1 / 59 \cos(2\,(59)\,\pi x) + 1 / 61 \cos(2\,(61)\,\pi x) - 1 / 63 \cos(2\,(63)\,\pi x) + 1 / 65 \cos(2\,(65)\,\pi x) - 1 / 67 \cos(2\,(67)\,\pi x) + 1 / 69 \cos(2\,(69)\,\pi x) - 1 / 71 \cos(2\,(71)\,\pi x) + 1 / 73 \cos(2\,(73)\,\pi x) - 1 / 75 \cos(2\,(75)\,\pi x) + 1 / 77 \cos(2\,(77)\,\pi x) - 1 / 79 \cos(2\,(79)\,\pi x) + 1 / 81 \cos(2\,(81)\,\pi x) - 1 / 83 \cos(2\,(83)\,\pi x) + 1 / 85 \cos(2\,(85)\,\pi x) - 1 / 87 \cos(2\,(87)\,\pi x) + 1 / 89 \cos(2\,(89)\,\pi x) - 1 / 91 \cos(2\,(91)\,\pi x) + 1 / 93 \cos(2\,(93)\,\pi x) - 1 / 95 \cos(2\,(95)\,\pi x) + 1 / 97 \cos(2\,(97)\,\pi x) - 1 / 99 \cos(2\,(99)\,\pi x))$$

$$\text{OTriangolarePari}(x) = (-8) / \pi^2\, (\cos(2\pi x) + 1 / 3^2 \cos(2\,(3)\,\pi x) + 1 / 5^2 \cos(2\,(5)\,\pi x) + 1 / 7^2 \cos(2\,(7)\,\pi x) + 1 / 9^2 \cos(2\,(9)\,\pi x) + 1 / 11^2 \cos(2\,(11)\,\pi x) + 1 / 13^2 \cos(2\,(13)\,\pi x) + 1 / 15^2 \cos(2\,(15)\,\pi x) + 1 / 17^2 \cos(2\,(17)\,\pi x) + 1 / 19^2 \cos(2\,(19)\,\pi x) + 1 / 21^2 \cos(2\,(21)\,\pi x) + 1 / 23^2 \cos(2\,(23)\,\pi x) + 1 / 25^2 \cos(2\,(25)\,\pi x) + 1 / 27^2 \cos(2\,(27)\,\pi x) + 1 / 29^2 \cos(2\,(29)\,\pi x) + 1 / 31^2 \cos(2\,(31)\,\pi x) + 1 / 33^2 \cos(2\,(33)\,\pi x) + 1 / 35^2 \cos(2\,(35)\,\pi x) + 1 / 37^2 \cos(2\,(37)\,\pi x) + 1 / 39^2 \cos(2\,(39)\,\pi x) + 1 / 41^2 \cos(2\,(41)\,\pi x) + 1 / 43^2 \cos(2\,(43)\,\pi x) + 1 / 45^2 \cos(2\,(45)\,\pi x) + 1 / 47^2 \cos(2\,(47)\,\pi x) + 1 / 49^2 \cos(2\,(49)\,\pi x))$$

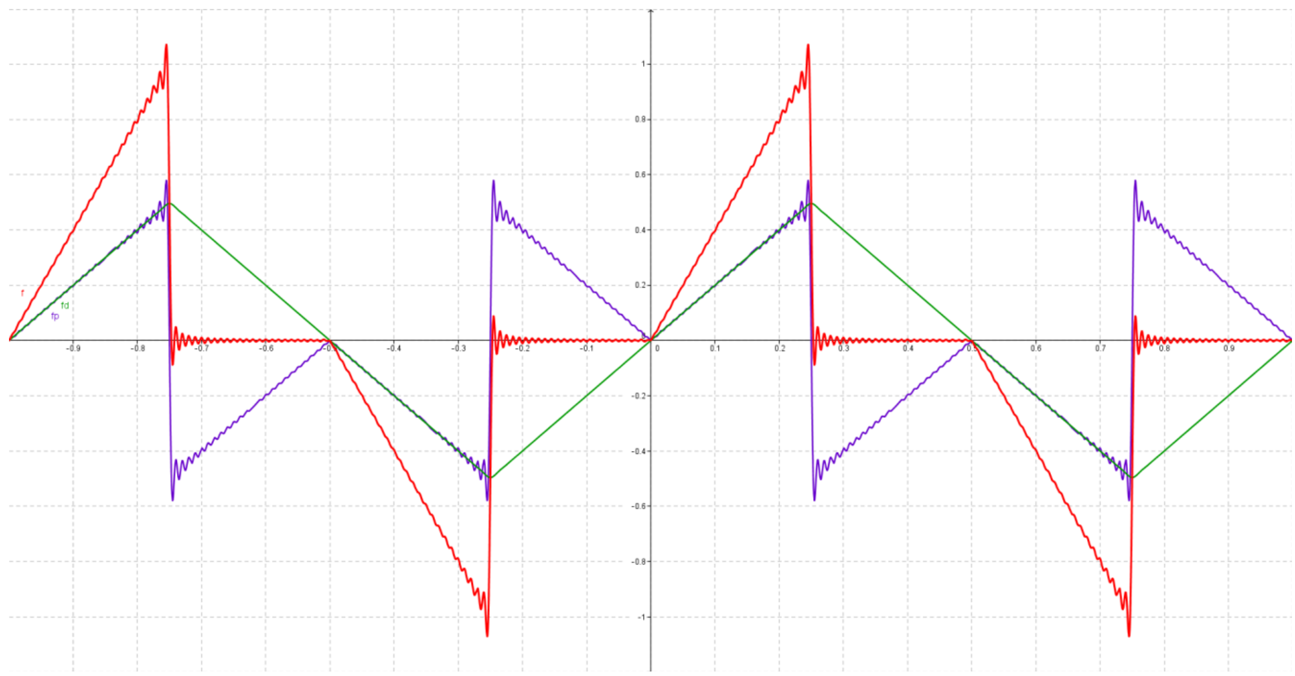
$$fp2(x) = \text{QuadraPari}(x) + \text{OTriangolarePari}(x)$$

$$fp(x) = 1/2\; fp2(x)$$

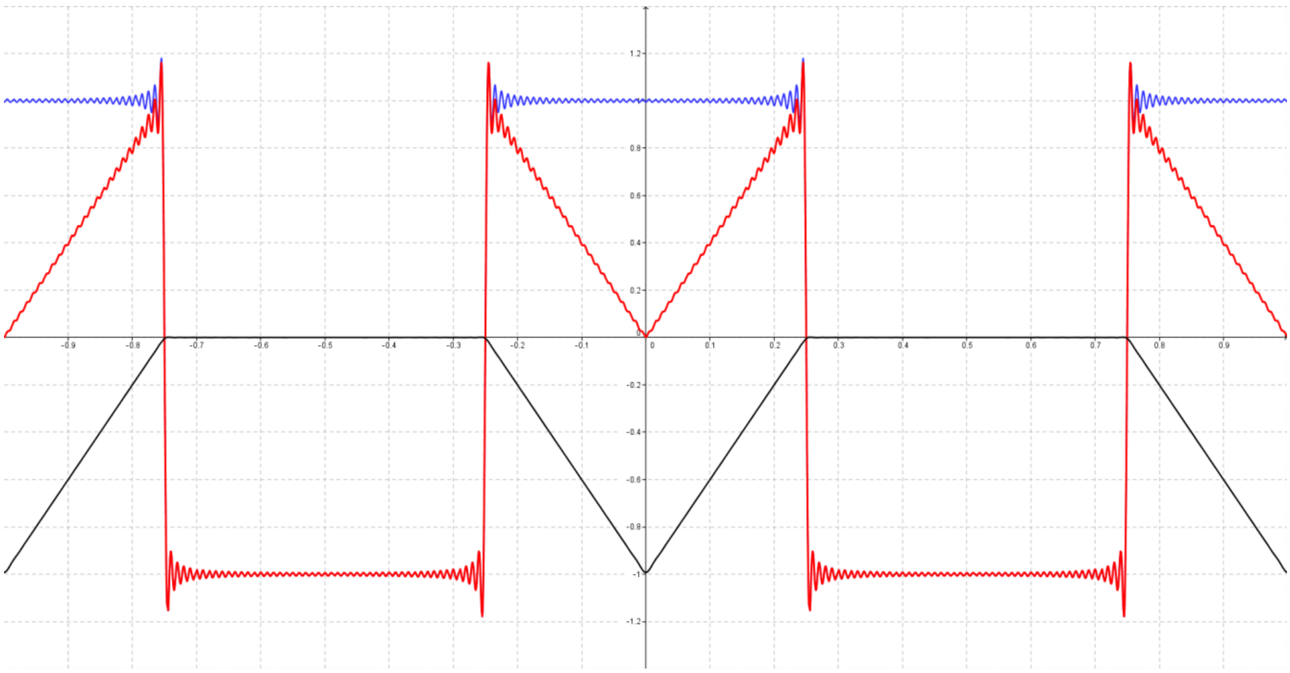


$\text{TriangolareDispari}(x) = 8 / \pi^2 (\text{sen}(2\pi x) - 1 / 3^2 \text{ sen}(2 (3) \pi x) + 1 / 5^2 \text{ sen}(2 (5) \pi x) - 1 / 7^2 \text{ sen}(2 (7) \pi x) + 1 / 9^2 \text{ sen}(2 (9) \pi x) - 1 / 11^2 \text{ sen}(2 (11) \pi x) + 1 / 13^2 \text{ sen}(2 (13) \pi x) - 1 / 15^2 \text{ sen}(2 (15) \pi x) + 1 / 17^2 \text{ sen}(2 (17) \pi x) - 1 / 19^2 \text{ sen}(2 (19) \pi x) + 1 / 21^2 \text{ sen}(2 (21) \pi x) - 1 / 23^2 \text{ sen}(2 (23) \pi x) + 1 / 25^2 \text{ sen}(2 (25) \pi x) - 1 / 27^2 \text{ sen}(2 (27) \pi x) + 1 / 29^2 \text{ sen}(2 (29) \pi x) - 1 / 31^2 \text{ sen}(2 (31) \pi x) + 1 / 33^2 \text{ sen}(2 (33) \pi x) - 1 / 35^2 \text{ sen}(2 (35) \pi x) + 1 / 37^2 \text{ sen}(2 (37) \pi x) - 1 / 39^2 \text{ sen}(2 (39) \pi x) + 1 / 41^2 \text{ sen}(2 (41) \pi x) - 1 / 43^2 \text{ sen}(2 (43) \pi x) + 1 / 45^2 \text{ sen}(2 (45) \pi x) - 1 / 47^2 \text{ sen}(2 (47) \pi x) + 1 / 49^2 \text{ sen}(2 (49) \pi x))$

$\text{fd}(x) = 1/2 \text{ TriangolareDispari}(x)$



f(x) = fp(x) + fd(x)



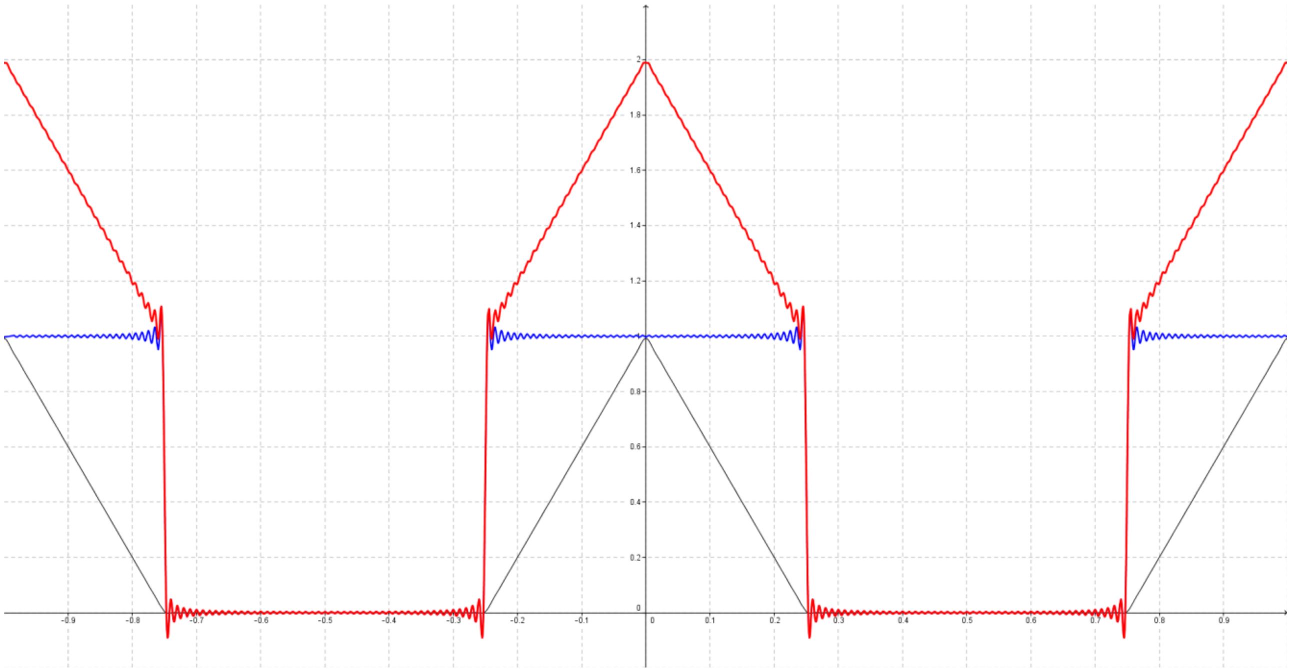
QuadraPari(x) = $4 / \pi (\cos(2\pi x) - 1 / 3 \cos(2 (3) \pi x) + 1 / 5 \cos(2 (5) \pi x) - 1 / 7 \cos(2 (7) \pi x) + 1 / 9 \cos(2 (9) \pi x) - 1 / 11 \cos(2 (11) \pi x) + 1 / 13 \cos(2 (13) \pi x) - 1 / 15 \cos(2 (15) \pi x) + 1 / 17 \cos(2 (17) \pi x) - 1 / 19 \cos(2 (19) \pi x) + 1 / 21 \cos(2 (21) \pi x) - 1 / 23 \cos(2 (23) \pi x) + 1 / 25 \cos(2 (25) \pi x) - 1 / 27 \cos(2 (27) \pi x) + 1 / 29 \cos(2 (29) \pi x) - 1 / 31 \cos(2 (31) \pi x) + 1 / 33 \cos(2 (33) \pi x) - 1 / 35 \cos(2 (35) \pi x) + 1 / 37 \cos(2 (37) \pi x) - 1 / 39 \cos(2 (39) \pi x) + 1 / 41 \cos(2 (41) \pi x) - 1 / 43 \cos(2 (43) \pi x) + 1 / 45 \cos(2 (45) \pi x) - 1 / 47 \cos(2 (47) \pi x) + 1 / 49 \cos(2 (49) \pi x) - 1 / 51 \cos(2 (51) \pi x) + 1 / 53 \cos(2 (53) \pi x) - 1 / 55 \cos(2 (55) \pi x) + 1 / 57 \cos(2 (57) \pi x) - 1 / 59 \cos(2 (59) \pi x) + 1 / 61 \cos(2 (61) \pi x) - 1 / 63 \cos(2 (63) \pi x) + 1 / 65 \cos(2 (65) \pi x) - 1 / 67 \cos(2 (67) \pi x) + 1 / 69 \cos(2 (69) \pi x) - 1 / 71 \cos(2 (71) \pi x) + 1 / 73 \cos(2 (73) \pi x) - 1 / 75 \cos(2 (75) \pi x) + 1 / 77 \cos(2 (77) \pi x) - 1 / 79 \cos(2 (79) \pi x) + 1 / 81 \cos(2 (81) \pi x) - 1 / 83 \cos(2 (83) \pi x) + 1 / 85 \cos(2 (85) \pi x) - 1 / 87 \cos(2 (87) \pi x) + 1 / 89 \cos(2 (89) \pi x) - 1 / 91 \cos(2 (91) \pi x) + 1 / 93 \cos(2 (93) \pi x) - 1 / 95 \cos(2 (95) \pi x) + 1 / 97 \cos(2 (97) \pi x) - 1 / 99 \cos(2 (99) \pi x))$

OTenda(x) = -Tenda(x)

Tenda(x) = $0.25 + 4 / \pi^2 (\cos(2\pi x) + 2 / 2^2 \cos(2 (2) \pi x) + 1 / 3^2 \cos(2 (3) \pi x) + 0\cos(2 (4) \pi x) + 1 / 5^2 \cos(2 (5) \pi x) + 2 / 6^2 \cos(2 (6) \pi x) + 1 / 7^2 \cos(2 (7) \pi x) + 0\cos(2 (8) \pi x) + 1 / 9^2 \cos(2 (9) \pi x) + 2 / 10^2 \cos(2 (10) \pi x) + 1 / 11^2 \cos(2 (11) \pi x) + 0\cos(2 (12) \pi x) + 1 / 13^2 \cos(2 (13) \pi x) + 2 / 14^2 \cos(2 (14) \pi x) + 1 / 15^2 \cos(2 (15) \pi x) + 0\cos(2 (16) \pi x) + 1 / 17^2 \cos(2 (17) \pi x) + 2 / 18^2 \cos(2 (18) \pi x) + 1 / 19^2 \cos(2 (19) \pi x) + 0\cos(2 (20) \pi x) + 1 / 21^2 \cos(2 (21) \pi x) + 2 / 22^2 \cos(2 (22) \pi x) + 1 / 23^2 \cos(2 (23) \pi x) + 0\cos(2 (24) \pi x) + 1 / 25^2 \cos(2 (25) \pi x) + 2 / 26^2 \cos(2 (26) \pi x) + 1 / 27^2 \cos(2 (27) \pi x) + 0\cos(2 (28) \pi x) + 1 / 29^2 \cos(2 (29) \pi x) + 2 / 30^2 \cos(2 (30) \pi x) + 1 / 31^2 \cos(2 (31) \pi x) + 0\cos(2 (32) \pi x) + 1 / 33^2 \cos(2 (33) \pi x) + 2 / 34^2 \cos(2 (34) \pi x) + 1 / 35^2 \cos(2 (35) \pi x) + 0\cos(2 (36) \pi x) + 1 / 37^2 \cos(2 (37) \pi x) + 2 / 38^2 \cos(2 (38) \pi x) + 1 / 39^2 \cos(2 (39) \pi x) + 0\cos(2 (40) \pi x) + 1 / 41^2 \cos(2 (41) \pi x) + 2 / 42^2 \cos(2 (42) \pi x) + 1 / 43^2 \cos(2 (43) \pi x) + 0\cos(2 (44) \pi x) + 1 / 45^2 \cos(2 (45) \pi x) + 2 / 46^2 \cos(2 (46) \pi x) + 1 / 47^2 \cos(2 (47) \pi x) + 0\cos(2 (48) \pi x) + 1 / 49^2 \cos(2 (49) \pi x) + 2 / 50^2 \cos(2 (50) \pi x) + 1 / 51^2 \cos(2 (51) \pi x) + 0\cos(2 (52) \pi x))$

Guglie(x) = QuadraPari(x) + OTenda(x

Le guglie del castello scaligero di Malcesine



QuadraPari(x) = $4 / \pi (\cos(2\pi x) - 1 / 3 \cos(2 (3) \pi x) + 1 / 5 \cos(2 (5) \pi x) - 1 / 7 \cos(2 (7) \pi x) + 1 / 9 \cos(2 (9) \pi x) - 1 / 11 \cos(2 (11) \pi x) + 1 / 13 \cos(2 (13) \pi x) - 1 / 15 \cos(2 (15) \pi x) + 1 / 17 \cos(2 (17) \pi x) - 1 / 19 \cos(2 (19) \pi x) + 1 / 21 \cos(2 (21) \pi x) - 1 / 23 \cos(2 (23) \pi x) + 1 / 25 \cos(2 (25) \pi x) - 1 / 27 \cos(2 (27) \pi x) + 1 / 29 \cos(2 (29) \pi x) - 1 / 31 \cos(2 (31) \pi x) + 1 / 33 \cos(2 (33) \pi x) - 1 / 35 \cos(2 (35) \pi x) + 1 / 37 \cos(2 (37) \pi x) - 1 / 39 \cos(2 (39) \pi x) + 1 / 41 \cos(2 (41) \pi x) - 1 / 43 \cos(2 (43) \pi x) + 1 / 45 \cos(2 (45) \pi x) - 1 / 47 \cos(2 (47) \pi x) + 1 / 49 \cos(2 (49) \pi x) - 1 / 51 \cos(2 (51) \pi x) + 1 / 53 \cos(2 (53) \pi x) - 1 / 55 \cos(2 (55) \pi x) + 1 / 57 \cos(2 (57) \pi x) - 1 / 59 \cos(2 (59) \pi x) + 1 / 61 \cos(2 (61) \pi x) - 1 / 63 \cos(2 (63) \pi x) + 1 / 65 \cos(2 (65) \pi x) - 1 / 67 \cos(2 (67) \pi x) + 1 / 69 \cos(2 (69) \pi x) - 1 / 71 \cos(2 (71) \pi x) + 1 / 73 \cos(2 (73) \pi x) - 1 / 75 \cos(2 (75) \pi x) + 1 / 77 \cos(2 (77) \pi x) - 1 / 79 \cos(2 (79) \pi x) + 1 / 81 \cos(2 (81) \pi x) - 1 / 83 \cos(2 (83) \pi x) + 1 / 85 \cos(2 (85) \pi x) - 1 / 87 \cos(2 (87) \pi x) + 1 / 89 \cos(2 (89) \pi x) - 1 / 91 \cos(2 (91) \pi x) + 1 / 93 \cos(2 (93) \pi x) - 1 / 95 \cos(2 (95) \pi x) + 1 / 97 \cos(2 (97) \pi x) - 1 / 99 \cos(2 (99) \pi x))$

QuadraPariM(x) = $1 / 2 (1 + \mathrm{QuadraPari}(x))$

Tenda(x) = $0.25 + 4 / \pi^2 (\cos(2\pi x) + 2 / 2^2 \cos(2 (2) \pi x) + 1 / 3^2 \cos(2 (3) \pi x) + 0\cos(2 (4) \pi x) + 1 / 5^2 \cos(2 (5) \pi x) + 2 / 6^2 \cos(2 (6) \pi x) + 1 / 7^2 \cos(2 (7) \pi x) + 0\cos(2 (8) \pi x) + 1 / 9^2 \cos(2 (9) \pi x) + 2 / 10^2 \cos(2 (10) \pi x) + 1 / 11^2 \cos(2 (11) \pi x) + 0\cos(2 (12) \pi x) + 1 / 13^2 \cos(2 (13) \pi x) + 2 / 14^2 \cos(2 (14) \pi x) + 1 / 15^2 \cos(2 (15) \pi x) + 0\cos(2 (16) \pi x) + 1 / 17^2 \cos(2 (17) \pi x) + 2 / 18^2 \cos(2 (18) \pi x) + 1 / 19^2 \cos(2 (19) \pi x) + 0\cos(2 (20) \pi x) + 1 / 21^2 \cos(2 (21) \pi x) + 2 / 22^2 \cos(2 (22) \pi x) + 1 / 23^2 \cos(2 (23) \pi x) + 0\cos(2 (24) \pi x) + 1 / 25^2 \cos(2 (25) \pi x) + 2 / 26^2 \cos(2 (26) \pi x) + 1 / 27^2 \cos(2 (27) \pi x) + 0\cos(2 (28) \pi x) + 1 / 29^2 \cos(2 (29) \pi x) + 2 / 30^2 \cos(2 (30) \pi x) + 1 / 31^2 \cos(2 (31) \pi x) + 0\cos(2 (32) \pi x) + 1 / 33^2 \cos(2 (33) \pi x) + 2 / 34^2 \cos(2 (34) \pi x) + 1 / 35^2 \cos(2 (35) \pi x) + 0\cos(2 (36) \pi x) + 1 / 37^2 \cos(2 (37) \pi x) + 2 / 38^2 \cos(2 (38) \pi x) + 1 / 39^2 \cos(2 (39) \pi x) + 0\cos(2 (40) \pi x) + 1 / 41^2 \cos(2 (41) \pi x) + 2 / 42^2 \cos(2 (42) \pi x) + 1 / 43^2 \cos(2 (43) \pi x) + 0\cos(2 (44) \pi x) + 1 / 45^2 \cos(2 (45) \pi x) + 2 / 46^2 \cos(2 (46) \pi x) + 1 / 47^2 \cos(2 (47) \pi x) + 0\cos(2 (48) \pi x) + 1 / 49^2 \cos(2 (49) \pi x) + 2 / 50^2 \cos(2 (50) \pi x) + 1 / 51^2 \cos(2 (51) \pi x) + 0\cos(2 (52) \pi x))$

Casetta(x) = QuadraPariM(x) + Tenda(x)